ARISTOTLE AND MATHEMATICS

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JOHN J. CLEARY

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APORETIC METHOD IN
COSMOLOGY AND METAPHYSICS

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BY

JOHN J. CLEARY

E.J. BRILL
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For Hans-Georg Gadamer

ἀλλ' ἐκ πολλῆς συνουσίας γιγνομένης περὶ τὸ πρᾶγμα αὐτὸ καὶ τοῦ συζῆν ἐξαίφνης...

Plato, *Epistle* VII, 341C-D

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von

Hans-Georg Gadamer

In diesem Buch legt John Cleary die Frucht langjähriger Studien vor. Ein scharfsinniger, im »sed contra« geübter Kopf, gewinnt einem wichtigen Thema neue Erkenntnisse ab. Sie vermehren nicht nur unser historisches Wissen um die Klassiker der griechischen Philosophie. Vielmehr werfen sie auch auf die spannungsvolle Wirkungseinheit von Plato und Aristoteles und damit auf die geheimnisvolle Seinsweise der Zahlen und aller anderen mathematischen Gebilde neues Licht.

Es geht dabei nicht darum, die Fülle doxographischer Zeugnisse zu vermehren, die im Werk des Aristoteles die Mathematik betreffen. Das ist von vielen Seiten schon bearbeitet worden, insbesondere durch die Sammlung von Heath. Es gilt vielmehr das genaue Gegenteil aller doxographischen Forschung zu leisten, nämlich die ontologischen Sachprobleme, wie sie sich damals stellten und wie sie sich in der heutigen Mathematik stellen, auf der soliden Textbasis der platonischen und der aristotelischen Überlieferung zu klären. Gewiß war der Vater solcher Doxographie Aristoteles selber, aber für ihn war sie nur der vorbereitende Zugang zu den Sachfragen. Diese Sachfragen aber betreffen Grundprobleme der Metaphysik, an denen kein modernes oder postmodernes Denken vorbeikommt. Ist Metaphysik wirklich eine philosophische Hinterfragung der Physik oder ist sie in Wahrheit eher Meta-Mathematik? Was für einer Physik, was für einer Mathematik?

Der Verfasser nimmt damit eine Fragestellung neu auf, die am Ende des Altertums Proklos entwickelt hat. Das liegt in der Tat an der Wirkungsgeschichte des neuplatonischen Denkens, die durch die Jahrhunderte gepflegt worden ist und die insbesondere über Scotus Eriugena in der irischen Heimat des Verfassers eine stille und wirksame Präsenz besitzt.

Natürlich ist es weder die Mathematik unseres Jahrhunderts und auch nicht die moderne Wissenschaftstheorie, um die es dabei geht. Aber es ist, als ob an dieser Grundfrage der Philosophie die Zeitalter und die Kulturkreise ihre Autonomie einbüßten und alle zugleich

zur Prüfung stünden. Am Ende ist es eine unantastbare Wahrheit, daß die Mathematik die erste aller Wissenschaften ist und damit zum Geburtshelfer Europas geworden ist. Sie hat gleichsam eine Vaterstelle für alles, was seither Philosophie heißt – auch noch und gerade angesichts des planetarischen Emanzipationsdranges, der die Welt erschüttert. Ob es sich dabei um Emanzipation von der Kirche, von der Schule, von den Institutionen, von den Sitten oder von dem schicksalsvollen Lauf der Weltgeschichte handeln mag: Woher weiß man, daß zwei mal zwei vier ist? Oder dass es überhaupt keine größte Zahl gibt? Oder, daß im Vergehen der Zeit kein Stillstand ist? Also doch Wiederkehr des Gleichen?

Aristoteles sah sich zu scharfsinnigen Prüfungen genötigt, weil die werdende Wissenschaft der Mathematik auch die ersten Schritte der Philosophie mehr und mehr bestimmte. Die Mathematik hat im Zeitalter Platos gewaltige Schritte getan. Man denke an die Proportionenlehre des Eudoxus oder an die Begründung der Stereometrie des Theätet, die wir alle aus Euklid kennen, und an den Hintergrund der pythagoreischen Astrologie und Musiktheorie, die dem Geheimnis der Zahlen das ganze Universum geöffnet hat. So befand sich Aristoteles als Schüler der platonischen Akademie an dem Eingang, an dem der berühmte Spruch prangte: "hier darf niemand herein, der nicht die Geometrie beherrscht".

Nun wissen wir freilich nichts aus unmittelbarer Überlieferung, außerdem soweit sich die Anfänge der Mathematik in Platos Dialogen und in den Schriften des Aristoteles und den Kommentaren des Simplicius spiegeln. Gewiß sind das keine Zerrspiegel, aber ihren Widerschein richtig zu erkennen, ist eine mühsame Aufgabe, die an die Geduld des Forschers und des Lesers hohe Ansprüche stellt. Im Unterschied zu allen Kombinationen, die die berühmten Mathematik-Historiker und die klassischen Philologen aufgrund sorgfältiger Sammlung und Interpretation angestellt haben, ist die unmittelbare Teilhabe an der Bewegung des Denkens, wie sie nur von vollständig erhaltenen Texten ausgeht, ein unmittelbarer und kostbarer Zugang zu den Sachen selbst. Der platonische Timaios ist überdies nicht nur ein kunstvoll kombinierter Text, der einen einfach informieren soll, sondern vielmehr ein souveränes Geflecht von Einsichten, Einfällen, Spielen und Scherzen, so daß man nicht ohne Verblüffung feststellt, daß sich gleichwohl die Folgezeit, sogar für die geometrische Elemententheorie auf den Timaios beruft, die in Wahrheit dort recht deutlich als ein spielerischer Einfall markiert ist.

Nun, wir kennen die seltsame Unempfindlichkeit, mit der sich selbst Aristoteles an platonischen Texten mit logischem Scharfsinn festbeißt. Gewiss hat er da meistens Recht, und in unserem Zusammenhang zum Beispiel, wenn er gegen Plato darauf besteht, daß selbst eine Häufung ganzer Pakete von Dreiecken niemals ein Körpergewicht ergeben können. Sollte das Plato wirklich nicht gewußt haben?

In ähnlichem Stile argumentiert Aristoteles auch die Denkversuche vieler anderer unter seinen Vorgängern mit logischen Waffen in Grund und Boden. Gewiß ist dabei auch ein wenig Spielfreude des überlegenen Logikers am Werk. Aber am Ende wird doch in all diesen logischen Spielen und im Durchgang durch alle Aporien ein Weg gewiesen, wie man die scheinbar unlösbaren Widersprüche lösen kann. Das wird im Besonderen in dem 4. und 5 Kapitel des Buches von Cleary vorgeführt, wenn dort eine genaue Analyse des 13. Buches der *Metaphysik* mit größter Präsision durchinterpretiert wird. Daß Zahlen nicht sind wie sichtbare Dinge, hat Plato seinerzeit zu der Auszeichnung des Seins von Ideen inspiriert. Wie sich die ganze große Dimension sprachlicher Sinnbildungen in der gleichen Dimension der Ideen und doch zugleich als die vollste Wirklichkeit des Wirklichen erweist, das gibt auch den mathematischen Gegenständen Sinn und Wirklichkeit.

Heidelberg, August 1994

PREFACE

Hans-Georg Gadamer

In this book John Cleary presents the fruit of long years of research. Here an acute mind, skilled in 'sed contra' argumentation, wrests fresh knowledge from an important theme. This enlarges not only our historical understanding of the classical authors of Greek philosophy, but it also throws new light on the tension-filled 'unity of effects' of Plato and Aristotle, and thereby also on the mysterious mode of being of numbers and of all other mathematical objects.

However, this work is not intended to enlarge the stock of doxographical evidence which touches on mathematics in the work of Aristotle, since that has already been dealt with from many sides, especially in the collection by Heath. Instead, it manages to accomplish the exact opposite of all doxographical research; namely, from a solid textual basis in the platonic and aristotelian tradition to clarify the real ontological problems, as they then arose and as they continue to arise in modern mathematics. Certainly, Aristotle himself was the father of such doxography, but for him it was only the preparatory approach to the substantive questions. However, these real questions deal with fundamental problems of metaphysics, which no modern or postmodern thinking can escape. Does metaphysics actually consist of philosophical questions about the foundations of physics or is it really more like meta-mathematics? If so what sort of physics, and what sort of mathematics?

For this purpose the author resumes the problematic which Proclus had developed at the end of the ancient era. In fact, this is due to the 'history of effects' of neoplatonic thinking that has been cultivated throughout the ages and which, especially through Scotus Eriugena, still has a quiet but active presence in Ireland, the native land of the author.

Of course, this problematic has nothing to do with the mathematics of our century nor with modern theory of knowledge. But still it is as if, faced with this fundamental question, every age and civilization should lose its autonomy and be subjected simultaneously to scrutiny. Ultimately, it is an unassailable truth that mathematics is

the first of all the sciences and, thereby, became the midwife to the birth of Europe. It played the role of father, as it were, to all that has since been called philosophy—also and especially in view of the planetary drive for emancipation that is convulsing the world. Whether one concerns oneself with emancipation from the church, from the schools, from institutions, from custom, or from the fateful course of world history: How does one know that two times two is four? Or that there is absolutely no greatest number? Or that there is no standstill in the passage of time? Is there then an eternal return?

Aristotle felt himself compelled to conduct a searching examination of mathematics, since this burgeoning science increasingly dictated the first steps in philosophy. In the lifetime of Plato, mathematics made enormous progress; for instance, with the Eudoxean theory of proportion, and with the founding of stereometry by Theaetetus, which we all know from Euclid. The general background was Pythagorean astrology and harmonics, which opened up the whole universe to the mystery of numbers. In such circumstances Aristotle found himself as a student in Plato's Academy over whose entrance stood the famous dictum: 'Let no one ignorant of geometry enter here'.

Now, of course, we know nothing from the direct tradition, except in so far as the beginnings of mathematics are reflected in Plato's dialogues, in the works of Aristotle, and in the commentaries of Simplicius. Certainly that is not a distorting mirror, yet to recognize the reflection correctly is a laborious task, which makes great demands on the patience of the researcher and the reader. As distinct from all the combinations which the famous historians of mathematics and the classical philologists have employed by means of careful collection and interpretation, the direct participation in the process of thinking, which can proceed only by means of completely preserved texts, provides an immediate and precious access to the things themselves. Moreover, Plato's *Timaeus* is not just an artfully constructed text, which is intended simply to give one information, but rather a masterful network of insights, inspirations, playacting, and jokes. Hence, one notes with astonishment that the subsequent ages appeal to the *Timaeus* for the geometrical theory of elements, even though what is there is in truth quite clearly no more than a game.

Now we are familiar with the odd insensitivity which Aristotle himself displays in seizing on the platonic texts with such logical precision. Certainly, he is often correct, for instance, in the present

context when he insists against Plato that even an accumulation of whole packages of triangles can never yield a heavy body. But could Plato really not already have known that?

In a similar manner, with his logical weapons Aristotle radically criticises the ideas of many others among his predecessors. Here, of course, one finds at work a little of the playfulness of the superior logician. Ultimately, however, in all this logical play and in the passage through all aporiae, a way is indicated in which one can resolve the seemingly insoluble contradictions. This is demonstrated in Cleary's book, especially in chapters 4 and 5, where an exact analysis of Book XIII of the *Metaphysics* is made with the utmost precision. The realization that numbers are not like visible things inspired Plato to characterize the being of the Ideas. Just as the entire large dimension of verbal symbolism proves to be identical to that of the Ideas and yet is the fullest reality of the real, it also in a way provides meaning and reality for the mathematical objects.

Heidelberg, August 1994

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This book is the late-ripening fruit of many years of labor in the stony fields of classical scholarship, and in the course of my research I have incurred a plethora of intellectual and other human debts, which in many cases I must belatedly try to repay with thanks. My odyssey began at Boston University with a dissertation, written under the abstract gaze of the late John Findlay who bemusedly tolerated my obsession with the written works of Aristotle, despite the attractions of Plato's unwritten doctrine. In any case, I am most grateful for his patience, and I wish him eternal happiness in contemplating his beloved One. I also want to thank Alasdair MacIntyre, my academic mentor at Boston University, for all his help and guidance, but especially for a memorable guided reading of Aristotle's *Metaphysics* XIII–XIV. In this connection, I should also thank Kenneth Quandt for initially stimulating my interest in Aristotle's method of inquiry.

However, the most decisive influence of my student years was exercised by Hans-Georg Gadamer, whose regular visits to Boston College enabled us to begin a philosophical conversation that has continued to the present day. It is entirely appropriate, therefore, that this book should be dedicated especially to him, and I am deeply honored that he agreed to write the Vorwort in German, which I have also translated and used as my Preface. Despite some appearances to the contrary, the guiding philosophical spirit of my work derives from Gadamer's hermeneutics rather than from analytic approaches to Aristotle. In addition, I wish to acknowledge the general philosophical influence of the late Karl Popper who was always very generous with advice and encouragement.

Among the established scholars in the field of ancient philosophy who helped me in my work, I want to mention especially the late Gregory Vlastos who read my first chapter and made some incisive comments from which I benefited greatly. I also wish to thank David Furley and Tom Robinson for reading some of the book in rough drafts and for making some helpful remarks and suggestions. But I am particularly grateful to Julia Annas for taking the time and the trouble to read some of my work and to make extensive comments,

some of which I have incorporated as footnotes. Finally, I want to thank the late Hippocrates Apostle for his encouragement and for his generosity in allowing me to quote extensively from his fine translations of Aristotle's works.

Among the colleagues on both sides of the Atlantic from whom I have learned most through informal conversations about ancient philosophy, I should mention Arthur Madigan S.J. and Patrick Byrne at Boston College, Wolfgang Wieland at Heidelberg University, Gerard Watson at Maynooth College, Markus Woerner at University College Galway, and last but not least, John Dillon at Trinity College, Dublin.

Naturally, I wish to thank my wife, Breda, for her patient endurance during the long gestation and painful birth of my philosophical child.

Some of the research for this book was done during periods of sabbatical leave from Boston College and Maynooth College, and the final revisions were completed during my stay at Heidelberg University as a research fellow supported by the Von Humboldt Foundation.

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of Aristotle, ed. J. Barnes. Princeton: Princeton University Press, 1984.

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Aristotle's Select Fragments, translated by W.D. Ross. Oxford: Clarendon Press, 1952.

Aristotle's De Anima II–III, translated by D.W. Hamlyn. Oxford: Clarendon Press, 1968.

ABBREVIATIONS

Adv. Math.	Sextus Empiricus, Adversus Mathematicos
APr.	Aristotle's Prior Analytics
APst.	Aristotle's Posterior Analytics
Cael.	Aristotle's De Caelo
Cat.	Aristotle's Categories
Crit.	Plato's Critias
DA	Aristotle's De Anima
DK	H. Diels-W. Kranz, Die Fragmente der Vorsokratiker (Berlin 1951–2)
EE	Aristotle's Eudemian Ethics
EN	Aristotle's Nicomachean Ethics
Enn.	Plotinus' Enneads
GA	Aristotle's De Generatione Animalium
GC	Aristotle's De Generatione et Corruptione
in Ar. Gr.	Commentaria in Aristotelem Graeca
in APst.	in Aristotelis Analytica Posteriora Commentaria
in Cael.	in Aristotelis De Caelo Commentaria
in Eucl.	Proclus, in Euclidem
in Metaph.	in Metaphysica Commentaria
in Phys.	in Aristotelis Physicam Commentaria
Int.	Aristotle's De Interpretatione
KR	G.S. Kirk, J.E. Raven & M. Schofield, The Presocratic Philosophers (Cambridge 1983)
LSJ	H.G. Liddell, R. Scott & H.S. Jones, A Greek-English Lexicon (Oxford 1940)
Lach.	Plato's Laches
MA	Aristotle's De Motu Animalium
Mem.	Aristotle's De Memoria et Reminiscia
Met.	Aristotle's Metaphysics
Meteor.	Aristotle's Meteorology
PA	Aristotle's De Partibus Animalium
Parm.	Plato's Parmenides
Phd.	Plato's Phaedo
Phil.	Plato's Philebus
Phys.	Aristotle's Physics

Pol.	Aristotle's Politics
Rep.	Plato's Republic
Respir.	Aristotle's De Respiratione
Rhet.	Aristotle's Rhetoric
Sens.	Aristotle's De Sensu
Soph. El.	Aristotle's De Sophisticis Elenchis
Theaet.	Plato's Theaetetus
Tim.	Plato's Timaeus
Top.	Aristotle's Topics

INTRODUCTION

This book tries to situate Aristotle's thinking about mathematics within the broader context of his metaphysical and cosmological views as they developed in response to the mathematical cosmology of Plato's Academy. Apart from Socrates, the Pythagoreans were most influential for Plato's turn away from the empirical approach of the *physiologoi.* In studying Aristotle's views on the theoretical sciences, therefore, we are interested in his disagreements with Plato when he distinguishes physics from mathematics and first philosophy. Since he rejects Platonic Forms and Intermediates, Aristotle must provide an alternative account of the foundations for the paradigmatic Greek sciences of mathematics. This task calls for the development of a non-Platonic ontology that is grounded in Aristotle's own distinctive cosmology. Hence one of the major theses of my book is that this cosmological debate is essential for understanding Aristotle's philosophy of mathematics.

Another major thesis is my methodological claim that it is virtually impossible to understand Aristotle's views without systematically examining how they emerge from a dialectical testing of the opinions of predecessors and contemporaries. Since such an aporetic process constitutes an important part of his own philosophical method, I hold that it is necessary to follow him faithfully upon this way of inquiry if we are to see how he solves the puzzles set up by conflicting opinions. Aristotle's usual procedure is to begin an inquiry in philosophy by collecting the common opinions about his chosen topic, usually in such a way that they conflict in the manner of logical opposites. The result is a typically Socratic aporia, although Aristotle's ambition is in every case to break the impasse with some principle or reputable opinion that withstands dialectical testing. Thus, in the middle of many treatises, we often find him positing a new principle that dissolves the difficulties arising out of the conflict between opinions. Frequently, when he talks of 'saving the phenomena,' he means that an adequate solution will preserve the grain of truth in the common opinions, while conforming to the relevant facts. For this reason, therefore, I accept the necessity of 'going through the difficulties' (διαπορεῖν) with Aristotle before one can fully understand how

his solution is adequate to the problem. Of course, it is an entirely different question whether Aristotle's views on the foundations of mathematics are satisfactory from the point of view of modern philosophy of mathematics.

Within contemporary discussions about the foundations of mathematics, we can distinguish at least three general tendencies; namely logicism, intuitionism and formalism. While this is not a precise classification that covers all the leading thinkers, it will serve for the purposes of my brief remarks.[1] For instance, the leading exponents of logicism were Frege and Russell who each proposed to reduce all of mathematics to logic, though neither of them ever got much beyond arithmetic. Hilbert's project of axiomatization shows similar tendencies, but it is usually classified as formalism because he did not propose to reduce mathematics to logic. What logicism and formalism have in common is the emphasis on internal coherence as the primary characteristic of a mathematical system. This leaves open the possibility of there being many different mathematical systems, which are internally coherent but perhaps mutually inconsistent.

On the other hand, the intuitionist approach to mathematics is based on the Kantian insight that arithmetical and geometrical concepts are structured by the forms of perception of a transcendental subject. Therefore there is only one consistent mathematical system, according to intuitionists like Brouwer and Heyting, but it is not true by virtue of corresponding to some abstract set of objects. For instance, the integers are dependent on human thought and so their existence is guaranteed only by determinate thinking activity. This means that for the intuitionists there is no actual infinity of numbers but only a potential infinity that is generated by a finite series of additions. The modern platonist, by contrast, is at least theoretically committed to the existence of an actually infinite set of integers, since he holds mathematical objects to exist independently of the human mind. This sounds plausible as long as one does not ask how one can know or prove any of the propositions about actually infinite sets. In order for a mathematical system to qualify as valid and true, according to the platonist criterion, it must be both internally and externally coherent; thereby bridging the gap between human intuition and the world. For an intuitionist, on the other hand, there is no world in itself independent of the constructions provided through human intuition.

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[1] See Benacerraf's "Introduction" in P. Benacerraf & H. Putnam eds. (1983).

It seems clear that the Kantian presuppositions which underpin the intuitionist view render it anachronistic for the purposes of discussing Greek philosophy of mathematics. Although it is not obvious that the logicist view is similarly anachronistic, it may be argued that neither Plato nor Aristotle ever tried to reduce mathematics to logic. That seems to leave platonism as the only viable modern parallel to ancient views on the foundations of mathematics. However, the term 'platonism' should not lull us into a false sense of security about the dangers of anachronism, as the terms of the modern debate are likely to be shaped by modern philosophical presuppositions. For instance, all parties to that debate share some Cartesian presuppositions about the mind and its relationship to reality, which are quite different from those governing the ancient debate. In contrast to Descartes, both Plato and Aristotle give primacy to the cognitive object over the faculty of cognition. Thus, despite their many disputes over ontology and epistemology, they are robust realists in a way that is no longer possible for post-Cartesian philosophers. So I will not frame my discussion of Plato and Aristotle in terms of the modern debate in philosophy of mathematics, but instead I will draw on Proclus to recover the cosmological context for problems about the foundations of mathematics.

With reference to the distinction in ancient geometry between theorems and problems, for instance, Proclus[2] tells us that the followers of Speusippus called all propositions 'theorems' because their objects are eternal things. Since there is no generation among such things, the constructive language associated with problems was explained away as being only for the sake of understanding eternal objects *as if* they were in a process of generation. By contrast, the school of Menaechmus held that all mathematical inquiry consists in problems that have a twofold purpose: (1) to provide (through construction) something that is being sought; (2) to see (hence 'theorem') what some determinate object is in itself or what relation it has to others. Adopting his typical syncretic approach, Proclus insists that both schools are right in different ways and thus he tries to reconcile these conflicting traditions. But his syncretism does not distort the report of an ancient debate that originated in Plato's Academy.

Along with the methodological debate about theorems and problems, Proclus reports some epistemological and ontological debates about the foundations of mathematics within the Platonic tradition.

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[2] Proclus, *in Eucl.* 77.15–78.13 Friedlein.

For instance, there is the question about which faculty makes a judgement (κριτήριον) with respect to mathematical objects. While this might be a neoplatonic way of formulating the question, Proclus can appeal to the authority of Plato in proposing understanding (διάνοια) as the appropriate faculty. In *Republic* VI, for instance, understanding is associated with the mathematical method of using ungrounded hypotheses together with sensible diagrams so as to draw conclusions. By contrast, the dialectical mode of thinking called intellection (νόησις) ascends to unhypothetical first principles and makes no use of images or diagrams. Thus Proclus is drawing upon the Platonic tradition when he distinguishes the intellectual insight proper to dialectic from the more discursive understanding associated with the mathematical sciences.

Given the typical Greek assumption about the correspondence of knowledge with its object, it is quite natural for Proclus to raise a subsequent question about the mode of being of mathematical genera and species. Drawing on different strands in the tradition, he entertains the following possibilities: (1) that they are derived from sense objects either (a) by abstraction or (b) by collection; (2) that they exist prior to sense objects, as Plato and the neoplatonists held. As we might expect, Proclus rejects the first set of possibilities and accepts the second, while proposing his own theory of how mathematical objects are 'projections' of forms previously existing in the soul. From my point of view, of course, the contribution of Proclus to the debate is of less interest than the fact that he gives us a point of entry into the ancient debate by supplying the cosmological and metaphysical framework within which the debate took place. For instance, his commentary on Euclid's *Elements* is guided by the assumption that it is a cosmological treatise in the Platonic tradition, since it concludes with the construction of the five perfect solids. While this interpretation may be too neoplatonic in emphasis, it has some basis in the cosmological function of mathematics in Plato's *Timaeus*. Thus I will approach the views of Aristotle on mathematics from the perspective of cosmological and metaphysical speculation within the early Academy.

Using the same framework, I will also try to make some sense of Aristotle's reports on Plato's unwritten doctrines, which have been a source of great controversy among modern scholars. I will not attempt to give another reconstruction of this unwritten tradition, since that has already been done by the school of Tübingen and since my

book is not primarily about Plato. In any case, I am convinced that this whole tradition can be better understood from the perspective of Aristotle's criticism of Plato's mathematical cosmology. Such a cosmology itself represented an attempt to reconcile the older infinitism of the *physiologoi* with the finitism of mathematics, which are reflected in the Indefinite Dyad and the One, respectively. In Pythagorean terms, the One is a Limit which is given primacy over the Unlimited in a synthesis that Plato expresses cosmologically in the *Timaeus* and ontologically in the *Philebus*. But all of this has been worked out in elaborate detail by Gaiser (1962) and Krämer (1959 & 1964), who use later neoplatonic reports to develop what are merely 'hints' in the dialogues. Although they remark on how quickly the Platonic project of mathematizing reality is abandoned in the Academy, they tend to overlook Aristotle's radical criticism as a possible cause of this apostasy. Significantly, they concentrate on *Metaphysics* I and tend to ignore Book III as a source for objections to this project. I will adopt the opposite approach of concentrating on Aristotle's criticism of mathematical cosmology as an essential preliminary to elucidating his own views on mathematics.

In contrast to my approach, most scholars begin to reconstruct Aristotle's philosophy of mathematics from the use that he makes of mathematics to illustrate the notion of a demonstrative science in his *Posterior Analytics*. As a result, they feel hampered by the absence from the extant Aristotelian corpus of any work that deals exclusively with mathematics, though the ancient lists seem to include some works on mathematics, including one on the Pythagoreans. Thus, in view of this lacuna, it seems to them that the best plan is to select from each of the extant works passages that are of mathematical interest. This appears to be the approach of Thomas Heath,[3] who treats Aristotelian texts primarily as sources of information about contemporary Greek mathematics. His attitude seems to be that, although Aristotle himself did not make any original contributions to Greek mathematics, he might give us some insight into the great discoveries made by his contemporaries.

While it is true that he was not a creative mathematician, neither

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[3] In his posthumously published work, *Mathematics in Aristotle*, Heath (1949) writes a brief introduction on mathematics as a theoretical science, and then gives a series of passages of mathematical interest drawn from the works of Aristotle and grouped according to these works rather than by theme. Perhaps the most revealing statement in his introduction is the following: "The importance of a proper

was Plato yet he is still treated with reverence, presumably because 'platonism' is a respectable view in modern philosophy of mathematics. By contrast, 'aristotelianism' is not a position that philosophers of mathematics tend to espouse, though some theorists have invoked Aristotle's name in support of their own views on the foundations of mathematics.[4] Even when careful scholars like Julia Annas[5] take Aristotle seriously as a philosopher of mathematics, they usually find him hopelessly confused on issues that are thought to have been settled definitively by theorists like Frege and Russell.

Such is the influence of modern positivism that it even seduces partisans like Hippocrates Apostle[6] into extracting mathematical passages from their proper contexts in Aristotle, so as to reconstruct a comprehensive Aristotelian theory of mathematics as 'a science of quantities.' In opposition to all of these influential scholars, I hold that one cannot legitimately construct such a theory for Aristotle without first exploring the cosmological and metaphysical problems that led him to stake out his position in relation to other thinkers. Another way of formulating the difference in my perspective is that I treat Aristotle's philosophy as genuinely dialogical (in the Platonic sense), whereas modern scholars regard it as monological in spirit and in form.[7]

In order to lay the foundations for this alternative approach to Aristotle, my first chapter explores Plato's views on mathematics within the context of his cosmological speculation. I survey some important passages in the *Republic* where mathematics is made the crucial discipline for turning the soul away from the sensible and towards the intelligible realm. This corresponds with passages in the *Timaeus* which appear to give mathematical objects a mediating role between these

understanding of the mathematics in Aristotle lies principally in the fact that most of his illustrations of scientific method are taken from mathematics" (1949) 1.

[4] See Richard Martin's (1985) presidential address to the Metaphysical Society.

[5] Cf. 'Introduction' in Annas (1976).

[6] Cf. Apostle (1952, 1978–9 & 1991).

[7] Regretfully, I must fault the method of Apostle (1952) vii when he reverses Aristotle's usual order of inquiry, which involves first outlining and criticizing the views of others before proceeding to expound his own. By way of justification for his reversal, Apostle claims to be following Aristotle's scientific order of presentation, which proceeds from causes to effects. This means that he begins with the principles of mathematics, as set out in the *Posterior Analytics,* and pursues their application in the *Physics, Metaphysics* and *De Anima.* It is only at the end of his book that Apostle considers the views of Aristotle's predecessors and contemporaries concerning mathematics. As I hope to make clear in my book, this approach is completely at odds with Aristotle's dialectical method.

two realms. Here I focus specifically on the function of mathematics in the 'generation' myth narrated by Timaeus, who proposes it as a likely account of the origin and structure of the sensible cosmos. My hermeneutical strategy is to read the combination of Reason and Necessity in this account as Plato's synthesis of two distinct cosmological traditions; namely, the Pythagoreans who pay exclusive attention to arithmetical form, and the natural philosophers who give more attention to material principles. In addition, I take him to be fulfilling the Socratic demand (as represented in the *Phaedo*) for explanation in terms of the Good, since the ordering cause is given priority over the necessitating conditions.

My second chapter examines Aristotle's criticism of Plato's mathematical cosmology in terms of a sharp distinction between mathematics and physics. Here I focus mainly on Aristotle's *De Caelo* which provides a nice contrast with Plato's *Timaeus,* since it espouses quite a different cosmological viewpoint. For instance, with regard to the Platonic view that the world is of everlasting duration but had a beginning, Aristotle trenchantly refuses to accept the analogy with geometrical construction as a way of defusing the apparent contradiction involved in treating the universe as both generated and indestructible. His own view is that the world as a whole cannot be either generated or destroyed, even though the internal ordering of its parts may be changed in many different ways.

As one might expect from a natural philosopher, however, Aristotle makes the problem of change a crucial test for the cosmological views of his predecessors. The Eleatic denial of change is taken to be absurd, while being diagnosed as a failure to distinguish between sensible and intelligible entities. According to Aristotle's account, Plato did not make the same mistake (as Parmenides) of applying the argument from the sciences to the sensible world, though his mathematical cosmology is open to more serious objections. For instance, the claim that all bodies are generable through the composition and dissolution of planes is taken by Aristotle to undermine the foundations of mathematics because it implies that the basic triangles are indivisible magnitudes. In *De Caelo* I.5 and III.1, he insists on the cosmological importance of the related questions about the continuum (i.e. whether there is a minimum magnitude) and about the infinite (i.e. whether or not there is an infinite body). While these questions are also mathematical, it is obvious that Aristotle is primarily interested in their physical implications for the universe.

Thus his criticism of Plato's mathematical cosmology must be understood with reference to a set of problems connected with motion and change in the sublunary realm. For instance, against Plato's account of the construction of bodies out of mathematical planes, Aristotle objects that there is a difficulty in explaining how such bodies can have weight. Similarly, for any cosmology that relies on mathematical explanations there is a problem in explaining motion and rest; so Plato has to appeal to the World-Soul as a moving cause of the universe.

But, in the *De Caelo* at least, Aristotle eliminates the need for such a cosmic soul by positing a proper and natural motion for each of the bodily elements, including *aither* for the heavenly bodies. From this vantage point he criticizes both the Atomists and Plato for giving priority to random over natural motion, since he argues that even a teleological ordering by the Demiurge cannot be the first and eternal order of the universe. This is connected also with the question of whether the natural elements are generated from more basic entities such as atoms, or from the fundamental triangles posited by Plato. Since Aristotle thinks that either mode of generation would involve a void (or non-being), he claims that each element emerges from some potential state of another element. The analysis of these problems, therefore, shows him developing his own characteristic cosmology through the criticism of predecessors.

My third chapter assumes that a problem about the ontological status of mathematical objects was part of Plato's legacy to Speusippus, Xenocrates, and Aristotle, each of whom addressed it on their own philosophical terms. So, when Aristotle rejects the separate existence of Forms, he is implicitly undermining the Platonic foundations for mathematics. Yet there are passages in Aristotle's *Topics* which indicate that he has not yet worked out these implications, since he seems to opt for a Platonic ontology in respect to mathematics.

Despite showing divergences from Plato, some of Aristotle's early works like *On the Good* and *On Forms* also show the same ambivalence in his attitude towards mathematical Platonism. Furthermore, in his analysis of mathematics as a demonstrative science in the *Posterior Analytics,* he skirts the issue about foundations by saying that first principles are hypothesized or taken for granted. The challenge of Protagoras with regard to the truth of such hypotheses must be faced again because Aristotle has rejected Plato's answer in terms of separate ideal entities, and sense perception does not provide sufficient

warrant for the basic assumptions of mathematics. But yet his discussions of quantity in the *Categories* and *Metaphysics* V seem to treat mathematical entities as if they were independent subjects of attributes. While this conforms with the practice of mathematicians, it will not do for Aristotle because he regards quantity as being a different category from that of substance. As he points out in *Metaphysics* VI, however, this whole question about foundations is not the business of mathematicians but is rather a task for (first) philosophers.

Thus the fourth chapter of my book is pivotal, since it explores Aristotle's dialectical method of inquiry in metaphysical treatises where questions about first principles are being considered. My guiding rule here is that to understand Aristotle's problems we must carefully follow the aporetic procedure which begins with the conflicting opinions of predecessors as raw material for his inquiry. In fact, I argue that the terms for an adequate solution are set by this dialectical procedure because his goal of 'saving the phenomena' requires that the difficulties be resolved, while the grain of truth in previous opinions be preserved. Within Aristotle's own dialectical method, I distinguish between this first aporetic stage of reviewing the common opinions and a second elenctic stage, which involves the refutation of some (or all) of the conflicting views so that the initial impasse is broken. I claim that the first stage *only* is to be found in *Metaphysics* III, and that we must look to specific treatises (for example, XIII.1–3) for the second stage of dialectical inquiry.

With respect to the problem at hand, I concentrate on three major aporiae that have either a direct or indirect bearing on the ontological status of mathematical objects. The first involves the question of whether or not there are supersensible substances like Forms and Mathematical Objects, and this turns out to be the guiding aporia for most of *Metaphysics* XII–XIV. The second is a more specific aporia that guides the self-contained treatise at XIII.1–3 about whether or not mathematical objects are substances. Finally, there is the aporia about whether or not One and Being are the substances of things. Despite its unpromising appearance, the difficulties associated with this aporia show that it has a direct bearing on mathematical ontology, since the primary meaning of 'one' is to be a principle of number. The justification for spending a whole chapter on Aristotle's aporetic treatment of these issues is to be found in his explicit warning that one cannot identify or resolve a problem without becoming wholly familiar with the 'knots' that one must untie.

My fifth chapter is devoted entirely to a detailed analysis of *Metaphysics* XIII.1–3, where Aristotle considers the question about the ontological status of mathematical objects. Such a narrow focus at this stage of the book is justified both on methodological and on philosophical grounds. Methodologically, this little treatise is a perfect example of Aristotle's dialectical treatment of a metaphysical problem; while on the philosophical level it deals with the specific aporia about whether or not mathematical objects are substances. With respect to his method I claim that, as distinct from the even-handed review of opinions such as we find in *Metaphysics* III, we have here a partisan refutation of opponents for the purpose of clearing the way to Aristotle's own solution. I show that this second elenctic stage is to be found in XIII.2, where he systematically refutes both possibilities for mathematical objects as independent substances either *in* sensible things or separate from them.

As a result of this refutation, it is clear that these objects are somehow dependent on sensibles and in XIII.3 Aristotle undertakes the positive task of specifying their precise mode of being, while saving all the relevant phenomena (including the contemporary practice of mathematicians). Even though he concedes that mathematicians treat their objects of study *as if* they were self-subsistent entities, Aristotle insists that this is merely a logical separation made for the sake of scientific understanding and that it does not have the ontological implications which the Platonists drew from it. I show that such a solution implicitly depends on his logical theory of subtraction, which legitimates the consideration of sensible things *qua* numbers for arithmetic or *qua* continuous quantities for geometry. Contrary to traditional interpretations, however, I argue that Aristotle's position does not involve an epistemological theory of abstraction and that mathematical entities are not 'abstract objects' in the modern sense of entities that exist only in the mind.

In an attempt to place these views within a broader cosmological context, my sixth chapter considers the reasons why Aristotle rejects a mathematical ordering of the universe in favor of a hierarchy with the Prime Mover at its apex. This involves returning to the aporia about whether One and Being are the substances of things and examining how this is negatively resolved in *Metaphysics* XIII.6–9 through a dialectical inquiry into the view that Forms are identical with certain non-combinable numbers. This view is systematically refuted when Aristotle shows that it conflicts with our intuitions about numbers

and that it fails to provide any satisfactory account of their causal role in respect to sensible things. In *Metaphysics* X.1–2, he argues that 'one' is convertible with 'being' in that it has just as many meanings, though it has a different focal meaning under the category of quantity as the principle of number.

Thus, he rejects the One Itself along with Being Itself as general ontological principles. Furthermore, the putative failure of the Platonists to establish any causal role for Forms in relation to the sensible world is a decisive factor in Aristotle's rejection of these entities along with mathematical objects as supersensible substances. By contrast, he attributes such a mode of being to the Prime Mover precisely because it functions within his own cosmological system as the final cause of motion in the sensible universe. Therefore, like an army or a household, the Aristotelian cosmos is ordered for the best, but not according to the mathematical models of Plato or Speusippus.

Having established the cosmological and metaphysical frameworks for the problem about mathematical objects, I devote my final chapter to specific issues in Aristotle's so-called 'philosophy of mathematics.' For instance, there is some evidence that he was concerned with the epistemological problem of how the mind grasps mathematical objects and that he proposed a solution consistent with his own metaphysics; i.e. that, as entities dependent on sensible things, they are grasped through induction in much the same way as other perceptible forms. But there is little evidence that Aristotle developed an epistemological theory of abstraction, even though his logical theory of subtraction is crucial for the possibility of science.

This raises the logical problem of specifying the precise relationship of mathematical forms to sensible matter such that these can be separated intellectually by the mathematician, without giving rise to the sort of error that would occur if a physicist did the same for physical forms. The solution to this logical problem is contained in Aristotle's *qua* locution and the related method of subtraction, which is implicit in his distinction between sensible and intelligible matter. As shown in chapter 5, there is a corresponding ontological problem of establishing the exact mode of being for mathematical objects that fits with their logical and epistemological character.

In conclusion, therefore, I hold that Aristotle's views on the foundations of mathematics do not fit under any of the standard modern views; for instance, formalism, logicism, or intuitionism. Since all of these views are influenced by post-Cartesian epistemology, it is not

surprising that his object-oriented theory of knowledge should make a significant difference for the conception of mathematics as a science. What will most offend modern positivists,[8] however, is the impossibility of understanding Aristotle's so-called 'philosophy of mathematics' without reference to his cosmological views. One might have thought that his views would be more amenable to our academic specialization, especially since he is often cited as the originator of this approach to scientific inquiry. But the level of our disappointment gives some indication of the gap between ancient and modern approaches to the question of foundations for the sciences.

[8] Of course, people like Rudolf Carnap (1983b) 241–57 would dismiss as meaningless such metaphysical questions as that about the existence of number.

CHAPTER ONE

THE ACADEMIC BACKGROUND

In order to understand Aristotle's complex views on mathematics, one must take into account the historical and philosophical context within which it became a theoretical science with metaphysical and cosmological significance. The most important historical milieu, of course, was Plato's Academy whose members engaged in dialectical and mathematical research. Since both Theaetetus and Eudoxus were associated with the Academy, one may assume that the mathematical sciences played the leading role suggested by later idealizing reports. Furthermore, it is possible that Plato's oral teaching was just as important as the published dialogues for his influence on Speusippus, Xenocrates, and Aristotle.[1] But good hermeneutical practice requires that one give primacy to Plato's dialogues in clarifying the mathematical basis for his philosophical and cosmological inquiries. So I will examine passages from the *Republic* and *Timaeus*, and from other dialogues that provide insight into the role of mathematics in Plato's cosmology. Where obscurities can be clarified by means of doxography, however, I will refer to his so-called 'unwritten doctrines.'[2]

In connection with this unwritten tradition, the notorious lecture on the Good plays an important though ambiguous role. It seems to have been a special event open to ordinary Athenians, who came expecting to hear something to their benefit. Not surprisingly, they were puzzled to find Plato expounding at length on some technical mathematical matters, which appeared to them to have no relevance for practical affairs. However, tradition has it that some members of the Academy took notes and later published divergent accounts. This suggests that Plato's lecture sparked off an internal debate in the Academy about whether principles like the One and the Indefinite Dyad were suitable as principles of reality. Thus, for instance,

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[1] See Findlay (1974), Gaiser (1962), & Krämer (1959 & 1963).

[2] After making a comprehensive survey of the arguments for an esoteric Plato, Luc Brisson (1993) has recently concluded that there is no good reason to look beyond the dialogues which are sometimes misinterpreted by Aristotle for his own polemical purposes. While this is a sensible approach, I think it goes too far (with Cherniss) towards dismissing any evidence of an unwritten tradition.

Aristotle's lost dialogue *On the Good* seems to have outlined different principles from those proposed by Speusippus and Xenocrates. According to the report of Aristoxenus (*Harm. Elem.* II.30–1 Meibon), Plato talked about number, geometry and astronomy, and ultimately claimed that the Good was unity. In order to obtain some indication as to the content of his lecture, however, we must turn to the central books of the *Republic.*

I. *Mathematics and dialectic*

With reference to the educational curriculum prescribed for the philosopher-rulers in the *Republic,* I will consider two questions: Why is mathematics an ideal preparation for dialectic? And what (if anything) does this tell us about the ontological status of their respective objects? With regard to the first question, Vlastos (1988) has argued that Plato's discovery of the hypothetical method represents a decisive turn away from the negative Socratic elenchus of the early dialogues. This is evident in the *Meno* (86E4–87B2) where the geometrical method of investigating 'from an hypothesis' is used for an inquiry into whether or not virtue can be taught. In connection with the second question, Burnyeat (1987) has pointed out that the ontological status of mathematical objects was not actually clarified by Plato, and that it was his proposal to reform the mathematical sciences through greater axiomatization that prompted the subsequent debate about their foundations.

I.1. *Discovering theoretical objects*

Let us begin at *Republic* VI (508A4–511E) with Plato's famous analogy between the role of the sun in seeing visible things and that of the Good in knowing intelligible things. The complexity of the relationships involved is reflected in the several attempts made to clarify the analogy between the sun and the Form of the Good.[3] For instance, Plato explains that the sun not only makes objects visible but

[3] I think that Gadamer (1986) 89–90 points the right way when he distinguishes between the epistemological function of the Good as an immanent principle of intelligibility and its ontological function as a transcendent cause of Forms in Plato's thought.

it is also responsible for seasonal changes, without itself being changed. Similarly, he says (509B), the Good is not only responsible for objects of knowledge being known but also for their being and essence. He adds that the Good Itself is not being but is rather beyond being, just as the sun transcends the process of sublunary change. From this we may infer that the Good functions both as an immanent and a transcendent principle in relation to other intelligible objects. Drawing on reports of Plato's lecture on the Good, one might interpret it as a principle of unity both for each Form and for the whole system of Forms.

In order to clarify the sun simile, Plato uses the simile of the divided line to capture the distinction between the sensible realm over which the sun 'reigns' and the intelligible realm, governed by the Form of the Good (509D). Such a differentiation of realms is compared to a line that is divided into two unequal parts, each of which is divided again in the same ratio. Thus a comparison in terms of clarity and obscurity between the sensible and the intelligible realms yields a guiding ratio (AC : CB), which is then applied within each realm to yield two distinct proportions (AC : CB :: AD : DC and AC : CB :: CE : EB). In proportion theory this implies that the highest level of the sensible realm is equivalent to the lowest level of the intelligible (e.g. DC = CE). Adam[4] takes this to be 'a slight though unavoidable defect' in the line analogy because these two parts are not equal with respect to clearness. But, since Plato was no mathematical slouch, we should reconsider Adam's verdict in terms of what each section represents.[5]

The lowest section of the visible is held to consist of images, such as shadows and appearances in water and other smooth or bright materials. They are inferior in clarity to the contents of the second section; i.e. animals, plants, and artifacts that serve as originals for the images in the lowest section. Thus the analogical relationship goes as follows (510A): just as the object of opinion is to the object of knowledge with respect to clarity, so the image stands to the original which it imitates. Since the division so far has been made in terms of characteristic objects, one would expect a corresponding

[4] See Adam (1902) ii, 64.

[5] Wieland (1982) 201 explains the equality of the two middle sections in terms of the same field of objects being considered in two different ways; i.e. by *pistis* as originals which are reflected at the lower level of *eikasia*, and by *dianoia* as images of original intelligibles.

division between intelligible objects, but instead Plato makes the division only in terms of different modes of cognition.

In the lower section of the intelligible, the (dianoetic) soul is said to be using as images what were originals in the sensible realm, so that it is forced (ἀναγκάζεται) to inquire from hypotheses, while proceeding 'down' to a conclusion rather than 'up' to a first principle. In the higher section, by contrast, the (noetic) soul is held (510B) to proceed from hypotheses 'up' to an unhypothetical first principle, without using the images of the other section, while moving within the realm of Forms. Thus the division of the intelligible realm is made in terms of two distinct methods of inquiry rather than with reference to their characteristic objects and, as Burnyeat (1987) notes, Plato has not specified whether there are distinct kinds of objects corresponding to different modes of cognition.

The dianoetic activity of the soul is characterized by the use of sensible images and the necessity of moving from hypotheses to conclusions. The second feature is illustrated (510C–D) with reference to mathematicians who begin from hypotheses such as the odd and the even, without giving any account of them, and reach consistent conclusions about their subject of inquiry. In view of Plato's repeated claim that mathematicians are forced to begin from hypotheses, Burnyeat claims that he cannot have intended it as a critical comment on the shortcomings of mathematical method.

While Burnyeat may be right about Plato not rejecting the hypothetical method of mathematicians, he can hardly use Euclid's *Elements* to show that this was the *only* way in which they could have proceeded, since that is to ignore the older tradition of empirical mathematics. So it is with some reservations that I accept Burnyeat's suggestion that the problem being raised by Plato about mathematics concerns its ontology rather than its method. He conjectures that the reason why the divisions within the intelligible realm are delineated in terms of method rather than of subject-matter is that Plato had become aware of an unresolved problem about the ontological status of mathematical objects.

While I am indebted to Burnyeat for these helpful suggestions, I am not convinced that they make complete sense of passages in the *Republic* and elsewhere which discuss the subject-matter and method of mathematics. In the present passage, for instance, why should Plato emphasize that mathematicians are *forced* to begin from hypotheses, if he only means to acknowledge a fact about their actual practice?

Furthermore, what is the connection (if any) between the hypothetical method and the use of images by mathematicians, and especially by geometers? Since these two features of mathematical method are frequently mentioned in tandem, one might look for a closer link between them.[6]

Immediately after describing the method of beginning from hypotheses, Socrates talks (510D) about the way in which geometers use visible forms and give accounts of them, though they are really thinking about the originals which they resemble. He claims that they are giving their accounts 'for the sake of' the square itself and the diameter itself but talk about those figures that they draw. Appealing to a previous analogy, he explains (510E) that such constructed figures correspond to the originals of which the shadows in water are images, though they are now being used as images to inquire into those things that cannot be seen except in thought. Along with throwing light on the equality between two sections of the line, this explanation also suggests a direct connection between the reliance on images and the necessity of using hypotheses to investigate the contents of the intelligible realm.

Such a connection is confirmed by a subsequent (511A) recapitulation dealing with the intelligible genus which the soul studies when it is forced to make use of hypotheses that are not grounded in a first principle. Here the word order suggests that the soul is thus constrained because it uses as images those very things which in the sensible realm are regarded as originals. Once again, the equation of two sections of the divided line implies that the method of hypothesis and the use of images are related procedures, though it is unclear what sense is to be made of this connection. My suggestion is that the reliance of geometers upon sensible diagrams necessitates the use of hypotheses as starting-points. In the sensible world there is nothing which is simply a triangle, for instance, and so this intelligible entity functions as a hypothesis under which the diagram serves as an image of Triangle Itself.

Perhaps one can clarify this point indirectly by means of Plato's description (511B) of the noetic activity of the soul which does not treat hypotheses as starting-points but rather as stepping-stones for

[6] Ian Mueller (1992) has recently suggested that the necessity for geometers to proceed from hypotheses is connected with their use of images to understand intelligible mathematical Forms.

an ascent to the unhypothetical first principle of all being. Indulging in some word-play, Socrates stresses that the postulates of the mathematicians are not ultimate 'beginnings' but literally 'hypotheses,' since they lack the grounding provided by dialectic. It is significant that, when describing such grounding in terms of an unhypothetical principle, he insists that a dialectical account makes no use of anything sensible but rather confines itself to formal implications between Forms themselves. Indirectly, this confirms the connection between the use of sensible images by mathematicians with their method of positing ungrounded hypotheses and of drawing conclusions from them.

Such a link is also suggested by the *Phaedo* (100A) where the method of hypothesis is introduced to guard against possible confusion arising from the fact that sensible things often present contrary appearances. For instance, one may become confused about whether two is generated by addition or division unless one somehow keeps Twoness separate from what participates in it. The suggested way of avoiding such confusion is to set down some hypothesis and to explore its consequences from the point of view of their mutual consistency. If someone challenges the hypothesis itself, however, one can defend it by going up to a more adequate hypothesis. This is similar to Plato's distinction in the *Republic* between descending from hypotheses and ascending to the unhypothetical. While the latter approach belongs to dialectic, the former is typical of mathematics, which makes use of ungrounded hypotheses and relies upon images in the form of diagrams. In the *Phaedo* passage, Plato warns against the confusion that may result from treating mathematical objects in a quasi-physical manner, and this is consistent with his distrust of constructive language and of an excessive reliance on geometrical instruments. The mathematician must avoid the confusion of conflicting predicates by determining one predicate through an hypothesis and by fixing it in a visible diagram.

Burnyeat argues that, since Plato continually emphasizes that mathematicians are forced to adopt their particular procedure, he cannot be criticising them for their method *qua* mathematicians. While this sounds plausible from the point of view of Aristotle's division of the theoretical sciences, there is little evidence that Plato made such a division. So, even if his criticism of mathematical method was based only on its neglect of philosophical groundwork, this would still mean for him that there is something wrong with mathematics as a *theoretical* science. But Plato also seems to accuse the mathematicians of

failing to understand the conceptual foundations of their own disciplines, and therefore of being open to sophistical objections such as those lodged by Protagoras.

My interpretation is based on a different reading of Plato's insistence that mathematicians are forced to employ hypotheses; namely, that their reliance upon sensible images brings with it the threat of confusion, so that they are forced to take refuge in the intelligible realm by means of hypotheses.[7] But practitioners of these sciences do not feel the need for a clarification of foundations, and so they simply posit assumptions as starting-points, while relying more or less on sensible diagrams. Although there is an internal tension between these two basic procedures, yet it is precisely this feature of mathematics that makes it suitable for Plato's purpose of 'converting' the soul to the intelligible realm.

I find support for my interpretation in *Republic* VII, which uses that memorable image of the cave as an allegory for the human condition.[8] While the parallel with the divided line simile is not exact in every detail, Plato obviously intended them to correspond in general, since Socrates explicitly compares (517A–B) the visible realm with the inside of the cave where the fire plays the role of the sun. Given their parallel distinctions between originals and images, it is fairly easy to line up the correspondences between the cave and the sensible section of the divided line, though it is more difficult to do so for the intelligible section and the world beyond the cave.[9] Should we take visible things in that world, illuminated as they are by the light of the sun, to correspond with Forms in the intelligible realm that are similarly lit up by the Form of the Good? Socrates hints at such a correspondence when he says (517B–C) that this Form is the

[7] Mueller (1992) 185 takes Plato to mean that mathematics is an attempt to understand the intelligible world by reasoning *about* sensible things rather than (as modern philosophers of mathematics might say) an attempt to reason about the intelligible world by using sensible things. Such a reading indirectly supports my interpretation.

[8] On the purely literary level, this allegory shows the attraction of images for the human soul (one remembers this part of the *Republic,* while forgetting the arguments), along with the need for hypotheses to focus our attention on the relevant information contained in the image.

[9] At 516A–B there is a vague suggestion of a possible correspondence between the lower section of the intelligible realm and the shadows and reflections in water which the prisoner must first observe on coming into the daylight, before he is able to look at the things themselves. But this seems to be merely a passing remark, since it is not recalled or developed at 517A–B where the correspondences are explicitly discussed. Perhaps Plato's silence here is an indication of an unresolved problem.

last and most difficult to grasp, though it must be acknowledged as the cause of all that is right and beautiful. In the sensible world the Good is said to produce both light and the sun itself, while in the intelligible world it is directly responsible for 'truth and intelligence.' This obscure statement suggests that the Form of the Good has two different though related modes of causality in the sensible and intelligible realms, respectively.

But when one compares the account of the ascent from the cave with the analogy of the divided line, there arises a problem about what kind of objects are studied by the mathematical sciences. One can address this problem within the context of Plato's educational curriculum by asking why he gives (518B) to mathematics the crucial role of turning the soul around (περιάγειν) from the sensible to the intelligible world. The leading question here (521D) is about which of the disciplines has the power to draw the soul from the realm of Becoming into the realm of Being.

After considering the traditional discipline of music, which he had earlier made part of the basic training of the guardians, Socrates rejects it as unsuitable for his present purposes because it does not lead towards knowledge of Being. Instead he proposes to Glaucon that they look for the common thing that is used by all the crafts and sciences, and which everyone must learn at the beginning. This is the deceptively simple ability to distinguish and count things, which is called number and calculation (522C).[10] The military-minded Glaucon seeks a justification of this basic art in terms of its usefulness in war for arraying and counting troops.[11] But this is merely a diversionary skirmish because the real purpose of the discussion is to identify what it is about arithmetic when used correctly that draws the soul towards intelligible reality.

For purposes of clarification, Socrates introduces (523A–B) an important distinction among sense perceptions between those which do not call upon intelligence and those which do. The explicit grounds for this distinction is that the first type of sense impression is adequately judged by perception, whereas the second type yields conflict-

[10] Although there is no systematic distinction between counting and calculating in Plato, the first seems to refer to numbers in themselves, whereas the second considers the relations between numbers.

[11] LSJ lists some older meanings of ἀριθμεῖν as being linked with the counting of soldiers in the battle array. This is the point of the reference (522D) to Palamedes' mockery of Agamemnon as a general who was unable to count.

ing information, so that the soul must call upon intellect to yield a sound judgment. By way of clarification, Socrates refers to the perception of three fingers side by side on a single hand, since this can be used to illustrate both kinds of perception. Taken by itself, the perception of a finger raises no doubts in the soul, whatever its situation or color or size in relation to other fingers. Thus Socrates declares (523D) that the soul of the many[12] is not forced (ἀναγκάζεται) to call upon intelligence to decide what is a finger, because sight never indicates to it that the finger is at the same time its opposite. By contrast, sight does *not* have a sufficiently stable perception of the size of a finger such that it makes no difference whether it is situated in the middle or at the extremities. For instance, when compared with the thumb the index finger appears to be large but, at the same time, it appears to be small compared with the middle finger. Similarly Socrates finds that the same thing often appears to be both hard and soft, and so he concludes that the senses by their very nature must deal with such opposites.

With reference to the ascent of the soul, therefore, it is only such sensory impressions that provoke the soul to reflect on the true nature of the things perceived. In the case of touch, for instance, Socrates says (524A) that the soul must be puzzled (ἀπορεῖν) as to what perception means by hard if the same thing also appears to be soft. It is noteworthy that Plato identifies the apparent relativity of sensory opposites as a stimulus for the soul to engage in theoretical reflection. In fact, his claim is much stronger than this when he repeatedly says that the soul is *forced* to call upon intelligence to clarify what the senses mean by these opposites that they find in the same thing.[13] When taken together with the simile of prisoners in the cave, these statements strongly suggest that for the majority such reflection is an involuntary activity thrust upon them by conflicting sense im-

[12] Adam (1902) ii, 110 notes that this qualification seems to leave open the possibility that some philosophical natures may find intellectual stimulus even in the perception of a finger.

[13] The grammatical construction that is consistently used with ἀναγκάζεται throughout this passage does not support Burnyeat's reading of it as indicating that, since geometers *must* use hypotheses, they are not being criticized for doing so. In fact, the construction emphasizes that the soul, puzzled by contrary perceptions, is *forced* to call upon intelligence and so resorts to hypotheses in order to gain some foothold in the noetic realm. My reading leaves open the possibility that Plato is critical of ordinary mathematicians for failing to give any further account of the intelligible foundations of their disciplines.

pressions. But, more significantly, Socrates says (524B) that the first step towards theoretical reflection is taken when the soul calls upon thinking to determine whether the opposing perceptions are one or two in number.

The older Pythagoreans thought that number is given clearly in perception, as when one sets out rows of figures in different configurations such as the well-known tetractys. By contrast, Plato emphasizes (524C) that the one and the many are commingled in sense perception, just like the large and the small in the perceptible finger. Thus, when the soul responds to puzzling phenomena by distinguishing between largeness and smallness as two distinct things, it has taken its first step into the realm of the intelligible with the aid of calculation.

Socrates explicitly identifies (524C4–5) a state of puzzlement as the original source for the leading question about what is largeness and smallness. He explains that such a question is at the root of the general distinction between the sensible and intelligible realms, and that this is what he was trying to get at through his previous distinction between perceptions that call upon intelligence and those which do not.

But the strategy of his argument becomes clearer when Glaucon is asked to which class the perceptions of number and of unit belong. Everything hinges on the answer to this question because if the unit "in and by itself" is adequately perceived by the senses then it would not serve as a track[14] towards intelligible reality. On the other hand, if the one always appears along with its opposite (i.e. the many) then the soul would be undecided and would be forced (ἀναγκάζοιτο) to wonder (ἀπορεῖν) and inquire (ζητεῖν) by stirring up intelligence to ask what is the unit itself. Since Glaucon agrees (525A–B) that we always see the one together with the indefinitely many, it follows that the study of the one (i.e. calculation or arithmetic) is a discipline that 'converts' the soul to reality.

Having decided that arithmetic belongs among the studies sought for the 'conversion' of the soul, Socrates proposes (525C) that those guardians who are to rule should study arithmetic until they are able to contemplate the nature of numbers in themselves. This proposal

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[14] It is difficult to capture the linguistic suggestion in the Greek that the soul would be drawn towards Being just as a ship is hauled overland by a machine (ὁλκός), since neither are capable of proceeding under their own sails, as it were. This simile reinforces the impression that the soul is under some duress in its ascent to the intelligible realm; cf. also 521D.

draws attention to the tension between theoretical and pragmatic goals that has already been hinted at earlier. Socrates insists here (525D) that the guardians should use arithmetic purely for theoretical purposes since in that mode it actively leads the soul upwards and forces (ἀναγκάζει) it to consider numbers themselves, and not as embodied in visible things. In this connection, Socrates refers to the behavior of contemporary experts in arithmetic who ridicule anyone who thinks that the unit can be divided. Thus, he says (525D–E), if you divide their unit they will multiply it, thereby rejecting any suggestion that it may have parts.[15]

Yet such a defensive strategy still leaves these practitioners open to the following question: What kind of numbers are you talking about, in which each unit is equal to every other without the slightest difference and contains no parts? The point of the question is to get at the hidden assumptions that lie behind the practice of contemporary arithmeticians who are imagined (526A) as answering that these units can be grasped only by thinking and not in any other way.[16] This fictional exchange reinforces the suspicion that these mathematicians are unable to give an adequate ontological account of their subject matter. Thus, *pace* Adam,[17] Plato does not here provide an account of mathematical objects as intermediate entities between Forms and sensible things. While such an account may be reconstructed from Aristotle's reports on the so-called 'unwritten doctrines,' all one finds in the Platonic dialogues are vague hints.[18] But Euclidean geometry does require a plurality of ideal figures which are neither sensible things nor unique Forms.[19] Therefore Burnyeat is right in saying that the precise ontological status of mathematical

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[15] van der Waerden (1954) 49–50 & 115–16 claims that calculation with fractions probably led to the theory of proportions found in Euclid's *Elements* Book VII, but he thinks that these fractions were later eliminated because of the theoretical indivisibility of the unit, as shown by *Republic* 525E. By contrast, both Mueller (1981) 88 & 92–3, and Fowler (1987) 139–40 think that the addition and multiplication of fractions is completely absent from Euclid.

[16] Mueller (1992) takes this passage to show that the arithmetician is really concerned with intelligibles; i.e. with units and numbers that can only be thought and not subjected to any quasi-physical process like division.

[17] Cf. Adam (1902) ii, 115–16.

[18] See Wedberg (1955) for a sensible and balanced attempt at reconstructing the unwritten doctrine from hints scattered throughout the Platonic dialogues. See Aristotle's *Metaphysics* I.6 for his most extensive report on the same doctrine.

[19] This is especially clear from the ἔκθεσις in a theorem when Euclid says something like: 'Let ABC and DEF be two scalene triangles with two sides and the included angle equal.' Obviously, he cannot be talking about the unique Form of

objects emerges as a question for subsequent inquiry rather than as a doctrine in these central books of Plato's *Republic*.

I.2. *Geometry and astronomy as propaideutic*

The issue is clarified further by Socrates' leading question (526E1–2) as to whether geometry makes it easier for the soul to behold the Form of the Good. Since that goal is promoted by whatever compels (ἀναγκάζει) the soul to orient itself towards true Being, he adopts (526E5) this as a criterion for any discipline to qualify as propaideutic. In examining the contemporary practice of geometry, therefore, Socrates finds (527A–B) an internal conflict between its real purpose and the way in which geometricians talk.[20] For instance, they are like practical men when they talk about 'squaring' and 'applying' and 'adding,' as if they were giving their accounts for the sake of action, whereas the whole discipline is pursued for the sake of knowledge. Thus Socrates describes (527A5) such talk as 'quite absurd' and, like the banausic uses of arithmetic, as 'rather vulgar.'[21] Without making any effort either to clarify or resolve this internal conflict, he asks Glaucon to agree that geometry is studied for the sake of knowledge of 'what always is' rather than of what comes to be and perishes (527B).[22] On this basis, Socrates concludes (527B) that it should be the sort of study that draws the soul towards the truth and produces philosophical thought by directing upwards what is presently directed downwards. Here we find an implied criticism of contemporary practitioners of geometry for their failure to recognize

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Triangle, since that is neither scalene nor isosceles nor equilateral. It is a moot question, however, whether he is talking about 'intermediates' or about sensible things considered in a certain way.

[20] Shorey (1935–37) 170 notes that the very etymology of 'geometry' (γεωμετρία) implies an absurd practical conception of the science; cf. *Epinomis* 990C: γελοῖον ὄνομα.

[21] Adam (1902) ii, 118 translates ἀναγκαίως as 'in beggarly fashion' and there are good grounds for this in ordinary Greek and in the particles τε καί, which are certainly not adversative; cf. Denniston (1934). When read in this natural way, the whole clause provides little support for Burnyeat's (1987) claim that Socrates is accepting constructive language as being 'unavoidable,' given the nature of Greek geometry.

[22] While this appeal to two distinct realms is straightforward Platonic doctrine (cf. *Tim.* 27D–28A), the blunt demand for consensus as to which realm geometrical objects belong conceals the possibility of a debate such as arose subsequently in the Academy over whether 'problem' or 'theorem' is the appropriate language for the purposes of geometry; cf. Proclus, *in Eucl.* 77.15 ff.

its real purpose of obtaining knowledge of Being and, hence, its true function as a propaideutic discipline for dialectic.[23]

Historically, it seems that Plato's proposed reform of the mathematical sciences led to their reorganization and axiomatization. Perhaps inspired by the work of Theaetetus, some mathematicians within the Academy compiled *Elements* that were later incorporated into the work of Euclid. Although most pre-Academic geometry has been lost due to this Platonic reform, one may conjecture that it was quite empirical in its procedure and proofs; i.e. that it relied heavily on constructions and on appeals to perception.[24] Similarly in astronomy there must have been considerable reliance upon observation, since the empirical traditions of Babylonia and Egypt were familiar to the Greeks.[25] Just as in the case of the more 'abstract' mathematical disciplines like arithmetic and geometry, Plato proposed to reform this empirical discipline in terms of its theoretical purposes.

He draws attention to the question about the real purpose of astronomy by having Socrates upbraid (527D–E) Glaucon for wishing to make this discipline seem useful in order to satisfy ordinary people who would not accept the most important reason for pursuing astronomy; namely, rekindling an intellectual fire that has been quenched through banausic pursuits. Instead of trying to persuade such people, Socrates advises Glaucon to pursue these studies for the sake of purifying his own soul. So this whole discussion about the theoretical purposes of inquiry leads to the subsequent proposals for the reform of contemporary astronomy.

In the meantime, however, the order of discussion is deliberately interrupted (528A–B) to correct a mistake in the previous order of inquiry. Socrates says that they were wrong to move directly from plane geometry to astronomy, which involves the solid in motion, without considering the solid in itself. Instead they should have studied the 'third increase' after the second; i.e. the cubic increase that

[23] Burnyeat's claim that Plato is *not* being critical of the practice of contemporary Greek mathematics does not square with the tradition that he objected to the use of all mechanical devices in geometry, except for the straightedge and compass; cf. Plutarch, *Quaestiones Convivales* 718e–f. Such an objection fits with his critical remarks here about the absurd constructive language of geometers, given the theoretical goals of the discipline.

[24] This conjecture is supported by Szabo (1978) 189, and von Fritz (1955) 94.

[25] Cf. Neugebauer (1957) chs. 4 & 5, van der Waerden (1961) chs. 1 & 3, & Fowler (1987).

has depth.[26] Glaucon offers the excuse that such studies have not yet been discovered but, in view of the work of Theaetetus on the five perfect solids, this remark may not reflect the actual historical situation. Adam thinks however that the problems of stereometry had not yet been discovered or solved, and he cites the Delian problem as a leading example of such unsolved problems. But the solution to that problem, involving the discovery of two mean proportionals, is presupposed in the *Timaeus* for the continued geometrical proportion between the four elements within the World-Body.[27] So the remark may be seen as a dialogical fiction,[28] since Socrates uses the occasion to outline (528B) two major reasons for the neglect of stereometry; namely, lack of support within the city and lack of guidance for such research.

Therefore, one should interpret this digression on stereometry in the light of Plato's proposal to reform the mathematical sciences from the viewpoint of their theoretical foundations. The lack of support for stereometry among the ordinary citizens is consistent with their low estimation of all theoretical disciplines, but this one is particularly neglected because it does not appear to have any of the practical uses of arithmetic and geometry. The absence of leadership that is cited as a reason for the feeble state of stereometry should not be taken as a historical remark about any lack of outstanding researchers but rather as a comment about the lack of *proper* direction in the field. What is needed is someone with a vision of the true theoretical nature of the discipline who will pose the right questions and reorient the whole field of research.[29]

A similar project is envisaged by Socrates in the discussion of astronomy, which now occupies its proper place as fourth in order within the mathematical sciences (528E). Once again, a rather comic exchange with Glaucon exposes a misunderstanding of the proper

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[26] See *Laws* X (894A) for similar language and the implied 'generation' of solid bodies through the consecutive 'growth' of each of the dimensions into the other.

[27] Lasserre (1964) 114–116 refers to the *Platonicus* dialogue written about 240 B.C. by Eratosthenes, in which it is claimed that the Delian problem had been solved in Plato's Academy. Yet Plato's *Timaeus* (32A–B) is compatible with the historical tradition that the problem had already been formulated and solved by Hippocrates of Chios.

[28] See Shorey (1935–7) 176 note a.

[29] This fits with the reputedly divine origin of the Delian problem, which gave some historical impetus to research in stereometry; cf. Heath (1921) & van der Waerden (1961).

purpose and subject-matter of astronomy. Mindful of Socrates' previous reproaches, Glaucon hastens to assure him that this discipline obviously forces (ἀναγκάζει) the soul to look upwards, by contrast with other disciplines. On the contrary, says Socrates, those who try to lead us up to philosophy make us look downwards on account of the way they practice astronomy. The implied criticism of contemporary astronomers becomes obvious when he compares them to people gaping up at ornaments on a ceiling (529A–B). Just like Glaucon, these practitioners mistakenly think that they can acquire knowledge with their eyes rather than with their minds. There is no knowledge of such (sensible) things and therefore the empirical astronomer is looking downwards rather than upwards (529B–C). What this statement suggests is that Plato himself sees the contemporary practice of astronomy as seriously misdirected when it concentrates on observing the sensible heavens. If this is the case then it has more general implications for his views about the reform of the mathematical sciences and about their appropriate objects.

The proposed reforms of astronomy are introduced (529C) so that the discipline will function better in orienting the soul towards intelligible reality. The first step seems to be the most difficult because it involves persuading practicing astronomers that the real objects of their inquiry are not heavenly bodies but intelligible (i.e. invisible) objects. In this case Socrates does not mention any sensible opposites that would force their souls to seek help from the intellect, as in the case of arithmetic and geometry.[30] Even though he concedes that the heavenly bodies are the most beautiful and most exact of sensible things, he insists (529C–D) that they fall short of the real beings which can be grasped only by reason. Such realities are the motions by which real speed and real slowness express true number and true shapes.[31]

But this proposal for reform seems unlikely to persuade those at

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[30] Perhaps Plato thought that the anomalous motion of the planets is the sort of confusing sensible impression that forces the mind to seek an explanation in the intelligible realm by means of the hypothesis of perfectly uniform circular motions. According to a tradition relayed by Simplicius (*in Cael.* 488.7–24), Plato set this problem of 'saving the phenomena' for contemporary astronomers, and Eudoxus was the first to propose a solution in terms of his theory of homocentric spheres.

[31] Fowler (1987) 121–30 suggests that what lies behind Plato's strange approach to astronomy here is the anthyphairetic understanding of astronomical ratios developed by people like Eudoxus on the basis of cyclical calendars such as the solar and lunar calendars compiled by the Egyptians.

whom it is directed because Socrates is suggesting (529D–E), in effect, that astronomers use the heavenly bodies only as models for the sake of studying other things, just as one uses blueprints to represent the original. The persuasive force of this proposal depends on the parallel with geometry and its use of sensible diagrams. Socrates says (530A) that if an experienced geometer were to see even the most beautifully crafted plans, he would admire them but would think it absurd to seek in them the truth about the equal or the double or any other symmetry. Similarly, when the real astronomer observes the movements of the stars, he will view them as the beautiful workmanship of the Demiurge but he will refuse to accept that the length of days, months, and years never deviates anywhere, since they are material and visible. So he will try to discover the truth by studying astronomy through problems, just as the real geometer does, and 'let things in the sky go' (530C1). As with geometry, the proposed reform of astronomy is designed to make the naturally intelligent part of the soul useful for theoretical activity.

Although this is a very controversial passage, I will not rehearse the many interpretations it has been given.[32] One of the thorniest issues is whether Plato intended to banish observation of the sensible heavens completely from astronomy or whether he meant to propose some less radical reform. The issue has been admirably treated by Vlastos (1980), and I only want to take issue with him on a minor point. Against the stronger reading of ἐάν as 'abandon,' he objects that it would involve the liquidation of astronomy rather than its reform because at least some factual discoveries about the heavenly bodies are presupposed by this science. If this was what Plato really meant, he argues, then it would be unlikely that Eudoxus could have been so influenced by him and still construct a system of homocentric spheres to 'save the phenomena.'

But such an historical argument is rather suspect, given that Eudoxus was an independent thinker who disagreed with Plato on issues like the separation of Forms from sensible things.[33] The fact that Eudoxus

[32] Both Vlastos (1980) and Mourelatos (1980) have summarized the competing interpretations.

[33] See Wright (1973–4) & Mittelstrass (1962) who argue convincingly that Eudoxus was more likely to have influenced Plato with respect to astronomy, rather than the other way round, since he was the practicing expert in the discipline. But yet Plato's proposal for reforming the discipline with respect to its axiomatic foundations may have led Eudoxus to adopt a more theoretical line of inquiry.

was associated with the Academy does not prove that he shared Plato's attitude toward astronomy, though it might suggest that he found the intellectual atmosphere conducive to his research. On this basis Vlastos argues that Eudoxus could not have been an associate if Plato's attitude was so radically anti-empiricist as some commentators suggest.[34]

However, Vlastos does not rest his case on historical conjecture but rather on a careful analysis of the passage and its central analogy between the existing methodology of expert geometers and that proposed for a reformed astronomy, especially with reference to the proper attitude towards sensible diagrams. Even though geometers begin by constructing a visible diagram, they cannot base their proof on observation if they are to move beyond true opinion to knowledge. In order to do this, they must 'put aside' the diagram and find the geometrical principles from which to derive the required conclusion as a deductive consequence. Vlastos submits that the same is true for Plato's proposal that the visible heavens be used as a model for the sake of understanding the real motions of the heavenly bodies. So the accumulated empirical data about the diurnal and annual motions of the sun can serve as a preamble to a genuinely scientific inquiry about its 'true' motions.

But if the methodological parallel with geometry is to be taken seriously, the problem facing a scientific astronomer will be to identify a set of assumptions from which to deduce consequences that will accord with the phenomena. In the *Timaeus,* for instance, the sun's complex motion is analysed into two perfectly circular motions that are combined in a closed spiral and this is consistent with most of the observable phenomena. However, unlike modern scientific conjectures, these hypotheses are not held to be refuted by a direct appeal to incompatible phenomena.[35]

In the case of harmonics, Plato directs a similar criticism at those practitioners who keep their ears fixed upon audible sounds (530D). The likely target is the school of Harmonists, who used the quarter-

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[34] Cf. Mourelatos 1980. Mittelstrass (1962) 122–3 also takes Plato to be separating 'true' astronomy from any dealings with the heavenly phenomena. By contrast, Dicks (1971) 107–8 sees him as making a proposal for the systematization of astronomical knowledge which does not exclude observations, as long as these are integrated into the system.

[35] Giora Hon (1989) has shown that in ancient astronomy the function of observations in relation to theory was not that of testing but rather of corroborating. This means that observations played the secondary role of illustrations, by contrast with their primary role as tests in modern astronomy.

tone as a measure because they regarded it as the smallest interval still perceptible to a trained ear. According to Socrates such people are laboring in vain, just like the empirical astronomers, because they measure heard sounds and harmonies against each other rather than against ideal harmonies. The criticism applies not only to practicing musicians, who torture their instruments and dispute about whether the quarter-tone is the smallest discernible interval, but even to those Harmonists who search for numbers in the consonances that they hear (531B–C). Their mistake is that they do not make their way up to problems, which would involve asking about which numbers are consonant and why they are so.[36] The language here emphasizes that the arithmetical approach through problems is considered to be more explanatory for the discipline of harmonics than the Harmonist method of listening for consonances in the concrete continuum of sound. In general, Plato's argument for a theoretical approach in all these disciplines is that only in this way can we gain access to intelligible entities as the ultimate causes of sensible things.

I.3. *Dialectic as the mistress-science*

While the propaedeutic function of mathematics with respect to dialectic is clear, the same cannot be said for the ontological relationship between their respective objects. One seeks in vain for further clarification of this matter in *Republic* VI–VII, where Socrates talks about the propaideutic function of the mathematical disciplines. At 531D, for instance, he says that if the inquiry about these sciences grasps their intercourse and kinship with one another and if they are considered in so far as they are akin, then their investigation does promote the kind of subject-matter that is sought and the labor will not be in vain. Socrates emphasizes (531D–E) that this mighty labor is a necessary preamble to the science of dialectic.

It seems however that this projected science is differentiated from the mathematical sciences in terms of its methodological approach

[36] Lasserre (1964) credits Archytas with having shown the incommensurability of the audible fractions of the whole tone, thereby undermining the empirical solutions which the Harmonists had proposed. For example, by proving that the semi-tone of the perfect fourth is not strictly equal to half the perfect whole-tone, he showed that mathematics contradicts not only the testimony of the ear but also the assumption of the Harmonists that a perfect half-tone must exist. The problem of the relation between mathematics and reality was discussed by Archytas of Tarentum in *Elements of Music*, parts of which make up the so-called *Section of the Canon*.

rather than in terms of its characteristic objects. For instance, mathematicians are said (532A) to be unable to give a reasoned account of their objects and so they do not really know what they ought to know, though this is the task which dialectic fulfils. Dialectic stimulates the intellectual capacity of the soul to find the true reality of each thing without help from the senses, and not to give up before grasping the Good Itself. Socrates draws an explicit parallel here between the allegorical journey up out of the cave and the intellectual path through dialectic towards a vision of the Good. Since this is familiar territory, let us focus on just one detail that bears on our guiding question about the ontological relationship between the objects of mathematics and of dialectic.

When describing the ascent from the cave into the daylight, Socrates says (532C1–2) that the prisoner is unable to look at the original living creatures but only at the 'divine images' and shadows of things in water. Then he concludes (532C4–7) that the mathematical sciences (as previously discussed) have the power to lead the best part of the soul upwards to the contemplation of what is best in reality, just as the clearest sense in the body (presumably sight) gazes on the brightest thing in the visible world. It seems that here a parallel is being suggested between the objects of mathematics and the 'divine images' in water, especially since the use of images was previously associated with mathematical method. Yet, since Plato does not draw any such parallel, we must confine ourselves to his subsequent elaboration on the methodological differences between dialectic and the other disciplines.

When Glaucon demands (533A) to be shown the power of dialectic itself, Socrates doubts his ability to comply because it no longer involves images but rather the truth itself. On the other hand, however, he insists that this power is available only to someone who is already trained in the mathematical disciplines. While underlining the role of mathematics as propaideutic, Socrates also emphasizes (533B) that dialectic must be differentiated by the method that it uses to grasp what each thing really is. By contrast, all the other crafts concern themselves with the opinions of men or with other natural things that are equally transient. Even the mathematical disciplines which have some purchase on intelligible reality, Socrates describes (533C1–3) as 'dreaming about being' so long as they use hypotheses without reflection and fail to give a reasoned account of them. The upshot of this implied criticism is that the mathematical

disciplines cannot be called true sciences because they begin from what is unknown and proceed through intermediate steps to an agreed conclusion. *Pace* Burnyeat, the crucial point of the criticism seems to be that the failure of mathematicians to reflect upon the foundations of their discipline undermines their status as theoretical sciences.

In support of his criticism, Socrates asserts (533C–D) that dialectic is the only discipline to follow the route of 'destroying' hypotheses by going beyond them to a principle, so that they may be firmly grounded. This science gently draws the eye of the soul, which in fact is buried in a kind of barbaric bog,[37] and leads it upwards by using the mathematical disciplines as means of spiritual conversion. After noting the propaideutic character of these disciplines, Socrates insists that they have only been called 'sciences' out of force of habit and that some other name is needed to indicate that they are clearer than opinion, though more obscure than science. While warning against any dispute about names, he recalls as suitable the term διάνοια that was used previously to demarcate such disciplines.

This recollection leads Socrates to summarize (533E–534A) the whole discussion in terms of the divided line as follows: the first section is called knowledge (ἐπιστήμη), the second reasoning (διάνοια); the third belief (πίστις), and the fourth fantasy (εἰκασία). Again we should notice that the division is made in terms of the respective faculties of the soul rather than their corresponding objects. In a subsequent summary, however, Socrates says (534A2–3) that the combined faculties that go under the general name of opinion (δόξα) are concerned with the realm of Becoming, whereas intelligence (νόησις) in general deals with the realm of Being. He also repeats the proportions of the divided line with reference to the faculties of the soul, while pointedly refusing to spell out the proportion between their respective objects in case it should involve many more arguments. Thus Burnyeat (1987) rightly sees in this whole passage Plato's implicit recognition of a problem about the ontological status of mathematical objects, which could have been the topic for an 'unwritten chapter.'

Despite the absence of explicit statements about distinctive subject-matters, however, it is still possible to pursue the distinction

[37] This is my feeble attempt to capture the alliterative and punning features of the Greek (ἐν βορβόρῳ βαρβαρικῷ), which seems to have strongly pejorative connotations.

between dialectic and mathematics through Plato's descriptions of their respective methods. At the conclusion of the present passage in the *Republic,* for instance, Socrates proposes (534B) to call that man a dialectician who can give a reasoned account of the essence (οὐσία) of each thing, whereas he denies that anyone without such an account can claim to have intelligence (νοῦς) about a subject. From this we may infer that the mathematician can still be described as having understanding (διάνοια) of his subject-matter. But another clear implication is that such a person is not a scientist in the strict sense because he cannot defend his hypotheses by appealing to higher principles, such as the One and the Good. The dialectician, on the other hand, can resist all attempted refutations because he argues according to the real nature of things rather than according to opinion. Since the unphilosophical mathematician cannot do this, the implication is that he only manages to grasp an image of reality through opinion, and again the dream metaphor is used (534C–D) to characterize the epistemological condition of such a person.

Some further light can be thrown on this condition by contrasting it with the state of knowledge of a dialectician, who considers the first principles of reality. At *Philebus* 23C, for instance, with reference to the question about the best way of life, Plato introduces a fourfold division of the universe which is presumably the sort of thing that a dialectician would know. This division consists of the familiar Pythagorean principles of Limit and Unlimited, along with a mixture of these two and a cause of that mixture. Socrates illustrates (24A) the nature of the Unlimited in terms of comparatives like hotter and colder where the more and the less range in an indefinite continuum. Being boundless in this way means being absolutely unlimited both in quality and in quantity. However, once a definite limit has been set to the continuum, it loses its more-or-less character and becomes a definite mixture. In linguistic terms, therefore, the Unlimited is reflected in adverbs like strongly or slightly, whereas the Limit is expressed by terms like equal, double, or indeed any ratio. Hence, as Socrates puts it (25E), the mixture of Limit and Unlimited involves some ratio that puts an end to the conflict of opposites, and makes them well-proportioned and harmonious through the introduction of number. For instance, in the case of the high and low in pitch, or the swift and slow in motion (the Unlimited), numerical ratios introduce limit and thereby establish the whole art of music. It is quite in keeping with Plato's Pythagorean approach to reality that

music provides the paradigm of a harmonious mixture of two generating principles, the Limit and the Unlimited.

But perhaps he is moving beyond the Pythagorean tradition when he introduces as an additional principle the cause or maker of that mixture, and gives it priority in being on the grounds that what makes something is prior to what is made. The dialogical purpose of introducing the fourfold division was to classify the lives of pleasure, reason and the mixture of both. Obviously, the mixed life falls under mixture, and Philebus readily agrees that pleasure belongs to the class of the Unlimited, since he does not wish to have pleasure limited in any way. This leaves open the possibility that reason belongs either to the class of Limit or to that of cause. The matter is decided indirectly through a discussion of cosmology where reason is described (28C) as the king of heaven and earth.

In this regard, the crucial question (28D) is whether the universe is controlled by an irrational and blind power, which operates by mere chance, or whether it is governed by a wondrous regulating intelligence. Protarchus finds it blasphemous to suggest that the world is ruled by chance. Therefore, it is only if reason is assumed to be ordering (διακοσμεῖν) the whole that we can adequately explain the ordered universe, containing the sun, the moon, the stars, and the revolution of the heavens. This assumption is taken to represent the general consensus of wise men, though there is some clever person (ἀνὴρ δεινός) who asserts that the universe is devoid of order (ἀτακτῶς).

The argument against this Democritean view of the world draws (29E) on the following parallel between the microcosm and the macrocosm: just as the animal body is composed of the four elements, so is the body of the universe. But it is the cosmic elements that nourish and sustain the particular elements that compose the human body. Similarly, our bodies could not have a soul if the body of the universe did not also have a soul, since it would be absurd to suppose that the cause of soul in us should fail to provide soul for the whole universe. Here (30A–B) we have the bare bones of an argument for the existence of the World-Soul, which plays such a prominent role in the *Timaeus*. Indeed, we should keep that dialogue in mind when Socrates says (*Philebus* 30C) that, along with Limit and Unlimited, there exists in the universe a powerful presiding cause which ought to be called wisdom and reason (σοφία καὶ νοῦς). But, since wisdom and reason presuppose soul, the highest divinity (traditionally called Zeus) has a royal soul and a royal reason by virtue of the power of the cause (i.e. the fourth division of reality). It is quite

revealing for Plato's method of inquiry that he makes Socrates insist (30D) that this conclusion is supported by the view of previous thinkers who declared that reason always rules all things. All of these hints point us towards Plato's cosmology in the *Timaeus* dialogue.

II. *Plato's mathematical cosmology*[38]

In order to clarify the function of mathematics in Plato's cosmology, I will treat the *Timaeus* as a rudimentary physical inquiry which anticipates Aristotle's teleological approach.[39] Given this common basis, one can delineate more sharply some important differences between them as to the role of mathematics in their respective cosmologies.

While paying attention to Plato's appropriation of earlier traditions of physical inquiry, I will focus mainly on the mathematical aspects of the *Timaeus*. For instance, his account of the 'works of Reason' seems to reflect a Pythagorean cosmology that is dominated by the search for abstract numerical relations as the principles of order. By contrast, the account of 'things that happen of Necessity' seems indebted to the views of *physiologoi* like Empedocles and the Atomists. But Plato outstrips his predecessors by showing how their partial views can be integrated into a complete cosmos that is guided by principles of order and harmony. My general hermeneutical strategy, therefore, is to read the dialogue as a dialectical synthesis in which conflicting opinions are reviewed in the search for fundamental principles that will 'save the phenomena.' For instance, I do not accept the common view that the works of Democritus were completely unknown to Plato.[40] But first let us consider how Plato viewed the contribution of Anaxagoras to teleological explanation in cosmology.

II.1. *The demand for teleological explanation*

At *Phaedo* 96A the question of whether human souls are really indestructible and immortal is said to require an account of the causes of

[38] I am grateful for written comments on previous drafts of this section, which I received from David Furley, Tom Robinson and the late Gregory Vlastos. While I have benefited greatly from their remarks, they are not responsible for any deficiencies in my discussion.

[39] Vlastos (1975) 30 & 62 notes that Aristotle's cosmological system consolidated the teleological methodology championed by Plato.

[40] Taylor (1928) 3 & 10 thinks that Plato knew nothing about Democritus, since he takes very seriously the report of the latter's complaint about being unrecognized

generation and corruption in general. Within this dialogical context Socrates recounts an earlier enthusiasm for natural science (περὶ φύσεως ἱστορία), which was driven by the belief that it would be magnificent to know the reasons for everything; e.g. why a thing comes into being, why it perishes, and why it exists.

But the experience of grappling unsuccessfully with these questions left Socrates totally confused. For instance, with regard to the question of what causes a person's growth, it had always seemed obvious that he grew because he ate and drank, with the result that flesh was added to flesh, bone to bone, and so on for the other parts of the body. Just as this account seemed appropriate, so it had also appeared sufficient to say that a man was taller than another 'by a head' (τῇ κεφαλῇ), or that 10 was more than 8 by the addition of 2, or that 2 metres was longer than 1 meter 'by the half' (διὰ τὸ ἡμίσει).[41]

After dabbling in physical inquiry, however, Socrates no longer feels sure of knowing the reason (αἰτία) for any of these things. Significantly enough, he chooses an arithmetical example to illustrate his puzzlement; i.e. when 1 is added to 1 does the first 1 become 2, or does 1 and 1 together become 2, because of addition (διὰ τὴν πρόσθεσιν). The puzzling point here (97A5) is that, when the units were apart from each other, each was 1 and there was as yet no 2, but as soon as they approached each other there seemed to be a cause for 2; namely, the union in which they were put next to each other. On the other hand, if we divide the unit, should we say that the division is the reason for the generation of 2? But this would involve us in the contradiction of explaining the same thing by means of two opposite causes; namely, bringing together as opposed to separating. Such contradictions undermine any physical explanation of how a unit comes into being or perishes or even exists. So this passage implies that the empirical approach to mathematics should be replaced by the hypothetical method that uses only intelligible or ideal entities.

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in Athens when he visited there. This view has been disputed by Vlastos (1975) who thinks that Plato himself bears a heavy but unacknowledged debt to Democritus. Yet the victory of Platonism in mathematics has been so complete that only a few people like Mourelatos (1984 & 1987) entertain the idea that Democritus may have offered mathematical arguments for atomism.

[41] Hackforth (1955) 124n1 notes that the strange way of expressing double length is due to Socrates' wish to put this instance on all fours with all the other examples; so that A is bigger than B because of something having happened to A. But this sophistical way of putting things leads to confusion.

In his state of intellectual confusion, Socrates heard someone reading from a book of Anaxagoras which said something about Mind (νοῦς) as a cause that arranges all things. Socrates approved (97B–C) of Mind being the cause of everything because it must do all its ordering in the way that is best for each individual thing. Hence, if one wanted to discover the cause of anything coming into being or perishing or existing, the right question to ask was how it was best for that thing to exist or to act or be acted upon. Since these are identical with questions posed earlier (96A), Plato is implying that it was Socrates rather than Anaxagoras who anticipated the teleological approach to physical inquiry.

This is confirmed by the inference which Socrates draws from a mere hint of Anaxagoras' theory that Mind is the cause of the cosmos. According to such an account, he reasons (97D3), the only thing a man had to think about was what is the highest good. It is no accident, however, that he infers that one must also know what is bad, since knowledge is always of opposites. The crucial point is that Socrates had adopted a teleological perspective *before* he read Anaxagoras, given that he always inquired about good and evil. For instance, Socrates says (97D) that such calculating led him to believe that he had found in Anaxagoras a teacher about the cause of things that corresponded to his own thinking. Here are two of the questions he wants (97E) to ask: 1) Whether the earth is flat or round, and why is it better for it to be as it is? 2) Whether the earth is in the center of the universe and why it is better for it to be there? Except for the new evaluative dimension, the Socratic questions reflect the typical inquiries of the older *physiologoi.*

Undoubtedly, Socrates is here represented as expecting teleological explanations, and this is confirmed by the account (98B–C) of how his hopes were dashed on reading Anaxagoras because he made no use of Mind as a cause for setting things in order (τὸ διακοσμεῖν) but, instead, he gave as causes air and *aither* and water, and a host of other absurdities (ἄτοπα). The language here conveys Plato's own dismissive attitude towards Anaxagorean explanation, which is compared with a misguided explanation that might be given for Socrates' decision to stay in jail rather than escape as his friends had urged. Thus Anaxagoras is likened to someone who says that Socrates acts in this way because of Mind and then tries to explain his actions in terms of the bones, sinews, joints, and all the other parts of the body necessary for sitting in a certain way. For Plato this is just as ridiculous as

giving the cause of Socratic conversation in terms of sounds, air currents, streams of hearing, and so on.

As Socrates puts it (98D), the trouble with such accounts is that they neglect the true causes; namely that, since the Athenians thought it better to condemn him, he also thought it better to sit in jail. In other words, he thought it more just and proper to stay where he was and submit to the punishment handed down by the court rather than run away. If it had been up to his old bones, Socrates jokes, they would long ago have fled to Megara or Boeotia, driven by their opinion of what was best. This characteristic joke makes the point that his decision to accept the death penalty would be unintelligible without appealing to the fact that Socrates thought it right and proper to submit to the law. By contrast, it would be quite absurd to appeal to such things as bones and sinews, unless one meant to say that without having such things, Socrates could not have carried out his decision. Obviously Plato has someone like Anaxagoras in mind when he criticises (99A) as very careless all descriptions of mental actions in physical terms, rather than in the appropriate terms of choice of what is best. For him this represents a failure to distinguish between the real cause and that without which the cause would not be a cause. This looks like the modern distinction between a cause and its necessitating conditions, which roughly corresponds to the distinction between final causes and auxiliary causes at *Timaeus* 46C, where necessitating conditions are treated as types of causes.

At *Phaedo* 99B, however, Plato says quite clearly that the *physiologoi* mistakenly think that the sort of cosmological explanations which they offer are causes. For instance, one man (Empedocles?) is reported to have said that the earth is kept in position by the whirl generated with the revolving heavens, just as a heavy ball is held in the center of a whirlpool by centripetal forces. Some other thinker (Anaximenes?) views the earth as a flat lid that is supported on a base of air. Once more, this explanation for the stability of the earth appeals to familiar empirical analogies, and it must be such a tendency that Plato has in mind when he describes these people as searching for a stronger and more immortal Atlas who is better able to hold things together, while having no inkling of the divine power which puts things in the best possible positions. He means that their preoccupation with the observable forces of the physical world has blinded the *physiologoi* to the possibility of invisible and intelligible forces that for Plato must be understood in terms of the intentional action of a moral agent.

This is the fundamental reason why he represents Socrates rather than natural philosophers like Anaxagoras as having anticipated the possibility of teleological explanations in cosmology. Such a possibility is precisely what Plato himself tries to realize in the *Timaeus* dialogue by synthesizing and transforming the Pythagorean and Ionian traditions through his own teleological cosmology.

II.2. *The works of reason*

According to the program of that dialogue, Timaeus speaks first about the origin of the cosmos and then about the generation of mankind. While being ostensibly a Pythagorean, this obscure figure is also used as a fictional mouthpiece for other cosmological views. Although his style of speech is didactic rather than dialogical, one cannot assume that Timaeus always represents Plato's own view and, as Gadamer[42] has shown, it is important for understanding the dialogue to advert to shifts in the mode of discourse. For instance, the solemnity of the proem underlines as genuinely Platonic the initial distinction (which guides the discussion of the realm of Reason) between that which is always Being and that which is (always) Becoming.[43] The first is grasped by thought with the aid of reasoning because it is always unchangeably real; whereas the second is an object of belief which is grasped by opinion with the help of unreasoning sensation because it changes and is never really real.

The subsequent discussion of the realm of Reason is guided by this ontological and epistemological distinction until it is emended for the discussion of things generated by Necessity. It is argued that the existence of generated things requires the agency of some Cause, since without it their existence is impossible. Thus there is an active agent in Plato's cosmology but not an omnipotent creator, since the Artificer (ὁ δημιουργός) must keep his gaze fixed on an unchanging

[42] Cf. Gadamer (1980) 156–93. He notes a distinct difference between the mythical narrative with respect to the realm of Reason and the almost technical discourse of the realm of Necessity. On this difference he builds an important argument about the absence of the Demiurge from the mathematical structuring of the Receptacle.

[43] It has been suggested by Whittaker (1969) 181–85 that the second ἀεί be excised because it has poor MSS authority. This suggestion has recently been revived by Robinson (1987) 103–19 in support of his literal interpretation of the dialogue. But John Dillon (1989) 63 points out that the ἀεί did not bother the Christian Aristotelian, John Philoponus, who took it as a reference to the state of constant dependence of a created universe. So its philosophical implications are not clear or decisive either way, and one could argue for its retention on stylistic grounds as

model if he wants the generated object to be beautiful.[44] For if the craftsman were to use a generated model, the end result would not be beautiful. The idea of beauty here functions as a guiding principle for the Demiurge in shaping the visible world.

The initial distinction is put to work when Timaeus addresses (*Tim.* 28B) the primary question about the whole Heaven or the cosmos; namely, whether it has been generated or not. His answer is that it has been generated because it is visible and tangible and possessed of a body.[45] All such things are sensible and are therefore generated, since they are grasped by opinion with the help of the senses. The initial distinction functions here as a criterion for deciding to which ontological realm any object will belong. Assuming that the cosmos has been generated, Timaeus claims that there must be some cause of its generation, but he seems to balk at the double task of discovering and explaining the Maker of the cosmos. It is unclear whether Timaeus intends to undertake either project, though he does say explicitly that it would be impossible to explain the cause of the universe to all men.[46] Yet that still leaves open the possibility that it can be made intelligible to people with mathematical insight.

Subsequently, Timaeus raises a more decisive question about the cosmos; i.e. whether its Architect copied from an unchanging model or from one which is itself generated. On grounds of piety, it is argued that the Architect of the cosmos must have fixed his gaze on an eternal model, since the cosmos is beautiful and the Demiurge is good.[47] This confirms the dominance of intelligible purpose in the origin and structure of the visible cosmos.

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being needed for the balance of a sentence which perhaps plays on the ambiguity between 'continuously' and 'continually' as meanings of ἀεί.

[44] Vlastos (1975) 26 ff. sometimes talks about the Demiurge as the creator of the cosmos but clearly he does not mean that there is creation *ex nihilo,* since he recognizes the existence of a primordial chaos. In a personal communication, Vlastos defended such talk of a 'creator' by drawing the analogy with Michelangelo as the creator of a sculpture, which allows for the fact that the Demiurge must work with pregiven material, just like a craftsman.

[45] Robinson (1987) 119 thinks this argument involves a category mistake, since the universe is not like any ordinary sensible object that may be perceived by the senses and categorized as a whole. It is possible that Plato was familiar with the Atomist distinction between 'universe' and 'cosmos,' even though he uses these terms as synonyms here.

[46] Vlastos (1975) 25 claims that the ordering activity of the Demiurge is 'supernatural' and therefore does not fall under the class of events that can be given a natural explanation. But Plato proceeds as if such activity can be made intelligible as mathematical construction.

[47] Plato's description of the Demiurge as 'lacking in envy' reflects conscious op-

The proem concludes with a methodological digression (29B–D) which stresses the importance of starting every discussion at its natural beginning. In connection with the model and its image Timaeus affirms that the accounts given of each will be akin to the different objects that they explain. Thus, when dealing with eternal objects that are grasped by reason, one should give an account that is itself rational and also permanent, insofar as it is fitting for such accounts to be unchanging and irrefutable. But such a correlation dictates that only a likely story can be given for the visible cosmos which is a copy of the eternal model.

Therefore, Timaeus promises only a plausible account of the origin and development of the visible world, within which he later (*Tim.* 47E3 ff.) distinguishes between 'the works of Reason' and 'what comes about of Necessity.' The general plan of his discourse is first to analyse the works of Reason, then the products of Necessity and, finally, to describe their cooperation in the generation of mankind.[48] It appears that the so-called 'works of Reason' are those aspects of the visible world, especially the sensible heavens, that show a rational and intelligible plan. By contrast, those things that are generated through Necessity seem to result largely from mechanical causes. Later in the dialogue (45A–47E, 69A ff.), Timaeus tries to reconcile these contrasting aspects by subordinating Necessity to Reason. For instance, sense perception can be explained in purely material terms as a product of Necessity but it is truly intelligible only in terms of its rational purpose. It is noteworthy that Plato here anticipates some teleological aspects of Aristotle's *Physics*.[49]

II.3. *The role of the demiurge*

Within the first section (29D–47E) of the discourse, the primary reason given for an ordered universe is the existence of an Architect

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position to the old mythopoetic tradition about the gods. With regard to the rationality and intelligibility of the cosmos, Plato is firmly on the side of the Ionian *physiologoi*, even though he staunchly opposes them on other questions; cf. Vlastos (1975) 27 ff.

[48] Vlastos (1975) ch. 2 assumes that the section on Reason is exclusively concerned with the structure of the heavens, whereas the section on Necessity deals with the structure of matter. While this seems plausible as a general division, it does not explain why each section offers a different mathematical analysis of the four elements and their interrelationships.

[49] But, if this is the case, one might be legitimately puzzled as to why Aristotle does not acknowledge that Plato had already discovered the final cause or 'that-for-

who, being good and lacking in envy, desires that all generated things should come as close to perfection as possible. Having posited this supreme cause of the generation of a world order,[50] Timaeus argues for the presence of soul in the visible cosmos by appealing to the *ethos* of a good Architect; namely, that it is never right for him to do anything but what is best. Yet if this cosmos is to be the best and the most beautiful, the good Architect must have fixed his gaze on an eternal model; i.e. the Living Creature of which all other ideal living creatures are parts, individually and generically. Again we find the familiar ontological distinction between the intelligible and sensible realms, along with the new thesis that the part/whole relationship is similar in both realms.[51] According to Timaeus himself, the reason for this similarity is that 'the god' used the intelligible Living Creature as a model in order to make the visible cosmos as complete as possible. Hence it is constructed as a single and visible living creature that embraces within itself all the living creatures that are naturally akin to it.

The description of the archetypal world as a Living Creature implies the organic unity of the visible world that imitates it, but Timaeus also argues that the visible heaven is unique and not multiple or indefinite in number. His argument is based on the claim that its eternal archetype is unique because that which embraces all intelligible living creatures cannot be one of a pair, otherwise there would be yet another unified Living Creature embracing these two as parts of it. So the visible cosmos is a single copy of this unique Living Creature.[52]

Thus Timaeus concludes that the Maker of the cosmos generated a unique visible cosmos in order that this living creature might closely resemble the complete Living Creature in respect of its uniqueness.[53] This suggests that, independent of the Maker, there is an eternal

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the-sake-of-which' (τὸ οὗ ἕνεκα). My second and sixth chapters will explore the reasons why Plato's mathematical cosmology is rejected by Aristotle as a genuinely teleological world-view.

[50] Prior (1985) 87–126 treats Plato's introduction of the Demiurge as a response to the objections in the *Parmenides* against the causality of Forms with reference to the world of appearances.

[51] R. Mohr (1985) 9–52 analyses this relationship, as does W.J. Prior (1985) ch. 3.

[52] There has been some doubt among scholars about the legitimacy of this argument for the uniqueness of the cosmos; cf. D. Keyt (1971) 230–35; R. Parry (1979) 1–10; R. Patterson (1981) 105–119. In defense of the argument see R. Mohr (1985) ch. 1 and W.J. Prior (1985) ch. 3.

[53] This argument for the uniqueness of the original model of the world is remi-

model which guides his ordering of the visible cosmos. Indeed, the whole argument seems to depend upon an implicit analogy with the master craftsman who makes a perfect copy in one attempt.

II.4. *The mathematical ordering of the World-Body*

Using his initial distinction between Being and Becoming, Timaeus claims that whatever has been generated must be visible and tangible. Since generated things are grasped by opinion with the help of sense perception, they must have perceptible bodily form because nothing is visible without fire nor tangible without something solid. But, he argues, it is not possible to put the two elements together without a third to serve as a bond, and the best bond combines its elements into the sort of unity which is most perfectly exemplified in proportion.[54]

Once again the beauty and perfection of the visible cosmos are held to depend upon mathematical proportion being the unifying bond for the elements. To illustrate the complex unity of proportion, Plato borrows an arithmetical example from the older Pythagorean tradition in which the theory of proportion for rational numbers was first developed in connection with musical harmony.[55] The example also illustrates continued geometrical proportion, which was regarded as the primary and best kind of proportion because all of its terms are interchangeable; i.e. we may take them in the order given (2 : 4 :: 4 : 8) or reverse the order (8 : 4 :: 4 : 2) or completely interchange the order (4 : 8 :: 2 : 4 or 4 : 2 :: 8 : 4), without losing that equality of ratios between numbers which constituted the original order. The unity and perfection of such continued proportions may have been what originally inspired Pythagoreans like Philolaus to seek the order of the visible cosmos in numbers and their relations.[56]

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niscent of Plato's argument in *Republic* X (597B–E) for the uniqueness of the Form of bed, except that 'the god' is there said to be the artificer of the real bed, which is a unique Form. But in the *Timaeus* there is no suggestion that the Demiurge is the maker of the eternal Living Creature, which serves as the unique model for the visible universe.

[54] τοῦτο δὲ πέφυκεν ἀναλογία κάλλιστα ἀποτελεῖν, *Tim.* 31C4–5.

[55] See Burkert (1962), Szabo (1978) & van der Waerden (1954) on the relationship between music and the early theory of proportion.

[56] See Philolaus Fr 4 (KR 427): καὶ πάντα γα μάν τὰ γιγνωσκόμενα ἀριθμὸν ἔχοντι· οὐ γὰρ οἱόν τε οὐδὲν οὔτε νοηθῆμεν οὔτε γνωσθῆμεν ἄνευ τούτου. See also Aristotle, *Physics* III.4, 203a10 & *Metaphysics* I.5, 986a22.

Timaeus is also speaking like a Pythagorean when he distinguishes between 'cubes' and 'squares' with reference to numbers. Within that tradition rational numbers with two or less factors could be represented either as straight lines (for prime numbers), or as rectangles (e.g. 6 is a rectangle with sides of 2 and 3), or as squares (e.g. 4 or 9). But a number with at least three factors might be represented as a solid (e.g. 24 is a parallelpiped with sides 4, 3 and 2). This sort of language is used with reference to numbers in the seventh book of Euclid's *Elements* (Defs. 17, 18), which may derive from an earlier Pythagorean treatise on numbers.[57]

But how is the Pythagorean distinction between solid and plane numbers relevant to the bond between fire and earth in the construction of the universe? Timaeus explains (*Tim.* 32A–B) that if the cosmos had been a plane then one middle would be enough but, since it is a solid, two means are needed. This whole explanation depends on the purely mathematical point that between two 'solid' numbers (e.g. 8 and 27) one cannot find a single geometrical mean that would also be a rational number, though one can insert two geometrical means (e.g., 12 and 18) to make a continued geometrical proportion (e.g., 8 : 12 = 18 : 27).[58]

Yet the argument itself is not quite perspicuous when it presents the plane and the solid as competing possibilities for the dimensions of the cosmos. The use of two Greek words, ἐπίπεδον and βάθος, suggests the analogy with square and cubic numbers upon which the plausibility of the argument rests.[59] It would appear however that such analogical arguments were typically Pythagorean, if we are to judge from examples like the equation of justice with the number 4 on account of the latter's reciprocity. Instead of such simple-minded accounts, Timaeus suggests (*Tim.* 32B6–7) that when constructing the visible and tangible heaven the Demiurge had good mathematical reasons for placing water and air as means between earth and fire; namely, that he tried to place them in the same proportion to one

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[57] See Heath (1925) ii, 294–5 for a review of the evidence which indicates that this passage from the *Timaeus* is dependent upon the Pythagorean tradition.

[58] In fact, this is not true if the 'cubic' numbers also turn out to be 'square' (e.g. 64 and 729).

[59] It may be an unintended consequence of the analogy with numerical proportion that the two means (e.g. 12 and 18 in the case of extremes like 8 and 27) will not themselves be 'cubic' numbers, and so will not correspond to 'solids' like air and water, posited as physical bonds between fire and earth as extremes, unless the former are rectangular parallelepipeds.

another in order to achieve unity and perfection in the visible cosmos. This refers to the older meaning of proportion as equality of ratio: fire is to air as air is to water, and water is to earth as air is to water (i.e. fire : air :: air : water :: water : earth). Such continued proportion plays a central role in Plato's account of how the good Architect used the four elements to construct the body of the visible cosmos as a harmonious whole.

II.5. *Physical reasons for the unity of the cosmos*

Plato insists that the harmony between the four elements of the World-Body constitutes the principle (Love) that guarantees the unity and indissolubility of the cosmos. By mentioning only the Empedoclean principle of unity, he underlines the absence of a principle of dissolution from the body of the cosmos that is structured according to mathematical proportions.[60] Another reason given (*Tim.* 32C) as to why the generated cosmos will not be destroyed is that the good Architect has used up all of the material available, so that nothing is left outside that might have a destructive effect on it. Since this explanation presupposes a limited supply of material, Plato seems to be rejecting the Atomist assumption of an unlimited quantity of matter in an infinite void.

On the other hand, some Parmenidean influences are evident in the claim that the good Architect made the visible universe spherical because this is the most complete and most uniform of all shapes. Although Timaeus does not offer any elaborate explanation as to why a sphere is more perfect and more uniform than any other figure, a hint may be found in his brief description of it as "equidistant in all directions from the center to the extremities." On account of such perfection the sphere was used as *the* cosmic figure in Parmenides' description of Being.[61] Thus, as a copy of eternal Being, the cosmos is constructed as perfectly as possible by being given a spherical shape.

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[60] Skemp (1942) 63 finds the same implicit references to predecessors but thinks that φιλία here may be a Pythagorean refinement on the Empedoclean moving power. However, he finds a close parallel between the function of Νεῖκος in the Empedoclean cosmic system and that of ἀνάγκη in the *Timaeus*. But this seems unlikely in view of the absence of a principle of dissolution from Plato's account of the Four Kinds in the *Philebus*.

[61] τετελεσμένον ἐστὶ, πάντοθεν εὐκύκλου σφαίρης ἐναλίγκιον ὄγκῳ, μεσσόθεν ἰσοπαλὲς πάντῃ, Fr. 8, 42–3 DK.

Furthermore, the body of the generated universe is held to be perfectly smooth on the outside. Timaeus claims that the cosmos has no need of eyes, since there was nothing visible left outside, and he gives a similar argument for the absence of hearing and respiration in the body of the cosmos.[62] In addition, he argues (*Tim.* 33C–D) that the cosmos was designed to feed itself on its own waste and to act and be acted upon entirely by itself and within itself. Thus, the good Architect constructed a completely self-sufficient entity because this state is better than one of dependency.[63]

Throughout his description of the World-Body, Timaeus emphasizes the static consistency and self-identity of reason, in contrast to the wandering multiplicity of opinion. Thus spherical motion is attributed to a World-Body that is itself rationally constructed to exclude the irrational motions (up / down, forward / backward, right / left) as far as possible. Such an exclusion is based on the claim that there is nothing outside the cosmos, so that neither can there be any of those directions in which it could move. This argument assumes that these directions are relative to the structure of the visible universe and hence can have no reference to anything outside of it.[64]

II.6. *The numerical structure of the World-Soul*

According to this account (*Tim.* 34B), therefore, the body of the generated cosmos was made smooth, uniform, equidistant from its center—a complete body compounded of perfect bodies by the good Architect who wanted to make the visible universe as divine as possible. Presumably its divinity is enhanced by the soul that is placed in the middle and stretched throughout the whole, so that the body was surrounded on the outside by soul.[65] Timaeus insists, however,

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[62] This is the sort of functional argument which Aristotle also uses extensively under the general principle that 'Nature makes nothing in vain.' ἡ δὲ φύσις οὐδὲν ἀλόγως οὐδὲ μάτην ποιεῖ, *Cael.* II.2, 291b14. See also *Cael.* I.4, 271a33; *PA* II.13, 658a9, III.1, 661b24; *GA* II.5, 741b5, II.6, 744a36; *Pol.* I.8, 1256b21. For an earlier parallel see Empedocles' description of the cosmic sphere when it is completely ruled by Love; cf. Fr 29 DK.

[63] From Homer onwards, self-sufficiency is consistently identified with the state of freedom (ἐλευθερία) of the nobleman (καλοκἀγαθός), whereas dependency is seen as the state of being in need that is characteristic of the slave (δοῦλος); cf. LSJ. 278, 447, 532, 1505–06.

[64] This denial of absolute directions in the universe is important for understanding Plato's account (*Tim.* 62C ff.) of relative weight, which also involves the denial of absolute weight.

[65] It may also have been traditional to think of the soul as a quasi-physical entity

that the body of the universe is neither prior in birth nor in excellence to the soul, since this would invert the order of cosmic rule.[66] The function of the soul is to be the ruler of the body and, thereby, to serve as a principle of limit on a cosmic scale. Yet the construction of the World-Soul by the Demiurge must be done in a different way and with different materials from those of the World-Body, otherwise the two would be identical. These materials are said to be a mixture of the Same and of the Different compounded with the help of Being, each of which in turn is an intermediate mixture of what is indivisible and eternal and of what is generated and divisible. Hence the World-Soul is constructed out of a kind of material that is intermediate between the realms of Being and Becoming.[67] The nature of this material is left unclear, since Timaeus talks obscurely about the Demiurge having difficulty blending it from intermediate Sameness and Difference to make one form, so that he had to enlist the aid of Being (οὐσία).[68]

In Timaeus' description of how the mixture is divided by the Demiurge, abstract concepts (e.g. Sameness, Difference, Being) are treated like malleable materials to be used in the formation of the

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that extends throughout the body; cf. Snell (1953) ch. 1. Judging by the arguments in the *Phaedo,* however, Plato had rejected such materialist concepts in favor of a separable and intelligible entity that is akin to the Forms. Thus Timaeus may here be using a Pythagorean notion of soul that is akin to Philolaus' principle of Limit, which itself is a concrete entity thoroughly embedded in the Unlimited; cf. Burkert (1962) 255.

[66] This topic of the proper order of priority among beings is an aspect of the Platonic tradition which Aristotle develops, even while reaching very different conclusions; cf. Cleary (1988b).

[67] Cornford (1937) gives a very clear analysis of the two-stage mixture which seems to be involved in generating the material of the World-Soul. Prior (1985) 99 notes that this distinction between two kinds of Being, Sameness, and Difference is unparalleled in Plato and so he concludes that it must be the result of Plato's recognition that Forms are non-spatial entities that are distinct from physical objects, which therefore must have a different sort of Being, Sameness, and Difference.

[68] Some commentators (e.g. Cornford) think that these three constituents of the World-Soul correspond to the so-called greatest kinds of the *Sophist* (244–45), while others (e.g. Owen) think that they can be found already in the *Republic.* But perhaps the most important issue is what significance should be attached to such constitutents. Shorey (1933) argues that the soul must be constituted from the three highest kinds because it must recognise them everywhere. But the same argument could be made for all the forms which the soul must recognise and, obviously, they are not all intended here. In any case, it is not clear that the World-Soul is subject to the same epistemological conditions as the human soul. After the formation of the World-Body, its soul may be presumed to perceive itself, the Forms, and the physical world; while making judgments of existence, sameness, and difference about all three. I am indebted to Tom Robinson for this latter point about the World-Soul.

World-Soul.[69] But I want to draw attention particularly to the fact that the resulting mixture is divided according to the following series of numbers: 1, 2, 3, 4, 8, 9, 27. This series is composed of two interlocking series of doubles and triples, respectively; i.e. 1, 2, 4, 8, and 1, 3, 9, 27. Timaeus calls them double and triple intervals (διαστήματα) when he talks about finding two means for each interval, which suggests that the harmonic and arithmetical means between the numbers are being thought of in terms of musical intervals.[70]

Hence, in order for the soul to grasp all existing things, it must contain a harmony between solid numbers with two means, so that it extends throughout the whole solid body of the world. Previously, in the construction of the World-Body, water and air were inserted as geometrical means between fire and earth so that the greatest unity and perfection would belong to the body of the visible cosmos. Similar conditions apply to the harmonic intervals in terms of which Timaeus gives his account of the generation of the World-Soul. All of this construction is based on the Pythagorean tetractys (1, 2, 3, 4) which yields the ratios forming the perfect consonances; i.e. 2:1 (octave), 4:3 (fourth), 3:2 (fifth). The whole point of the original distinction between consonant and dissonant intervals made by Archytas of Tarentum seems to have been that only harmony reveals the mathematical order of the universe. This is just the sort of ontological mathematics that influenced Plato and his nephew Speusippus, and which was subsequently criticized by Aristotle and his pupil, Aristoxenus of Tarentum.[71]

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[69] Taylor (1928) 106 ff. notes that this whole passage was a source of disagreement among early Academicians, such as Xenocrates and Crantor. While the former defined soul as a self-moving number (ἀριθμὸν κινοῦνθ' ἑαυτόν), Crantor regarded the soul simply as being aware of things with reference to motion. According to Plutarch (περὶ τῆς ἐν Τιμαίῳ ψυχογονίας—1013a), both of them refused to take literally Plato's account of the composition of soul, which they insisted was being given for the sake of inquiry (θεωρίας ἕνεκα).

[70] In the so-called *Sectio Canonis,* the first nine propositions draw some conclusions in terms of ratios from the theory of proportions, and these are then translated into statements about intervals; cf. Fowler (1987) 147–8. This suggests a careful distinction between λόγοι and διαστήματα, despite Szabo's (1978) 99 ff. claim that musical intervals expressed as ratios between numbers were called διαστήματα (i.e. distances between two points) in the most ancient Pythagorean theory of music. But, since a διάστημα really had two end-points that were assigned numbers, Plato is close to the original meaning of the word. It is also relevant to note that this is the term reportedly used by Speusippus to cover all material principles; cf. Taran (1981) 313–14.

[71] Cf. *Harm. Elem.* II, 32: "Some of our predecessors introduced extraneous reasoning and, rejecting the senses as inaccurate fabricated mental principles, asserted that height and depth of pitch consist in certain numerical ratios and relative rates

Timaeus speaks (*Tim.* 36B6) in a very concrete fashion about some kind of soul stuff that the Demiurge splits lengthwise into two strips that constitute the circles of the Same and the Different. One must here imagine two such strips being laid crosswise and then being bent around to form two circles interlocking at oblique angles opposite the initial point of intersection.[72] The outer circle is designated as the characteristic movement of the Same, and it represents the celestial equator around which the whole cosmic sphere turns to the right; i.e. from East to West.[73] By contrast, the inner circle represents the movement of the Different as being to the left along the diagonal; i.e. the plane of the Zodiac which inclines to the plane of the celestial equator like the diagonal of a rectangle to its side. In astronomical terms the circle of the Different is the Ecliptic which touches the Tropics of Cancer and Capricorn.

According to Timaeus, the Demiurge gave supremacy to the uniform movement of the Same and left it single and undivided. This is consistent with the rational principles of order laid down so far, but it seems to leave unexplained the 'wandering' motion of the planets. The tradition that Plato set this as a leading task for the astronomers of the Academy fits with this quasi-Pythagorean attempt to save the phenomena of planetary motion in terms of concentric circles.[74] It would promote concord if these circles were in proportion according to harmonic intervals, just as the circle of the Different is divided into seven unequal circles according to each of the

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of vibration—a theory utterly extraneous to the subject and quite at variance with the phenomena." We shall see in my second chapter that Aristoxenus echoes Aristotle's general criticism of Plato.

[72] Timaeus explicitly compares this crosswise structure to the formation of the Greek letter X, which suggests (contrary to Guthrie's translation) that the angle of intersection is not a right angle but rather an acute angle (e.g. 24 degrees) such as Greek astronomers discovered between the plane of the ecliptic and that of the celestial equator; cf. van der Waerden (1954). Oenipides (early 5th century) is credited with calculating the value for the obliquity of the zodiac as being equal to the angle subtended by the side of a regular fifteen-sided polygon; cf. Proclus, *in Eucl.* 283 & Theo Smyrnaeus, *Exp. math.*, 148 & 198 Hiller.

[73] Dicks (1970) 121 claims that when Timaeus makes the circle of the Same revolve to the right, this is purely in deference to the Pythagorean tradition that the right is superior to the left. This seems correct given Plato's (*Tim.* 62D ff.) denial of absolute directions in the cosmos.

[74] Burkert (1962) 310–1 claims that Pythagoreans like Philolaus broke away from the prevailing view of Anaximander and Parmenides by grouping the planets with the sun and the moon rather than with the stars. Thus he argues that Plato is following this Pythagorean tradition when he talks about the seven circles of the zodiac associated with these heavenly bodies; cf. *Tim.* 36D, 38C–D. But placing the

intervals of the double and triple intervals. The real significance of his list of double (1, 2, 4, 8) and triple (1, 3, 9, 27) intervals, along with their respective harmonic and arithmetical means, becomes clear when each of the planets are linked with one of these numbers; though the intended mathematical ordering becomes perspicuous only when their relative speeds are made proportional to one another (*Tim.* 36D).

The motion of the outermost sphere is that of the fixed stars, and it corresponds to the movement of the Same; i.e. the revolution of the sphere as a whole in uniform motion from East to West, which involves every fixed star in the heavens. This revolution has supremacy in that it carries around with it all the contents of the sphere, including the planets, even though these have opposite motions of their own. Of course, the Earth at the center would also participate in the movement of the Same, unless this were counteracted by the Earth's unique motion.[75] Since self-movement is its primary characteristic, Timaeus attributes a soul to each one of the planets, as well as to the earth and to the universe as a whole. Considered as one motion of the World-Soul, the movement of the Same reflects the supremacy of Reason regulating all its other motions, including its judgments and desires.

From a physical point of view, the motion of the Different is distributed among the orbits of the seven planets, which share a motion that is contrary in direction to that of the fixed stars, and whose movement (of the Same) they also share by virtue of its supremacy. Thus, every planet has a composite motion of at least two different movements, without counting a possible motion due to its own unique soul.[76] From this one might construct an explanation for the complex motions of the planets in terms of compounded circular movements that are simple and uniform in themselves. For instance,

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earth at the center of the universe represents a major divergence from Philolaus who posited a central fire; cf. Aristotle, *De Caelo* II.13, 293a18 (DK 58B37) & Aetius II.7,7 (DK 44A16).

[75] From the available textual evidence (*Tim.* 40B–C) it is difficult to decide whether or not the Earth is being given a unique motion, and so there is a long-standing debate among commentators about the issue. This depends somewhat on whether we read εἰλλομένην or ἴλλομενην, which Aristotle takes (*Cael.* 293b30–3, & 296a26–7) to mean that the earth had its own rotation. Claghorn (1954) 71 ff. surveys the various views on the question and opts for that of Cornford which attributes two contrary motions to the earth such that they cancel each other and the earth is at rest relative to the cosmos.

[76] Timaeus describes the composite motion of the sun as a 'spiral twist' (στρέφουσα ἕλικα); cf. *Tim.* 39A6–B1. Vlastos (1975) 55–57 gives a convincing geometrical re-

Timaeus says later (38D) that the sun, Venus, and Mercury, move in circles of equivalent speed, even though the latter two have a 'power' contrary to that of the sun. Although this looks like an *ad hoc* device to explain why Venus and Mercury sometimes overtake and sometimes fall behind the sun, yet presumably the same kind of vital power could be attributed to the other planets in order to explain the phenomenon of retrogradation.[77] In the *Timaeus,* however, Plato leaves unexplained this third motion (besides the movements of the Same and of Difference) which contributes to the complex motion of the planets. But since they are divine living creatures, perhaps he assumes that it is the self-motion of the planets which enables them to counteract or reinforce the movements of the Same and the Different.[78]

This seems quite consistent with a narrative which tells of the visible universe being modelled after the eternal Living Creature, since the constructed cosmos will imitate the model better if each of its parts has soul as a principle of life. Thus, on completing his account of the fabrication of the World-Soul, Timaeus emphasizes again that it permeates the body of the cosmos from the center to its extremities. In contrast to the visible body of the cosmos, the soul is invisible and, since it partakes of reason and harmony, it is the best among all those intelligible and everlasting things which are generated.

Ontologically considered, the World-Soul is a compound blended from the natures of the Same and of the Different and of Being. In addition, it has been divided and bound together in due proportion and it revolves back upon itself (*Tim.* 37A5–6). Therefore, according to Timaeus, whenever the soul touches anything whose Being is dispersed (i.e. participating in Difference) or whose Being is undivided (i.e. having Sameness), she is set in motion and reports the many

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construction of this motion, showing that it can be explained as the combination of two rational circular motions. He admits, however, that this does not explain the 'wandering' motion of the planets, though he denies that Plato took the easy option of attributing it to their particular souls.

[77] Burkert (1962) 323–4 argues convincingly that this passage does not refer either to the theories of homocentric spheres or of epicycles, though he thinks that Plato later learned and accepted the Eudoxean theory; cf. *Laws* 897D ff.

[78] Against this, Tom Robinson (1970) & (1987) claims that the definition of soul found in the *Phaedrus* cannot be assumed to apply to the concept of soul in the *Timaeus*. Mohr (1985) accepts Robinson's argument and hence claims that the World-Soul in the *Timaeus* functions as a maintainer of order against the natural tendency of the corporeal to be chaotic. If this were the case, however, one would expect Aristotle to exploit the inconsistencies in Plato's account of the soul as an origin of motion.

ways in which something is the same or different with respect to something else. Here the epistemological principle of 'like knows like' fits neatly with the ontological structure of soul.

Furthermore, since soul has an intermediate status between the realms of Being and Becoming, it can become acquainted with entities belonging to both realms. For example, whenever the soul is in contact with the sensible or with 'the circle of the Different,' opinions and beliefs are generated which are firm and true. By contrast, while touching the rational or 'the circle of the Same,' the soul acquires rational understanding and knowledge. Opinion is quite different from rational knowledge because the sensible universe is an inexact copy of the eternal Living Creature that was used as a model by the Demiurge. On account of its intermediate place, however, the soul is able to make contact with eternal and intelligible Being as well as with sensible and generated Becoming. This is one of the major implications of the principle of correlation between ontology and epistemology.

II.7. *Time, the moving image of eternity*

Having delineated the mathematical structures of the World-Body and of the World-Soul, Timaeus discusses (37C–D) the relationship between time and the visible motions of the heavenly bodies. According to his account, their motions provide a mathematical measure of time based on the stable structure of the cosmos. Since the Demiurge could not make the generated cosmos an exact copy of the eternal model, as a second-best option he made an everlasting image moving according to number which is called time (*Tim.* 37D5–9). Therefore time derives its everlasting duration from the gods and the original Living Being, even though their strict eternality places them outside time.[79]

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[79] Some commentators, such as Cornford (1937), Hintikka (1973), von Leyden (1935), and Whittaker (1968) think that Platonic Forms are (merely) everlasting, while others, like Cherniss (1957b) and Vlastos (1975) suppose that Forms are timelessly eternal or outside time, in some sense. Owen (1953) finds Plato wavering between these two views but I think that Mohr (1985) is right to see this as being due to the fact that the immanent standards of the early dialogues must share in the permanence of the Forms that they imitate closely. Prior (1985) argues convincingly that Forms and phenomena are sharply separated in the *Timaeus*, so that self-predication is ruled out for the Forms, while phenomenal things merely cling to Being through participation and by virtue of being 'in' the Receptacle.

From the warnings voiced through Timaeus (37E–38B) it is clear that Plato wanted to forestall careless speech about eternal entities such as Forms and mathematical objects. Since these are outside time, they must not be spoken about in the past or future tenses, nor should one attribute motions or changes to them like sensible objects. Perhaps these remarks are directed at those mathematicians who spoke loosely about objects undergoing changes or who used many instruments and proofs that seem to involve motion for mathematical objects. Proclus reports (*in Eucl.* 77–78) a debate on this issue within the Academy, which is also relevant to Aristotle's criticism of Plato's views on the status of mathematical objects.

Subsequently (38C–39E) Timaeus claims that, in order for time to be brought into being, the sun and the Moon and the five planets must have already been generated as a determination and guard for the numbers of time.[80] Although all the planets measure time by moving in their respective orbits, the sun functions as a 'conspicuous measure' of the relative speed or slowness with which they move. Hence the orbit of the sun provides mankind with a basic measure of time so that, as a result, he should learn to count and thereby develop mathematics through observing the daily motion of the sun which dominates the Heavens with its brightness.[81] Although the Moon is less conspicuous, it also enables mankind to count the months by noting the completion of its orbits. But very few men have discovered the revolutions of the other planets and so there are no common names for their orbital periods, since these are not usually computed and compared.

Timaeus claims, however, that the 'complete number of Time' would be the Great World-Year or, literally, 'the complete year'[82] when all eight orbits of the planets are lined up opposite one point on the revolution of the Same, in spite of their relative speeds. In

[80] Mohr (1985) 58 takes this statement to mean that the Demiurge uses the motions of these heavenly bodies to make clocks that measure temporal succession. He suggests that Plato has a rather technical sense of time in mind here; i.e. a sensible and immanent standard that is embodied as a clock.

[81] Cf. *Tim.* 39B–C & 47A–B. Such statements confirm my general claim that cosmology serves as the proper background for understanding the views of Plato on the foundations of mathematics.

[82] The interest in calculating the length of the so-called 'Great Year' was stimulated by the need for calendar reform which occupied the attention of Greek astronomers. Oenipides, for instance, is reported to have calculated its length as 59 years; cf. Dicks (1970) 88–9.

this way, one can conceive of a Great Year being completed when all the planets return simultaneously to their original starting-point. In the *Republic* (546B ff.), Plato calculates it at 36,000 years but his figure was not based on empirical observation so much as on mathematical considerations.[83] Such a calculation is made in the context of Plato's discussion of the so-called 'nuptial number' that should dictate eugenics within the ideal city. The whole discussion provides us with a characteristic example of Plato's attempt to interweave human affairs with the world-order that he finds in Pythagorean number theory.

III. *Reason and necessity*

At 40D Timaeus declares that he has given a sufficient account of the nature of the visible and generated gods; i.e. the heavenly bodies and their motions. In an ironic manner he accepts the accounts of the origin of the other (hidden) divinities given by the poets and theologians, even though their statements are neither probable nor necessary demonstrations. Thus his summary of mythological stories about Cronos and Zeus may be read as a subtle criticism of all such accounts, by contrast with his likely account which also uses similar modes of narrative. For example, the speech (41A ff.) of the Demiurge to the lesser gods is meant to give a plausible account of phenomena like the obvious difference between heavenly and sublunary bodies with respect to generation and corruption.

This can be shown by examining some major points in the fictional speech of the Demiurge. For instance, the declaration that the generated gods cannot be destroyed except by an act of his will seems to be a mythical explanation for the apparent eternality of the heavenly bodies. Secondly, the generated gods are charged with the task of generating three mortal kinds (creatures of air, water, and earth), so that the whole heaven may be perfected. This appears to conflict with what Timaeus (30C–D) said earlier about the Demiurge being the constructor of the cosmos as a perfect living creature, which contains within itself all the living creatures that are by nature akin

[83] In his classic discussion of Plato's 'nuptial number,' James Adam (1891) connects it with the reciprocal periods of incubation and duration of the cosmos, each of which lasts 36,000 years.

to itself. But the final qualification there only commits the Demiurge to generating living beings that are incorruptible in the same way as the cosmos itself. This is confirmed when he explains that he cannot be responsible for generating mortal kinds, otherwise they would be immortal like the lesser gods. However, the Demiurge does consent (41C) to produce the immortal and divine part of mortals, and to deliver it to the lesser gods for embodiment in what they have produced. This looks like Plato's attempt to give a plausible mythical account of the immortality of the human soul for which he argued more dialectically in the *Phaedo.*

According to Timaeus' mythical narrative (42E), the lesser gods obey the Demiurge and set about their task of making this mortal living creature (man) in imitation of the World Animal. For that purpose they borrow portions of the four elements and bind them together, though not with the sort of indissoluble bonds that the Demiurge used in producing the heavenly bodies. They bind the revolutions of the immortal soul (handed over by the Demiurge) within such physical bodies, which are typified by a number of different 'rivers' like nutrition and sensation. It is the fate of the soul to be thus bound up with a body whose 'floods' it cannot completely control, though it is never drowned in them. For instance, sensation is described as the tumult produced within each mortal creature as a result of the collision between its body and external bodies like fire, earth, and water.

Timaeus conjectures that these jostling sensations interfere with the revolutions of the soul produced by the Demiurge according to the movements of the Same and the Other. The result is the kind of irrational motion in every direction that we observe in children and in people driven by passion. In our ordinary sensations these disturbances show themselves in perceptual relativity and in such typical perceptual confusions as between left and right, small and great. However, Timaeus (44B–C) does hold out the hope that the soul's movements (the Same and the Other) can gain control over the irrational motions belonging to the body, and that such a rational state can be secured by the right educational nurture.

But it is more important for my purposes here to examine the program which Timaeus (44C–D) rather formally lays out for his subsequent discussion. For example, postponing discussion of the moral virtues, he promises to give a more exact account of the condition of the soul in the body, which is his present topic. On the other hand,

with respect to the prior topics concerning the generation of both the soul and the body, he adheres to the most likely account. The antithetical structure of the Greek sentence seems to suggest a contrast between two distinct projects in terms of different degrees of precision to be expected of them.

Timaeus continues his likely account by describing (44D) how the lesser gods constructed the human body. Accepting the human intellect as a product of the Demiurge, they bound it within a spherical body like that of the cosmos, which is appropriately called the head because it is the most divine and ruling part of the human body. He offers (44E) a somewhat comical teleological explanation to the effect that the gods connected the rest of the body to the head as its servant, in order that (ἵνα) it should not go rolling over the uneven surface of the earth.[84] Since the body was made by divine contrivance as a 'chariot' for the head, it sprouted four limbs as instruments of transport and so could travel through most places, bearing aloft the chamber of our most divine and holy part. Similarly, the superiority in dignity of forward motion is invoked to explain why the limbs are attached to the body in such a way as to facilitate this type of locomotion.

The implicit contrast here between explanation in terms of final causes and in terms of auxiliary causes can be developed further by considering Timaeus' subsequent account (45B) of the construction of the 'light-bearing' (φωσφόρα) eyes. According to his mythical account, the gods contrived a body from the kind of non-burning fire that gives off a mild light (φῶς ἥμερον), akin to the light of day (ἡμέρας). Here both linguistic similarity, and the principle of like-to-like, dictate that the material of the eye be the same as the medium in which it sees. Behind this account also lies some peculiar assumptions about the working of the eye, which may have been introduced to explain such visual activities as focusing. One of these assumptions is that the eye itself emits a stream of pure fire into the light of day, with the result that they coalesce to form a visual ray which extends from the eye to the object seen. Although the details of this theory of vision are unclear, it seems that visual rays are conceived of as homogeneous bodies that convey the motions of the external

[84] Taylor (1928) 275 notes that Timaeus is following Alcmaeon in treating the brain as the center of the sensori-motor system for the human body. In remarking that 'Timaeus' is having some fun at the expense of Empedocles, Taylor seems to have forgotten that such subtle irony is the work of Plato the writer.

objects back to the eye and into the soul. Thus, when night falls and the kindred fire vanishes, the inner fire goes forth into what is dissimilar and becomes quenched because it no longer shares the same nature with the adjoining air. This strange theory (which seems to have been adopted from Empedocles) is criticised by Aristotle (*Sens.* 437b11 ff.) on the grounds that, if it were correct, visibility should be very bad on cold and wet days.

But here I am less concerned with how well this theory saves the phenomena than with its status as an explanation. At *Timaeus* 46C–D such physiological explanations are identified as 'auxiliary causes' which the god uses as assistants in reaching, as far as possible, the form of the best. The language there clearly indicates the inferior status of these mechanical causes in relation to the teleological causes that guide the productive activity of the Demiurge and the lesser gods. The point is also made through the criticism of the *physiologoi* that Plato puts in the mouth of Timaeus. Instead of treating them as auxiliary causes, these natural philosophers have proposed as the primary causes of all things such processes as cooling and heating, solidifying and dissolving, and so on. But, Timaeus objects, these causes cannot involve rational or purposive planning because that is a unique property of invisible soul, and so cannot belong to visible bodies like the four elements. This objection implies that one must pursue first the sort of reasons that belong to an intelligent nature, and secondly those causes belonging to the kind of things that are moved by others and move others out of necessity (ἐξ ἀνάγκης).

In contrast with the *Phaedo,* it is noteworthy that here Plato accepts both as types of causes, even though he distinguishes those intelligent causes which are producers of things fair and good from those causes which lack intelligence and so always produce accidental and irregular effects. This distinction between causes is subsequently (46E–47A) illustrated in terms of seeing and light. Timaeus says that his previous discussion of the material structure of the eye dealt only with the auxiliary causes (ξυμμεταίτια) that help it to acquire the capacity for vision, so he must still clarify the most beneficial function for the sake of which the god bestowed eyes on human beings. According to his account, vision is the cause of the greatest benefit to us; namely, that none of the present cosmological accounts would ever have been given if men had not seen the heavenly bodies. In fact, he repeats his claim that the vision of night and day, and of months and years, has produced the art of number and thereby

has given mankind not only the notion of time but also the means of inquiry into the nature of the universe. As a result, mankind has procured philosophy as the greatest gift that has ever been bestowed by the divine, and it is this that Timaeus calls (47B) the greatest benefit of eyesight.

Finally, Timaeus closes the circle of explanation by bringing the microcosm (mankind) into correlation with the macrocosm (the whole cosmos). In his account (46B), the ultimate purpose of philosophy is clarified by the myth of the god bestowing vision on us so that (ἵνα) we might use the heavenly revolutions of Reason as guides for the revolutions of reasoning within us. Although these latter revolutions are perturbable, they are akin to the imperturbable revolutions of the heavenly bodies and so, sharing in true calculations, we may be able to stabilize the varying revolutions within us by imitating the absolutely unvarying revolutions of the god.

Undoubtedly, Plato intended this myth to be taken quite seriously, since it is the cosmological basis for the education of philosopher-rulers as envisaged in the *Republic*. As evidence for his seriousness, one should note that he has Timaeus repeat (47C) the same myth for the human voice and hearing; i.e. that they were bestowed by the gods for the same purpose (ἕνεκα) of developing human rationality. He claims that speech (λόγος) has made the greatest contribution to this goal, presumably because it facilitated dialectical conversation as the medium for philosophy. In addition, music was bestowed on mankind by the gods for the sake of harmony (ἕνεκα ἁρμονίας). But harmony also involves motions akin to those within the human soul, and so it is achieved only by those who make intelligent use of music for ordering the soul, and not for irrational pleasure. All of this is consistent with Plato's stipulations in the *Republic* about the use of music for the development of rationality in his ideal polis. However, it also implies that there is an irrational element both in mankind and in the cosmos itself.

III.1. *Things generated by necessity*

Just as in the *Republic* Plato recognizes an irrational element in the soul, so also in the *Timaeus* he acknowledges the presence in the cosmos of an erratic cause called 'Necessity' (ἀνάγκη), which is pictured as a submissive female being mastered by a dominant male through persuasion. The purpose of rational persuasion is to lead

every generated thing towards what is best, though Timaeus admits that such persuasion only works in the majority of cases. Therefore, to explain the refractory cases in a mathematically ordered cosmos, he now (48A) introduces the so-called 'wandering' (πλανωμένη) cause as a new principle.[85]

His explicit project (*Tim.* 48B3) is to obtain a vision of the true nature of the four elements and their properties, just as they were *before* the generation of the Heavens. Such an inquiry is held to be necessary because no one has so far explained the generation of these elements, even though everyone calls such things principles (ἀρχαί) and treats them as the elements (στοιχεῖα) of the cosmos.[86] Timaeus criticizes rather sharply those people who engage in the shallow word-play of comparing these elements to the letters of the alphabet (στοιχεῖα) which are compounded in forming syllables and words in Greek. It is possible that either Leucippus or Democritus used such a comparison to illustrate how a great variety of things can be generated from infinitely various combinations of atoms.[87] In any case, Timaeus complains about the inadequacy of the comparison, though he does not offer an alternative account of true first principles, since that would be too difficult a task to undertake with his present mode of exposition.[88] Instead he proposes to continue with his chosen method

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[85] πάλιν ἀρκτέον ἀπ' ἀρχῆς, *Tim.* 48B3. Cf. Ast (1835) and Brandwood (1976) for Platonic usage of ἀρχή.

[86] According to Aristotle, the Atomists used the term στοιχεῖα for the primary and atomic constituents of matter in order to draw the analogy with the manner in which combinations of letters can generate an infinity of words and sentences; cf. *Met.* 985b13–19. There seems to be no evidence that Empedocles compared his 'roots' with the letters of the alphabet but perhaps that became a popular analogy among his followers. Burkert (1959) 178 has shown that the analogy between letters of the alphabet and elements of a system was used by 5th century Sophists to justify their new methods of grammatical analysis, and even Democritus is credited with having written something on the subject. Perhaps he used grammatical analysis as a familiar parallel for his own unfamiliar analysis of the sensible world into atoms (letters) and void (spaces between them).

[87] Cf. DK 67A6, A14, 68A37; and H. Diels (1889) 13. Burkert (1959) has argued that στοιχεῖον cannot mean 'letter,' though these were sometimes used as synonyms because the latter exemplifies the original meaning of στοιχεῖον; namely, a fundamental part of some system, as in grammar or mathematics; cf. *Theaet.* 203B ff., *Rep.* 402A ff., & *Phil.* 18C7.

[88] In effect, what he suggests is that the method of giving a likely account (which is needed for dealing with sensible things) is unsuitable for dealing with absolute first principles. The methodological self-consciousness of this whole section (*Tim.* 47–68) of the dialogue is noteworthy, and it seems to hint at some 'unwritten doctrine' that might use more ultimate principles of explanation like the One and the Indefinite Dyad, which correspond with Limit and Unlimited at *Philebus* 23C ff.

of giving a likely account, which he insists will be no less plausible than other accounts concerning particular things and the whole world.

III.2. *The Receptacle*

Timaeus also acknowledges (48E) the need for additional distinctions to deal with the things that come to be of Necessity. Prior to this he was able to get by with two kinds of entity; i.e. a Form that is intelligible and always unchangingly real, and a copy which is both visible and subject to generation. But now the argument requires the description of 'a difficult and obscure form,' which Timaeus first (49A6) describes as a receptacle, just as if it were the nurse of all Becoming.[89] While he accepts (49B1–3) the need to clarify this description, there is the prior problem of making a clear distinction between the four elements. As Timaeus formulates it (49B3–5), the question is about what kind is to be called water rather than fire or some other name.[90] In other words, if the sensible phenomena are involved in a continual cycle of transformation, what is the basis for using such language in a trustworthy and sound manner?[91]

These questions seem to be directed at *physiologoi* like Empedocles who used the four elements as explanatory factors in their cosmologies, and so what follows may be taken as a facsimile of their explanations, illustrated in terms of the element usually called 'water.' It seems to us (ὡς δοκοῦμεν) that, when it is compacted, we see it becoming stone and earth.[92] On the other hand, when it is dissolved and dispersed, this same thing becomes wind and air, while through

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[89] Solmsen (1966) 130 thinks that Plato is reviving the 'archaic' dual conception of Hesiod when he describes the Receptacle in semi-mythical form as a 'seat' (ἕδρα) and a mother. The simile of the nurse captures the dual function of the Receptacle both as a place for generation and a source of nourishing material.

[90] Richard Mohr (1985) 85 ff. argues convincingly that the problem here is not about individuation but rather about identifying kinds, given that the sensible phenomena are taken to be in flux.

[91] οὕτως ὥστε τινὶ πιστῷ καὶ βεβαίῳ χρήσασθαι λόγῳ, *Tim.* 49B6–7. This passage shows that Plato is still following the method embodied in the 'second sailing' of the *Phaedo;* i.e. seeking knowledge of things through language rather than through the empirical analogies used by the *physiologoi.* Indeed, the *Timaeus* as a whole is greatly concerned with saving the phenomena of language; cf. esp. 58C–68D. There is a strong parallel with a passage in the *Cratylus* (438C–439E) where contradictions arising from the application of names to the phenomenal world are said to lead the inquirer beyond the sensible flux to the immutable objects of true signification.

[92] As Vlastos (1975) 80 points out, it is crucial for the proper interpretation of this passage that we take ὡς δοκοῦμεν as governing the whole physiological descrip-

combustion air becomes fire. But the reverse process was also thought possible by the *physiologoi*. For example, when contracted and quenched, fire returns once more to the form of air; while again air becomes cloud and mist when it is united and condensed. But, when these are further compressed, water flows out of them and again from water comes stones and earth. Thus it would appear (ὡς φαίνεται) that there is a circle of birth among the elements which leads to the following problem: if none of them ever remains the same in appearance, how can one give it the same name?

Timaeus accepts that it is indeed impossible to give the same name to elements which are changing in this way, though he provides another linguistic means of dealing with them. Whenever we perceive something like fire to be continually changing into another state, he suggests that we should never describe it as 'this' (τοῦτο) but as 'suchlike' (τοιοῦτον).[93] An implication of referring to something by means of a term like 'this' (τόδε) or 'that' (τοῦτο) is that one is assuming something definite (τι) as a referent. But sensible elements change and so cannot serve as stable referents for names like 'this' and 'that,' which would designate them as beings in themselves. Hence, by the criterion of permanence, sensible elements are posterior in being to Forms and the Receptacle.

Some commentators have compared this whole passage with Aristotle's *Categories* (3b1 ff.) where the distinction between substance and quality is based largely on the fact that the former provides a permanent subject of predication for the latter.[94] Even if the similarity in language is accidental, it still suggests that Plato made a corresponding distinction between substance and quality with reference

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tion that follows, just as if it were in quotation marks. This means that the description is of what is given on the level of superficial appearance, which will later be contrasted with what is real and intelligible.

[93] This provides some grammatical evidence for Mohr's interpretation, since τοιοῦτος is a demonstrative pronoun correlative with the interrogative ποῖος; both of which suggest that the question is about identifying kinds rather than about individuating particulars.

[94] Driscoll (1979) claims that at least five of the properties ascribed to primary substance in the *Categories* also belong to the Receptacle postulated by Plato. On the other hand, Mohr sides with Zeyl (1975) in his claim (contra Cornford) that τοιοῦτον can denote properties under any category. But, as Prior (1985) 126n29 notes, Aristotle calls individuals in the world of Becoming 'thises,' whereas Plato would call them 'suches.' Yet the nearest Aristotelian correlate to the Receptacle is matter, which is not strictly a 'this' for Aristotle unlike space for Plato, which he treats as a pure referent.

to their degrees of permanence and reality. The traditional four elements are designated as 'suchlike' things because they are continually whirled around in a circle of generation and corruption. In fact, according to Timaeus (*Tim.* 49E–50A), the only thing legitimately designated as 'that' or 'this' is that 'in which' (ἐν ᾧ) they appear when generated, and again that 'from which' (ἐκεῖθεν) they disappear when they perish. To illustrate these contrary qualities, Timaeus selects (50A1–6) 'hot' or 'white' as examples of 'suchlike' thing that cannot be designated as 'this' or 'that.'

By positing an entity on which these qualities depend, Plato may be opposing previous *physiologoi* like Anaximenes (DK 13B1) and Anaxagoras (DK B8) who treated 'the hot' and 'the cold' as independent though correlative powers (δύναμεις) that are displayed through action on other things. But yet one cannot treat the Receptacle merely as a material substratum like Aristotle's matter, since some of their defining characteristics are quite different. For instance, matter is that out of which the more complex material substances are worked up through the action of appropriate forms; whereas the Receptacle is always that in which sensible and material characteristics make their appearance. Just like the Forms which these characteristics imitate, the Receptacle itself has permanent being and its nature does not change, yet it lacks their intelligibility. Later, Timaeus gives an account of the elemental bodies in terms of their mathematical structure, so perhaps the place in which they appear can be made intelligible by the same means.[95]

Timaeus begins (50A6) to clarify this mysterious entity by drawing an analogy between the Receptacle and a mass of pliable material like gold which can be shaped into different figures. He asks his audience to imagine someone first moulding all sorts of figures out of gold, and then remoulding each of these figures into every other one. Presumably, this functions as an imaginary analogy for the continual cycle of generation and corruption of the traditional elements, which he has just explained as qualitative change within the Receptacle. Given the hypothetical situation of the gold continually changing shape, if one were to point to one of these figures and ask what it is (τί ποτ' ἔστι) then the safest answer with respect to truth

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[95] One possible implication is that for Plato space is three-dimensional, by contrast with Aristotle's two-dimensional concept of place, which was later criticised by Philoponus; cf. Furley (1991).

would be to say that it is gold. Although the question 'What is it?' begs an answer in terms of real being, it would be misleading to describe the triangular shape or any of the other shapes of gold as beings (ὡς ὄντα) because they are changing even while one is describing them, and so lack the permanence that belongs to true being.[96] Thus, as with the elements, we must be content if we can safely apply the term 'suchlike' to figures which are continually changing.[97]

In contrast, the same account and the same name must always be given for the Receptacle because it never diverges from its own capacity, which is to be always receiving things but never to take on the shape (μορφή) of anything entering it.[98] By nature it is a moulding-stuff (ἐκμαγεῖον) that is changed and transfigured by the things entering into it, so that it appears different at different times (*Tim.* 50B–C). Therefore the relevant point of the gold-analogy is that the Receptacle is an indeterminate though permanent medium, which is perfectly suited for receiving the images of the transcendent Forms. But the analogy throws no light on the wondrous (θαυμαστόν) way in which these images are stamped on that medium, although an account of this almost indescribable (δύσφραστον) process is promised (50C6–8).[99]

In summary (*Tim.* 50C–D), therefore, the 'likely account' involves three distinct kinds of thing: (i) that which is generated (τὸ γιγνόμενον); (ii) that in which things come to be (τὸ ἐν ᾧ γίγνεται); (iii) that from which (τὸ ὅθεν) the generated things are copied. Timaeus suggests

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[96] E.N. Lee (1971) has argued that Cherniss (1954) fails to establish an exact parallel between this passage and 49D–E, which prescribes that sensible changing elements should not be referred to as 'thises' but as 'suches.' Despite the strong parallels both in language and content, he is right to insist that the question is different here (50B), though his own reconstruction is not convincing.

[97] Mohr (1985) 99–107 suggests that Plato distinguishes between phenomena in continual flux, of which nothing can be reliably predicated, and phenomena as images of Forms, of which some quality (τοιοῦτον) can be predicated with some reliability.

[98] This statement seems to undermine any interpretation of the five perfect solids as intrinsic structures of space rather than of the stereometric bodies that occupy space. Plato's intuition seems to have been that space is completely amorphous, since it can accomodate bodies of indefinitely many shapes, even though he does attach great importance to the discovery by Theaetetus that there are only five possible ways in which it can be completely filled by regular figures. Indeed, I think it was precisely this discovery which led him to think that a constant pressure to eliminate the void goes along with the natural tendency of the elemental bodies to seek their own kind in a particular place in the universe; cf. Cleary (1995).

[99] Perhaps this is another oblique reference to the content of the 'unwritten doctrine.' According to Aristotle's brief report, the Forms and the Dyad of the Great and Small are conjointly the causes of sensible things; cf. *Metaphysics* I.6.

that these items might be compared to the three elements of human procreation.[100] While the Recipient in which things are generated may be likened to the mother, the source of generation is analogous to the father. In completing the analogy, Timaeus likens the offspring to whatever in the universe is naturally generated. According to the prevaling Greek theory of genetics, the male was the sole cause of generation by planting seed in the female, who merely provided the place for the nourishment of the infant. Some such theory lies behind the description of the Receptacle as the 'nurse' of generation, which provides the 'womb' for what is generated.

Displaying almost exaggerated caution, Timaeus says (51B1) that 'we will not be lying' if we describe the Receptacle as some invisible form which is unformed (ἄμορφον) and all-receptive (πανδεχές), and which is somehow intelligible but in a most perplexing and baffling way. In fact, he says (51B6–7) that it would be safer to say that the part of the Receptacle which has been made fiery appears as fire and, similarly, as water or as air or as earth, insofar as it receives copies of these. But, with respect to these so-called elements, Timaeus accepts the need for a more exact determination of their character by means of argument (λόγῳ). From what follows it appears that he is talking about some dialectical mode of argument having greater precision than the likely account that has so far been given.[101]

Since such a dialectical argument belongs to a different inquiry, however, Timaeus avoids a long digression by drawing (51D–E) a major distinction between the realms of Reason and True Opinion. His complex hypothesis combines both positive and negative aspects as follows: (i) If Reason and True Opinion are two different kinds then there exist self-subsisting Forms, imperceptible to us and intel-

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[100] In fact, sexual and biological metaphors for the 'generation' of the cosmos are used very frequently throughout the *Timaeus;* cf. 28C, 48A, 49A, 51A.

[101] This conjecture is partly confirmed by the dialectical character of the questions which Timaeus next (*Tim.* 51B–C) introduces: Is there such a thing as 'fire just in itself' or any of the other things which we are always calling 'things just in themselves'? Are we always talking idly when we say that there is some intelligible form (εἶδος νοητόν) of each thing and is this nothing more than a word? This set of questions is paralleled in the *Parmenides* dialogue (cf. 130D ff.) where Parmenides questions Socrates about the existence and range of the world of Forms over against the sensible world. With regard to chronology, I follow the scholarly consensus about the *Parmenides* being earlier than the *Timaeus,* and I accept William Prior's (1985) claim that the latter defends the theory of Forms against difficulties raised in the former. Most recently, this consensus has found support in Ledger's (1989) stylistic analysis, which concludes that the *Timaeus* is one of Plato's last dialogues.

ligible only; (ii) If, on the other hand, Reason and True Opinion do not differ in any way at all, then such things as we perceive through the body would have to be posited as the most stable realities. Since the latter conclusion is unacceptable, Timaeus affirms (*Tim.* 51E1–2) that reason and opinion are two different things because they have been generated separately and are unlike in condition. With respect to their origins, they differ because reason has been generated in us through teaching; whereas opinion arises through mere persuasion. Furthermore, they differ in reliability because reason can give a true account that cannot be shaken by persuasion; whereas opinion lacks such an account and can be shaken.

In the light of this distinction between Reason and Opinion, Timaeus can now characterize Form, Copy, and Receptacle as follows. Form is an ungenerated and indestructible object of reasoning, which neither receives anything else into itself nor goes into any other thing (*Tim.* 52A2–3). According to this account, what enter and leave the Receptacle are copies of the Form which bear the same name and are like to it. In contrast to the Form, a copy is a sensible and generated thing which is grasped by Opinion with the aid of perception. Furthermore, this second kind of entity is always in transition; at one time being generated in a certain place and at another time perishing from there (*Tim.* 52A–B).

In contrast to such generable and destructible things, the space (χώρα)[102] in which they appear is everlasting (ἀεί) and does not admit of destruction. Thus it is a third kind of entity that is barely an object of belief, since it is grasped without the senses by a sort of 'bastard reasoning.'[103] Timaeus explains (52B) that we look towards this entity in a dream state when we insist that nothing exists unless it is in some place and occupies some room, whether on earth or in the heavens. Since the Forms would be reduced to nothingness by this criterion of existence, the dream state might be that of such people as Zeno (DK 29A24), who assume that everything which

[102] Heidegger (1959) 65–66 considers it to be no accident that the Greeks had no word for 'space,' since they experienced the spatial not as extension but as χώρα, which signifies neither place nor space but that which is occupied by what stands there. Thus the place belongs to the thing itself, and each thing has its own place; so whatever is generated is placed in this local 'space' and emerges from it.

[103] Perhaps there is a parallel with Aristotle's view that matter can only be known by analogy since it is grasped by stripping away all categorial forms; cf. *Met.* 1029a11–25. Plotinus (*Enn.* II.4.10 & 12) also thinks that Plato is talking about how matter as an indefinite thing can only be grasped by negative abstraction.

exists is in some place.[104] Plato's use of the dream metaphor in the *Republic,* for instance, suggests that it may be read here as an ironical comment on those thinkers who must put everything in some place before they will acknowledge its existence.

But we have no guidance as to how we should understand the reference to 'bastard reasoning,' which is said to be the means by which space is known.[105] Perhaps Plato means that this type of cognition is an illegitimate combination of reason and opinion, which is somehow appropriate for grasping the Receptacle as something that is neither purely sensible nor fully intelligible. While it must be more akin to reason because its proper object is both everlasting and indestructible, yet the Receptacle and the Forms can be clearly distinguished by means of what Timaeus calls 'the accurately true argument' (52C–D), which argues that when one real entity is distinct from another, neither of the two can ever come to exist in the other because that would mean that the same thing would be both one and two simultaneously. The point seems to be that, in contrast to the Forms, the Receptacle is both one and two, since it receives into itself the images of the Forms.[106] Given the historical context of a Parmenidean challenge to Presocratic cosmologists, I suggest that the Receptacle is Plato's alternative to the Atomist void, as a plenum in which change takes place without absolute non-being.[107]

III.3. *Generation in the Receptacle*

Timaeus has merely given the ontological preconditions for a visible universe when he lists Form, Copy and Receptacle as three distinct

[104] Another possible target is Archytas of Tarentum, who reportedly made place (τόπος) prior to body according to the criterion of non-reciprocal dependence; i.e. place is distinct from body and exists in itself independently of all other entities. Since place imposes a limit on bodies, it is judged to be prior or more honourable. Thus, for instance, Archytas claims that the place of the whole cosmos is the limit of all beings (ὁ γὰρ τῷ πάντος κοσμῷ τόπος πέρας ἀπάντων τῶν ὄντων ἐστιν).

[105] Pierre Duhem (1913) i, 45 suggests that Plato's treatment of the Receptacle may have been influenced by the doctrines of Pythagoreans like Archytas and Philolaus. So, for instance, the so-called 'bastard reasoning' might be understood as the mixture of intelligible and sensible abstraction which they associated with mathematics.

[106] Hermodorus (apud Simplicius, *in Phys.* 247.30 ff.) suggests that for Plato matter as exemplified by the Receptacle is a paradigm case of the Indefinite Dyad.

[107] Duhem (1913) i, 36 regards the Receptacle as a composite notion, which combines Archytas' notion of place with the Atomist notion of the void. David Sedley

kinds of entity that existed 'before the Heaven was generated.' If Plato held the cosmos to be eternal, such talk about generation must be taken as metaphorical or as an instructional device analogous to 'generation' through construction in geometry.[108] Taken literally, however, the claim is that *before* it was taken over by the Demiurge the Receptacle was in a state of utter chaos and exhibited every variety of appearance from being liquified and ignited and also from receiving the shapes of earth and of air, along with the affections that go with these (*Tim.* 52D5–7).[109] The Receptacle is unevenly balanced because it is filled with dissimilar powers (δυνάμεις) that induce a swaying motion like that of a winnowing basket. The point of this simile is that the shaking of the Receptacle causes the separation of unlike kinds of quality, just as the wheat and the chaff are separated because of the difference in their weights. Indeed the whole description is reminiscent of the typical accounts of generation given by natural philosophers in terms of the motion of a vortex that separates things on the principle of like to like.[110]

In this way, he gives a preliminary account of why 'the four kinds' tend to be separated off into different regions of the universe. Even though the four elements possessed some traces (ἴχνη) of their true nature, they were in the sort of chaotic condition that one might expect in the absence of divine intelligence (*Tim.* 53B2–4). This may well be a critical remark directed at those thinkers who postulated

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(1982) 188 has suggested that there is some similarity between Plato's depiction of space in the *Timaeus,* and the Epicurean notion of intangible extension which is called 'void, 'place' and 'room' within different contexts; e.g. 'void' when it is not occupied, 'place' when it is occupied, and 'room' when bodies move through it without resistance.

[108] Following Crantor, this seems to have been the dominant interpretation of such statements among subsequent generations of Academicians. By contrast, many modern commentators (e.g. Cherniss, Vlastos, Robinson, & Mohr) have given strong arguments for taking literally the talk about the generation of order within a pre-existing chaos.

[109] It seems from these descriptions of the pre-cosmic chaos that the Receptacle is reflecting the images of Forms in a disorderly and phantasmagoric fashion. If these 'traces' (ἴχνη) of the four elements are already perceptible as disordered, this could mean that their subsequent ordering by the Demiurge does not involve a reduction to imperceptible triangles, but rather their clear determination into definite things with characteristic perceptible qualities. I am indebted to Myles Burnyeat for this suggestion.

[110] Both Empedocles (Fr. 360KR) and Democritus (Fr. 563–65KR) are credited with positing the vortex to explain the separation of basic elements in like to like fashion, but it is not clear whether Anaxagoras accepted the same mechanism together with the principle of like to like.

the random and mechanical interaction of basic elements, whether in a vortex or in a void.

All of these preliminary remarks prepare the way for a new account of the ordered generation of each of the elements taken over in a disordered state. Timaeus warns (*Tim.* 53B–C) his audience that the account will be unfamiliar, unless one is already acquainted with 'the ways of learning' which are necessary for his explanations. Subsequently, it becomes clear that these ways of learning are none other than plane and solid geometry which had made such progress in Plato's time in the hands of experts like Theaetetus and Eudoxus.[111]

From the assumption that fire, earth, water, and air are bodies whose form involves depth, Timaeus argues (*Tim.* 53C4) that in every case depth is necessarily bounded by surface, and that every rectilinear surface is composed of triangles. Taken concretely, the claim seems to be that triangles are prior in material composition to solid bodies because their limiting surfaces must be composed of a number of triangles. Although one cannot decompose the triangle into more elementary shapes in Euclidean geometry, one might also think of the line and the point as being prior in a logical sense to the triangle. When this sense of priority is applied to the composition of a solid, the bounding surface can be seen as prior because the notion of a solid presupposes some boundary.[112] In another way, the rectilinear surface of a regular solid itself presupposes the elementary triangles out of which it is composed because the area of any plane surface can be calculated as the sum of such triangles. Thus Timaeus seems to be claiming that triangles are logically and materially prior to surfaces and to solids. Aristotle reports (*Met.* V, 1017b17–20, 1019a1–4) that Plato himself attached some ontological significance to priority, and it becomes one of the major points of difference between them with respect to mathematical objects. So let us note

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[111] Theaetetus is usually credited with developing the theory of irrationals found in Euclid's *Elements* X, and with completing the construction of the five perfect solids that is contained in book XIII; cf. Heath (1925iii). Perhaps it was such mathematical activity within the Platonic circle that gave some historical plausibility to the anecdote about an inscription over the gates of the Academy to the effect: "Let no one who is ignorant of geometry enter here" (ἀγεωμέτρητος μήδεις εἴσιτω); cf. *in Ar. Gr.* XV, 117.29; XVIII, 118.18. But Fowler (1987) doubts the historical authenticity of this anecdotal tradition, which Riginos (1976) 138–40 cannot trace back any further than the 4th century A.D.

[112] The schema of point, line, plane, and solid, seems to have been used within the Academy as a paradigm for priority relations that are dictated by the criterion of non-reciprocal dependence; cf. Aristotle Fr. 28 = Alex. Aphr., *in Metaph.* 55.22 ff.

some ontological implications of this brief argument in the *Timaeus*.

One implication may be that, if priority is a criterion of reality, the elementary triangles will turn out to be more real than the solids which are obvious to sense perception. According to the same criterion, however, it also follows that such triangles cannot be ultimate principles, since they are themselves composed of lines and points. Therefore Timaeus hints (53D–E) at 'higher principles' (ἀρχὰς ἄνωθεν)[113] which are known only to divinely favored men, but he does not pursue any inquiry into these principles because he is committed to following the method of plausible reasoning combined with necessity. In keeping with this method, it is quite legitimate for Timaeus to posit less than ultimate principles for the purpose of inquiring into the nature and genesis of the traditional four elements. Here there is a compelling parallel with the application of mathematical hypotheses to the famous astronomical problem about the 'real' movement of the so-called 'wandering stars.'[114]

Timaeus seems to be adopting a similar approach when he posits the half-equilateral and the half-square triangles as the generating principles of fire and of the other elemental bodies. Furthermore, the parallel with mathematical construction is appropriate because the 'generation' of eternal elements may be taken as metaphorical in both cases. Just as mathematicians 'generate' mathematical objects out of more primitive elements for the purpose of analysis and instruction, so also Timaeus will 'construct' the elemental bodies out of primitive triangles to instruct his listeners about the mathematical order of the universe. Thus the parallel with mathematical method is an interpretative clue to the account given by Timaeus.

In this light it makes sense for Timaeus to formulate his leading question as follows: What are the four most perfect bodies which can be constructed such that, although unlike one another, they can be generated from each other by dissolution? He insists that the solution to this problem will yield the truth about the generation of earth and of fire and of the other elements which are their mean proportionals. The description of air and water as mean proportionals is already familiar from an earlier passage (32A) where Timaeus talks about the generation of the World-Body according to numerical

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[113] This could be another reference to the 'unwritten doctrine' that One and the Indefinite Dyad are the ultimate principles for generating a continuum in one, two or three dimensions.

[114] Cf. Simplicius, *in Cael.* 488.7–24.

proportion. But the same precise proportion between the four elements does not appear to hold throughout the *Timaeus.* If the extremes of earth and fire are identified with 'solid' numbers (e.g. 27 and 8), then it would seem that the mean proportionals (i.e. water and air) can also be given an exact number (e.g. 18 and 12). As we have seen, such continued geometrical proportion plays a central role in the mythical account of how the good Architect constructed the World-Body as a harmonious whole, but the same kind of proportion cannot hold between the traditional four elements within the realm of Necessity.[115] Indeed, Timaeus twice (*Tim.* 32B6, 32D1–3) warns his audience that, even within the realm of Reason, the good Architect only copied the eternal model *as best he could.* Along with the refractibility of the material chaos in the Receptacle, the fact that the structure of the elements is geometrical rather than numerical means that the problem of incommensurability requires a new 'generation' of the four elements.

Pursuing the method of combining the plausible with the necessary, Timaeus first postulates the two basic types of triangle out of which the four elements are constructed. Initially, the postulation appears rather arbitrary since he refuses to give any reasoned defense of his choice, except to claim (53D1) that all other triangles are derived from these two kinds. But it is difficult to see how the ordinary scalene triangle, for instance, is derived from either the half-equilateral or the half-square triangles. For constructing the regular solids, however, this choice makes sense because the faces of such figures (e.g. pyramid, cube, etc.) are either equilateral triangles or squares. Thus his strategy seems to be dictated by the character of the regular solids which Timaeus wishes to construct, since this explains his choice of a half-equilateral scalene over many other types of right-angled scalene triangles.[116]

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[115] Rivaud (1925) x, 77–80 has already noticed the incompatibility between the two different accounts given of the proportionality between the four elements. Pohle (1973) 311 thinks that the difficulty disappears if one gives up the expectation that the bond of geometric proportionality should hold at the level of the individual particles, since Plato also uses the names of the four elements as regional and generic designations. But this does not quite square with the fact that at 53E the project of finding numerical proportionality is explicitly recalled as a task, and that the subsequent discussion (54D ff.) includes a careful count of the triangles compounding the invisible 'isotopes' of fire, air, water and earth.

[116] As Taylor (1928) 370 notes, this specific triangle can be taken as 'the most beautiful' because its angles are in the simplest possible proportion; i.e. 1 : 2 (30 : 60) and 2 : 3 (60 : 90).

Furthermore, Speusippus says (Fr. 4 = DK 44A12) that the Pythagoreans gave pride of place to the equilateral triangle because of its unity and simplicity. He also tells us that the half-square triangle comes second in their estimation because it has only one difference of sides and angles and thus it is epitomized by the number 2. Presumably, this is a reference to the fact that the half-square triangle has two different lengths of sides and two different sizes of angle (i.e. 45 and 90 degrees), whereas the equilateral triangle has only one side-length and one size of angle (i.e. 60 degrees). However, their incommensurable sides force Plato to go beyond the numerical approach of the Pythagoreans, especially in constructing the 'cosmic figures.'[117]

Therefore, it seems that Plato's choice of basic triangles is largely dictated by his conception of the problem; namely, what structure can be attributed to the four primary bodies such that they can be generated from one another through dissolution. Guided by the method of analysis in geometry, he works back from the conclusions he wants proved to the assumptions he will have to make. So it is clear that Plato has already decided to identify the traditional four elements with four of the regular solids, and the problem is to construct these solids from elementary triangles in such a way that the phenomena of transformation between the elements are saved. As a result, Timaeus is made to reject (54B–D) the superficial appearance that *all* the elements can be transformed into one another, since his mathematical constructions dictate that he choose two different kinds of basic triangle. While the tetrahedron, the octahedron, and the icosahedron can all be constructed from half-equilateral triangles, one needs the half-square triangle to construct the faces of a cube. So geometrical considerations dictate that the element earth (identified with the cube) cannot be transformed into the others, and this exemplifies the *a priori* mathematical approach which Aristotle severely criticized as a method for doing physics.[118]

After giving these general hints about the transformation of the elements, Timaeus (54D–E) tries to explain the form in which each of them has been generated and the numbers from which it is

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[117] Popper (1945) i, 251 makes the interesting (though admittedly unproven) conjecture that Plato's choice of triangles is dictated by the geometrical fact that their hypotenuses represent $\sqrt{2}$ and $\sqrt{3}$, which he may have regarded (mistakenly) as basic elements for constructing all the other irrational numbers.

[118] Cf. *De Caelo* 306a2 ff.

compounded. First he tackles the form of fire which he calls the primary form, probably because it has the least number of component triangles. The triangle which serves as an element here is the half-equilateral with its hypotenuse twice as long as its least side. When a pair of these are put together along the diagonal, we have a complete equilateral triangle but Timaeus denies that we have as yet one complete face of the tetrahedron that is being constructed. Given that the side of a pyramid is an equilateral triangle, this denial seems rather arbitrary but its purpose becomes clear later (57C–D) when different gradations of size are introduced. Here Timaeus merely insists that one face of the pyramid is generated from three such equilaterals that are fitted together at their vertices around a single center, with the result that a single face of the tetrahedron is compounded from six of the basic half-equilateral triangles. When four of these sides are fitted together into a pyramid, the latter will be constructed out of a total of twenty-four basic triangles.[119]

Similarly, the octahedron can be analysed into the same half-equilateral triangles but there will be more of them involved in the construction. Since this second solid has eight sides, each of which is an equilateral triangle constructed from six half-equilaterals, its whole composition will contain forty-eight basic triangles. In the same way, the icosahedron is composed of one hundred and twenty elementary triangles, with each of its twenty sides being put together from six half-equilaterals.

Hence, adopting the basic triangle as a unit, the following numerical progression connects these three solids: 24, 48, 120. Since all three solids are compounded from the same basic triangle, it seems plausible to assume that the larger can be dissolved into the smaller bodies. In purely mathematical terms, the icosahedron can be dissolved into its basic elements which can then be reconstructed as five pyramids, and the octahedron can be similarly reduced to two pyramids. But, perhaps to conform with some physical phenomena, Timaeus later (56D–57C) puts forward a much more complicated account of the transformations of the primary bodies. Even on the mathematical level, however, the ratios between the first three solids that are mutually transformable do not appear to yield the contin-

[119] For an excellent set of diagrams illustrating the Platonic construction of these solids, consult Friedländer (1969). Nothing in my argument requires the reproduction of these diagrams here.

ued proportion that Timaeus earlier (31B–32C) asserted to hold between the four elemental bodies. For instance, assuming the basic triangle to be our unit, the tetrahedron is related to the octahedron in the ratio of 24 : 48 (or 1 : 2). In the same way, the tetrahedron is related to the icosahedron as 24 : 120 (or 1 : 5), and the octahedron is to the icosahedron as 48 : 120 (or 2 : 5). But 2 is not the geometrical mean between 1 and 5, so that the three solids are not in continued proportion. In fact, as Timaeus pointed out earlier (32B), we must look for two geometrical means because these numbers represent solids and there cannot be a single mean of this kind between numbers that are strictly 'cubes.' But the search for numerical proportion between the four solids now seems to have been abandoned, since it is effectively blocked by the cube being constructed from basic triangles that are different in kind.[120]

As Timaeus puts it (55B5) in his likely account, the first of the basic 'elements' ceased to be active when it had generated the three solids, since the nature of the fourth solid is generated from the half-square isosceles triangle. Four of these triangles are similarly combined to form a square, with their right angles fitted together around a central point. If we put six of these squares together at right angles to one another, we can construct a cubical body that has eight solid angles. Of course, just like the previously constructed solids, this cube will be shaping up the characterless Receptacle by means of its plane sides, as these passages seem to suggest.[121] We cannot read too much into these primitive constructions, however, since Timaeus is primarily concerned with the shape of the bodies that are constructed.

[120] Vlastos (1975) seems to think that the first analysis has to do with the heavenly bodies, whereas the second concerns the sublunary bodies. But, unlike Aristotle, Plato says nothing about the heavenly bodies being composed of different elements from those constituting the corruptible sublunary ones, though he does countenance a threefold division within the universe. In fact, Timaeus calls the elements earth, air, fire, and water in all the relevant passages; so the problem still remains as to how to reconcile the different mathematical analyses given in each place. The only hint of a solution is the remark at 56C about the numerical proportions, which govern the masses and motions of the four elements, to the effect that God realized these everywhere with exactness, in so far as the nature of Necessity submitted voluntarily or under persuasion.

[121] Cornford (1937), Vlastos (1975) & Mohr (1985) offer alternative interpretations of this construction. When introducing the constructions, Timaeus (*Tim.* 53C3 ff.) explicitly assumes that the traditional four elements are solid bodies (σώματα) with depth, which are bounded by the plane surfaces that are analysed into basic triangles. So perhaps this means that the elemental triangles provide regular boundaries for the chaotic bodily powers (δυνάμεις) that are said to preexist in the Receptacle.

Such uncertainty justifies Aristotle's complaint that an excessive concern with mathematical form leads Plato to neglect the material principle of physical bodies.[122] However the construction of the elemental bodies as described in the *Timaeus* does not conform with Euclid's constructions of the five regular solids.[123] Indeed the constructions given for all the elemental solids are more like empty three-dimensional models, though the question about their material content is left rather vague by Plato, as Aristotle's criticism shows.[124]

Having constructed four regular solids (tetrahedron, cube, octahedron, icosahedron) from the basic triangles, Timaeus assigns (55D7) each of them to one of the four traditional elements; fire, earth, air, and water.[125] Although this looks like an *a priori* mathematical approach to the physical world, Plato may be trying to account for some empirical phenomena. For instance, he assigns the cubic form to earth because it is the most immobile of the four kinds (i.e. figures) and the most malleable (πλαστικωτάτη)[126] of the bodies. It is quite clear from the context that 'kinds' (γένη) must refer to the four solids, whereas 'bodies' (σωμάτα) is ambiguous between physical and geometrical bodies. The subsequent passage explains the immobility of earth as a physical body in terms of the stability of its basic triangles, while its plasticity will be a function of its stability if one interprets it as the capacity to retain any shape into which it is

[122] In this regard, Schulz (1966) 65–86 claims that Plato does not neglect matter but rather reduces it to geometrical shapes in the Receptacle, by virtue of reducing all sensible characteristics of material body. Just as in the Atomist reduction, what is irreducibly real is the figure, ordering, and motion of the basic triangles, which are strictly two-dimensional by contrast with atoms which are three-dimensional.

[123] But, as Joan Kung (1989) 322 notes, Plato's construction of the tetrahedron at 54D5–55A5 may have been borrowed from Theaetetus, since it anticipates Euclid's *Elements* XIII, Prop. 13.

[124] In *De Caelo* IV.5, as part of his criticism of previous theories of weight, Aristotle assumes that Plato posited a single matter (μία ὕλη) for all the elemental bodies, namely the triangles (τὰ τρίγωνα), though he is well aware of the fact that two different kinds of triangle had been posited. In view of Aristotle's identification of the Receptacle with matter at *Physics* IV.2, 209b11–13 perhaps he has this in mind as the single matter which is shaped by different kinds of triangle to yield the basic units for the construction of the elemental solids.

[125] At 55C5–6 there is a brief and awkward passage in which Timaeus says that the last of the perfect solids, the dodecahedron, was used by the god in decorating (διαζωγραφῶν) the whole universe (τὸ πᾶν). Perhaps this is a reference to the twelve signs of the Zodiac, which itself may be treated as a dodecahedron inscribed within the spherical universe.

[126] LSJ (s.v. πλαστικός) lists as adjectival meanings: 'malleable, receptive of images, plastic (as in the plastic arts).

moulded. Timaeus claims that the most stable figure will be that whose bases are most stable, and he is referring to the faces of the cube which are each composed of four isosceles triangles.

Consequently, he claims (56A) to be preserving 'the likely account' by assigning the figure of a cube to earth, since it appears to be the most stable of the traditional elements. Among the other elements, water is more stable and so it stands to reason that it should be assigned a more stable figure like the icosahedron. Thus the earthy cube is most stable, while the watery icosahedron comes next, whereas the most mobile form is the fiery tetrahedron. Finally, between water and fire, comes the airy octahedron, which must also be quite mobile if the appearances are to be saved.

Timaeus also compares (56A–B) the three mutually transformable elements on the basis of the size, weight, and sharpness of the solid figures assigned to them. For instance, the smallest body is assigned to fire and the biggest to water, while air is made the intermediate, as usual. Given the absence of earth from this list, the comparison must be based on the number of half-equilateral triangles which are involved in the construction of each one of the solids that are compared; i.e. the tetrahedron has 24, the octahedron has 48, and the icosahedron has 120. If one were to neglect the differences in kind between the basic triangles and pay attention only to the number, then earth (which is compounded from 24 triangles) would be just as small in bulk as fire. But, significantly enough, this comparison is never made by Timaeus, even though later (57C6–D8) he distinguishes grades in the size of the fundamental bodies according to the number of triangles compounding them.

Furthermore, there is an order with respect to sharpness between these three elements, which is directly correlated with the number of triangles compounding their basic figures. For instance, Timaeus says that the figure (i.e. the tetrahedron) which has the fewest faces must naturally be most mobile, since it is in every way the sharpest and the most cutting of all the figures. It is also held to be the lightest because it is compounded from the fewest identical parts. Again one might doubt whether earth can be compared on this basis with other elements, given that its triangles are not identical with those compounding the others. However it is clear that the other figures compounded from the same basic triangles are being compared with respect to sharpness and lightness. In terms of such a comparison, the octahedron comes second after the tetrahedron while the icosahedron is

placed third. Thus, in accordance with the correct and likely account, the solid generated in the form of a pyramid is designated as the element or 'seed' of fire. In using the word σπέρμα here (56B5–7) Plato may be recalling the microscopic bodies posited by Anaxagoras and, thereby, implying that these can be given a better explanatory description in terms of solid geometry. First comes the generation of fire (i.e. the pyramid) from the basic triangles (as true elements), next comes air (i.e. the octahedron) and third comes water (i.e. the icosahedron). This reductive account may have been thought superior to that of Anaxagoras because it gives a more coherent explanation of transformations at the sensible level.

IV. *Saving the phenomena*

The first two parts of the *Timaeus* are primarily concerned with finding intelligible explanations. In practice, this means that Plato looks beyond sensible qualities to an underlying mathematical structure which is eminently intelligible and, for economy of exposition, I have concentrated almost exclusively on this explanatory aspect of the *Timaeus*. Finally, I will briefly consider some ways in which Timaeus uses his mathematical constructions to save the empirical phenomena.

At 61C, for instance, he begins to explain how tactile qualities like hot and cold affect our faculties of sense perception. Since we experience heat as a sharp sensation, it is usually assumed to act on our bodies by dividing and cutting. But a true explanation for its action must appeal to the mathematical figure corresponding to fire; namely, the tetrahedron. Hence by referring to the fineness of its edges, the sharpness of its angles, the smallness of its particles, and the swiftness of its motion, one can explain why fire is experienced as something intense and keen that sharply cuts whatever it encounters. In a similar fashion, Timaeus claims that one can rationally explain the opposite quality of coldness in terms of the shapes and motions of the fundamental figures of water or air. Here he appeals to the greater relative size of the fundamental particles of water, which gives them the capacity to drive out the smaller particles of fire, so as to compress the existing particles of water in the body. The natural reaction of the body to this contraction is what he calls 'shivering' or 'trembling.'

Such reductive explanations can be usefully compared with those of Democritus, who also tried to link the character of a sensation with the shapes of the atoms in the body that stimulates it. Since atoms were assigned all sorts of haphazard shapes, Plato may well be competing with atomism by giving his reductive explanations a more economical basis in 'pure' mathematics. As circumstantial evidence for my conjecture, it should be noted that Theophrastus compares and contrasts the Platonic and Democritean accounts of sensible qualities in terms of a reduction to atomic elements that lack such qualities. For instance, Timaeus explains (61D–E) the sensation of heat in terms of the dividing and cutting effects which the invisible tetrahedrons of fire inflict upon our bodies. He differentiates clearly between the sharpness (ὀξύ), which is a sensible quality (πάθος) of fire, and its hidden causes which must be rationally reconstructed (λογιστέον ἀναμιμνησκομένοις) in terms of the sharp edges and the acute angles of the tetrahedrons, combined with their smallness and rapidity of motion. These are the real reasons why fire is perceived as intense (σφοδρόν) and keen (τομόν), and why it sharply cuts whatever it encounters.

Plato offers this explanation of why fire naturally produces that affection (πάθημα) called heat (θερμόν), since its hidden structure makes it particularly fitted to divide and mince up (κερματίζειν) our bodies. Even though the etymological connection again seems quite fanciful, it reflects his repeated attempts to save both sensible and linguistic phenomena by means of such reductive explanations. It may also be seen as an implicit critique of Democritus, who is reported by Theophrastus (*De Sensibus* 68 = DKA135) to have attributed a spherical atom to fire, so as to account for its mobility. If this report is accurate, then Plato might charge Democritus with failing to account for the sharp sensation of fire.[127]

With respect to the weight of bodies, we can also make some direct comparisons between Plato and the Atomists because they give two competing accounts of this perceptible characteristic.[128] According to

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[127] As circumstantial evidence that Plato is competing with Democritus, one might cite the report of Theophrastus (*De Sensibus* 60 = DK 68A135) that only these two (prior to Aristotle) have dealt fully with the nature of sensibilia.

[128] This has already been studied exhaustively by Denis O'Brien (1981 & 1984) who also reads Plato as consciously improving on the Atomist account of weight, even though he hedges (1984) 289n17 on whether Democritus exercised any direct historical influence on Plato.

Theophrastus (*De Sensibus* 61 = DK 68A135), Democritus distinguished between heavy and light in terms of size (τῷ μεγέθει), yet in the case of compound bodies he held the lighter to have more, and the heavier to have less void. Although the report seems to distinguish between two different Democritean explanations of weight, I think we may assume that the weight of individual atoms was determined by their size, whereas the weight of compounds was dictated both by their size and by the amount of void in them. Thus he could explain the difference in weight between bodies of equal size, such as a cubic foot of lead and of wood, by saying that one body is denser (i.e. has less void) than the other. Since Plato denies the actual existence of a void, he cannot use such an explanation, though he does acknowledge (*Tim.* 56A–B) that elemental bodies differ in weight and that this must be due to their having a different number of basic elements. Yet, according to his numerical account, the perfect solids identified as fundamental particles of fire (tetrahedron) and earth (cube) have the same number of basic triangles (24), so two fiery and earthy bodies of the same size should have the same weight. But, since this conflicts with our sense experience, Plato must give a more sophisticated account of the weight of elementary bodies with reference to their proper places in the universe.

He begins (62C) by correcting a common error, which bears on the phenomenon of weight; namely, that the universe is divided into an absolute 'down' (κάτω) to which all things with bodily mass move, and an absolute 'up' (ἄνω) to which all things move only involuntarily.[129] This assumption is erroneous because the universe is spherical by nature, and so the center and the circumference are the only clearly defined opposites. But, in such a universe, one cannot use 'up' and 'down' in an absolute fashion without being guilty of using completely unsuitable names, since what is up and down changes as one moves around the central and spherical earth.

In order to explain how such names arose and why they are used habitually by people on this earth, Plato introduces (63B) a *Gedanken-experiment* according to which one observer is located just inside the

[129] In a footnote to this passage in his Loeb translation of the *Timaeus,* Bury (1929) 62n1 claims that the reference is probably to Democritus, but there is no good reason to think that the notion of an absolute 'down' correlated with the motion of heavy bodies is unique to the Atomists among Presocratic thinkers. What may be unique, however, is their view that no bodies move upward by nature but only by force of the whirl; cf. Simplicius, *in Cael.* 712.27 (DK68A61).

circumference of the universe, where fire has its proper place and where the bulk of that element is massed together. This man is supposed to be weighing parts of fire on a balance by separating them from the main mass of fire, thereby forcing them towards the region of air. In such a hypothetical situation it is thought to be obvious (δῆλον) that the smaller rather than the larger mass will be more easily forced away from its parent mass, so that the larger mass will be called 'heavier' and the smaller mass 'lighter.' But this is exactly parallel to the situation of another observer standing on this earth and weighing various earthy substances, pulling them into the dissimilar air by force and against nature (βίᾳ καὶ παρὰ φύσιν), since they naturally cleave to their own kind. Thus, because the smaller mass yields more easily, we call it 'light' and the region towards which we force it 'above'; whereas the larger mass is called 'heavy' and the region towards which it tends 'below.' The major point of this 'thought-experiment,' therefore, is that such terminology and the phenomena which it describes must be understood as being relative to the region being described; so it may not be totally anachronistic to call this Plato's 'relativity theory' of weight and direction.

But Aristotle rejected Plato's theory of relative weight, and posited absolute weight and lightness as being coordinated with absolute directions in which heavy and light elements move by nature. Similarly, he rejected the Atomist account of weight in terms of the size of individual atoms, and of the amount of void in composite bodies, because it presupposed a universe without proper places or absolute directions. While this provides only circumstantial evidence for a parallel between the sense-relative accounts of sensible properties given by both Plato and the Atomists, it is consistent with the reports given by Theophrastus. Hence, despite the paucity of evidence, it is possible to draw clear parallels between their respective accounts of weight both at the elemental level (where it is due to size or number of triangles and atoms, respectively) and at the level of compounds (where the gaps or voids between the elemental particles play a part).

Conclusion

In summary, therefore, the central question of the *Timaeus* is how purpose and contingency are related to each other both in the physical universe and in mankind. The Platonic way of formulating the

question is to ask whether Necessity is open to rational persuasion, and an answer is given in terms a mathematical ordering of the Receptacle and its chaotic contents. For instance, the Demiurge is represented as taking over disordered traces of the four elements within the Receptacle, and as reconstructing these elements as four geometrical solids from two basic kinds of triangle.[130] Without such identifiable structures, it is suggested, the so-called elements would be merely indefinite qualities floating about in the Receptacle. With the hypothesis about their mathematical structure, one can save the phenomena by making the realm of Necessity more amenable to the rational world-making of the Demiurge.

This interpretation reduces somewhat the power of the Demiurge as producer of order in the visible universe. As evidence for such a reduction, one could point to the many unresolved tensions in the dialogue between the realms of Reason and Necessity. For example, we might contrast the cogent logical argument at 31B for the uniqueness of the All with the later (55C–D) admission of five possible worlds. Timaeus also admits that it is simply a contingent fact about our world that it has the dodecahedron as the 'cosmic' figure which encompasses the four perfect solids of the traditional elements. Yet earlier the Demiurge was able to save the unity of the visible cosmos because all five regular solids can be circumscribed by a sphere. Although this was the privileged shape assigned to the cosmos by Timaeus in his discussion of the realm of Reason, it does not fit very well with his corresponding discussion of the realm of Necessity. For instance, even if the five perfect solids can be circumscribed by the cosmic sphere, there will be a void between the all-encompassing dodecahedron of the realm of Necessity and the Rational sphere. Furthermore, in his inquiry about the works of Reason, the unity of the visible cosmos was made to depend heavily upon a continued geometrical proportion that was postulated to hold between the traditional four elements. But, as I have already shown, this numerical proportion breaks down when Timaeus assigns to the four elements

[130] Gadamer (1980) notes the striking absence of the Demiurge from the whole account of the so-called products of Necessity. There is one noteworthy exception, however, where the God is said to mark out the elements into shapes by means of forms and numbers; cf. *Tim.* 53B4–5. This passage seems to assume as basic for the whole account that the divine Craftsman constructed the elements in the most beautiful possible manner, given the constraints of Necessity.

the perfect solids which are composed of two different basic triangles. What are we to make of this inconsistency between the two realms? Has the rational unity of the cosmos broken down so that it must be replaced by another kind of unity, which is a complex mixture of Reason and Necessity?

One way of making sense of these tensions within the dialogue is to separate Plato himself from the figure of Timaeus, who shows some blind spots in the dialogue that bears his name. For instance, Timaeus fails to acknowledge that the Pythagorean project of establishing numerical proportions between the elements has broken down within the realm of Necessity. He continues to hope for a completely rational world order, in spite of the recalcitrance of the Receptacle and its contents. This recalcitrance is best exhibited by the incommensurability that is built into the structure of the elements, through the basic triangles which constitute the perfect solids. However, through the structure of the dialogue itself, Plato shows himself to be aware of the bitter pill which the Pythagoreans had to swallow on finding the 'unspeakable' (ἄρρητος) and the 'irrational' (ἄλογος) at the very heart of their beloved mathematics. Since incommensurability is an integral feature of the things that appear in the Receptacle, the Pythagorean cosmology of whole numbers is not merely doomed to failure but it actually misrepresents the real structure of the contingent sensible world. This world does not correspond to the rational numerical model constructed earlier in the dialogue, but it is the best possible one which Reason can achieve by persuading Necessity. In the mathematical sphere, for instance, the results of such persuasion are to be found in the stereometry of Theaetetus and in the astronomy of Eudoxus, which bring the irrational under the control of reason.

Similarly, the animals which populate this universe and which are 'generated' by the lesser gods will have the same kind of structure; namely, the unity of soul and body. That is why Plato finds it appropriate to discuss the sensations of the body with reference to the realm of Necessity, while the activity of the soul is considered under the realm of Reason. Thus, in the final section of the dialogue (69A ff.), the proper proportion between soul and body is discussed in terms of the cooperation between Reason and Necessity. For instance, Timaeus locates the spirited part of the soul in the heart, while emphasizing that it must be directed by the head which is the location of reason. Of course, as we might expect, the appetitive part of the

soul is given a bodily location in the belly. Hence we can find here the physiological basis for the tripartition of the soul in the *Republic*. Indeed, if we pay attention to the dramatic situation established at the beginning of the *Timaeus*, there is ample evidence for this connection.[131]

[131] By contrast, Diskin Clay (1988) argues that all the apparent bridges between the two dialogues collapse when we attempt to cross them with our eyes fixed on the text. But, although it is a sound literary principle to insist on the integrity of each dialogue, I think it is rather extreme to insist that we cannot draw upon the philosophical content of the *Republic* to elucidate that of the *Timaeus*.

CHAPTER TWO

ARISTOTLE'S CRITICISM OF PLATO'S MATHEMATICAL COSMOLOGY

Given the dialectical character of Aristotle's philosophical method, it is reasonable to assume that his cosmological views were developed in response to those of Plato, the Pythagoreans, and other Presocratic thinkers. For instance, against the mathematical cosmology of Plato's *Timaeus,* Aristotle insists that it is physics rather than mathematics which studies the nature and motions of sublunary bodies. Yet he adopts Eudoxean astronomy as a guide to the motions of the heavenly bodies, while also accepting optics and mechanics as true for the sublunary sphere. For instance, Aristotle's claim that all heavy bodies fall towards the center of the universe seems to involve mathematical symmetry as part of his theory of natural places. So the similarities and differences between Plato's and Aristotle's cosmology can be clarified in terms of the distinction between physics and mathematics.

I. *Aristotle's response to the Timaeus dialogue*

Any assessment of Plato's influence on Aristotle's cosmology must begin with the *Timaeus,* which is mentioned more frequently than any other dialogue.[1] Thus the *De Caelo* is the locus classicus for Aristotle's criticism of the *Timaeus,* since both works may be seen as parallel cosmological treatises. In this regard one must take account of the problem raised by Jaeger[2] as to whether a particular treatise

[1] Cf. Bonitz (1870), 598a60–b19 & 761b55–60.

[2] Jaeger (1923) thinks that, since Book I of *De Caelo* brings *aither* within the realm of *physis,* it must have been written after *De Philosophia* which he dates at 347 B.C. But this conjecture depends on the dubious claim that the latter work still subscribes to the psychology of Plato's *Laws,* according to which *physis* is subordinated to the World-Soul. Furthermore, Jaeger (1923) 299 ff. claims that the confident tone of *De Caelo* I, 2–3 makes it likely that a version of the five elements theory had already been proposed prior to *De Philosophia* and that it is now being rewritten along mechanistic lines. Such speculation is challenged by Bos (1973) 131 as inconsistent and lacking in evidence.

presents an earlier or later phase in Aristotle's thought, even though it may be an impossible question to settle, if Aristotle reworked some treatises during his lifetime.[3]

I.1. *The program of De Caelo*

Given the discussion of the sublunary world in Books III & IV, the traditional title (i.e. 'On the Heavens') must refer to the cosmos, since the whole work outlines a world-system which (even without an Unmoved Mover) can be attributed to Aristotle.[4] For instance, the cosmos is held to have a finite spherical body, with the fixed stars at its extreme circumference and the earth at its center, while the places in between are occupied by other elemental bodies.[5] Contrary to Anaxagoras, Aristotle thinks that the sphere of the fixed stars (as well as those of the planets) is not composed of fire but of a fifth body popularly called *aither*. At *De Caelo* I.2 he argues for the existence of this simple body on the grounds that circular motion does not belong naturally to any of the sublunary elements, as their natures dictate that they move in rectilinear fashion away from or towards the center of the universe. Aristotle assumes that the heavenly bodies move eternally in perfect circles, so that this kind of motion must belong naturally to their matter. Since none of the four elements fits the bill, there must be a fifth body which is simpler and has a higher nature because circular motion is prior to rectilinear motion.[6] Thus

[3] In his 'Introduction' to the Loeb edition, Guthrie (1939) xvi describes the *De Caelo* as a transitional work that contains both earlier and later views. Bos (1973) 2 also holds that within this work one must distinguish earlier and later strata.

[4] Solmsen (1960) 272–3n24 thinks that the first mover is introduced at 300b21 but that the concept is different from that in the *Physics* and *Metaphysics,* since it is said to be itself in motion by nature. However this is conceptually more akin to Plato's concept of soul as self-moving by nature than to the notion of an *unmoved* mover. In support of his hypothesis about the later date of *De Caelo* I & II, Manuwald (1989) 115 argues that the unmoved mover is clearly assumed in II.12, and taken to be among the transcendent entities mentioned in I.9.

[5] Cf. *Cael.* 269a30 ff., 269b15–16.

[6] For Plato circular motion is characteristic of a completely rational soul, whereas Aristotle makes it a typical sign of the nature of *aither*. So Bos (1973) 58 suggests that this first element is introduced by Aristotle to replace the Platonic World-Soul as the agent responsible for the uniform circular motion of the World-Body. As evidence for a transition to this view, Bos points to *De Philosophia* (Fr. 27) where Aristotle seems to regard *aither* as the substance of the soul. If this is a response to the Platonic problem of how immaterial soul can act on the body, then perhaps Bos is right to emphasize that *aither* is treated by Aristotle as a first body.

the self-motion of the World-Soul in Plato's cosmology seems to be replaced by the natural circular motion of the primary body according to Aristotle's theory.[7]

I.2. *Geometrical determination of the universe*

In view of his criticism of Plato's cosmology, it seems odd that Aristotle begins *De Caelo* I in a quasi-Pythagorean manner. For instance, his argument for the completeness of a three dimensional body appeals (268a11–12) to the notion of the universe imitating the number 3, whose natural completeness is certified by its association with 'all' in ordinary counting and by its use in ritual worship. Similarly for continuous magnitudes, threeness is associated with body as the only complete magnitude because it is extended and divisible in all three dimensions. This reflects a Pythagorean correlation of numbers and magnitudes that sees completeness in numerical terms; cf. *Met.* I.5, 986a1 ff.

Despite these Pythagorean themes, however, Aristotle adopts an anti-Platonic stance in claiming that body is the only complete magnitude, since it is divisible in every way, by contrast with planes which are divisible in only two ways. One implication of this claim is that planes cannot serve as principles for bodies because the latter are not constituted from planes. So Plato is the intended target of Aristotle's firm rejection (268b1) of the transition (μετάβασις) into another genus that is involved in generating planes out of lines or bodies out of planes.[8] The point of his objection is that if this were its mode of generation then body would not be a complete magnitude since generation through 'emanation' (ἔκβασις) implies some lack in body; but, in fact, body is complete because it is divisible in every

[7] Cf. *De Philosophia* Fr. 22. Guthrie thinks that in this lost dialogue *aither* was treated as a divinity with nous rather than as a body with natural motion; so that this would represent Aristotle's earlier view of the motion of heavenly bodies as voluntary rather than as natural. However, Bos (1973) 62 rejects any interpretation of *De Philosophia* that makes voluntary and natural motion mutually exclusive, since he thinks this is due to an error somewhere in the tradition between Aristotle and Cicero. On reviewing the evidence, Hahm (1982) concludes that this was a very early work containing the four-element cosmology of Plato with no trace of *aither* as a fifth element.

[8] In *De Generatione et Corruptione* I.2 (316a3–4), however, Aristotle objects that putting planes together yields nothing other than (mathematical) solids, as evidenced by the fact that the proponents of such a theory do not even try to 'generate' the qualities of bodies.

way. Since Aristotle twice mentions the infinite divisibility of whatever is continuous, it is clear that his notion of the continuum is crucial for the break with Plato.

I.3. *The infinite divisibility of the continuum*

The general context for the problem of the continuum, as that is discussed in *Physics* VI, is provided by a treatise on locomotion that begins in Book V and concludes with the unmoved mover in Book VIII. Along with its central role in Aristotle's theory of motion, the analysis of the continuum also defines his position in relation to Plato and the Atomists. To identify more precisely Aristotle's disagreement with Plato, it is important to note that Aristotle treats the continuum more as a physical than as a mathematical problem.[9] His discussion of the continuum is central to the *Physics* not only because it deals with continuous bodily magnitudes but also because motion appears to be continuous, despite the difficulties presented by Zeno's paradoxes. Thus, at the beginning of his treatise on the continuum in *Physics* VI, Aristotle refers to definitions already given in V.3 of what is continuous (συνεχές), as distinct from what is in contact (ἁπτόμενον) and what is successive (ἐφεξῆς). Since these are obviously central concepts for his treatment of the continuum, we should examine their implications.[10]

In *Physics* VI.1, Aristotle briefly refers to continuous things (συνεχῆ)[11] as being those things whose limits are united, and he also defines (231a24) a continuum negatively as something which cannot be composed from indivisible parts. But the rationale for this definition cannot be properly understood without looking at *Physics* V.3 where a series of concepts are defined in preparation for a positive description of the continuum. For instance, things are said (226b21–3) to be together (ἅμα) in place when they are in one primary place, whereas things are said to be apart (χωρίς) when they are in different places. Only the first is compatible with the concept of contact (ἅπτεσθαι),

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[9] Wolfgang Wieland (1962) 281 brings this out very clearly in his masterly treatment of the continuum problem in Aristotle's *Physics*.

[10] In his 1924/5 'Sophistes Vorlesungen,' Martin Heidegger (1992) drew attention to the importance of these concepts for Aristotle's differentiation between a unit that is without position and a point that has position but no place.

[11] Wieland (1962) 283 attaches importance to the use of the plural here as an indication that continuity is a relational concept; i.e. it is not an attribute of one single body but rather a relation between parts of the body.

however, since things are in contact when their extremities are together in one place.

The crucial concept in this list, however, is that of succession (ἐφεξῆς) because it is the most general and so is logically prior to all the others. According to Aristotle's rather complex definition, a thing is in succession to another thing when it comes immediately after it either in position or in form or in some other definite respect. What 'immediately' means here is that there is nothing else of the same kind between them; e.g. two houses on a street may be in succession if there is a garden between them but not if there is another house there. It is also characteristic of things in succession that they constitute a non-reciprocal series of prior and posterior; e.g. one is prior to two but not vice versa.

Having defined the most general concept, Aristotle specifies the differences that make one series of things into a continuum, while the other remains a discrete succession. What makes some things contiguous (ἐχόμενον) is contact, i.e. their extremities are together in one place. But contiguity is not the same thing as continuity, since the extremities of two things may be together but not united to form a continuous thing. So Aristotle defines (227a10–12) as continuous only those things in contact whose limits become one and the same in such a way that they are contained in each other, as the word συνεχές implies. Here he appeals to the structure of the word itself to show that it fits those things which form a unity by means of contact. For instance, things organically united, and also things held together by glue or by rivets, are said to be continuous since the whole is one. From these examples it is clear that Aristotle considers the continuum to be observable in the sensible world both through visible bodies and their motions.

In this way, one might say that continuous bodies are prior in relation to perception, though they are not logically prior. After defining the continuous, Aristotle attributes logical priority to succession on the grounds that not everything in succession need be in contact; e.g. numbers are prior in definition to a continuum. This is important for distinguishing the unit from the point, even if they were independent substances, since points have position but units can only be in succession. Furthermore, there can always be something (e.g. line or points) between points, whereas there is nothing between the numbers one and two. Through the concepts of succession and contiguity, Aristotle here seems to be providing a physical

interpretation for the ontological hierarchy of unit, point, line, plane and solid, which was so central to the Pythagorean and Platonic cosmologies.

From these definitions we can better understand Aristotle's negative description (231a24) of the continuum as that which cannot be composed from indivisible parts. It is also clear why he illustrates this in terms of the claim that a line cannot be composed of points, since a line is continuous but points are indivisible. Contrary to the Pythagorean assumption, a conjunction of points does not form a continuum because their limits cannot be united in the way that Aristotle's definition requires. The reason is simply that indivisible things do not have limits as distinct parts, so that if two points are placed in contact they must coalesce into one. Therefore they are either indistinguishable from each other or they leave space for a line between them on which more points can be taken. Thus points cannot be next-door neighbours because other points can always be placed between them, and so they cannot form a continuum.

Even if a continuous line cannot be composed of points, it is natural to ask about how it is composed. But Aristotle rarely talks about the composition of a continuum out of (ἐκ) its material parts, except when he is criticizing the mistaken ideas of predecessors. It is important to see that his argument about points applies equally to the composition of any continuum from indivisibles. Since contact is a necessary condition for parts of a continuum, the crucial question is how indivisible parts can come into contact. Since indivisibles are without parts, Aristotle argues (231b2) that they must be in contact as whole to whole, just like points. Therefore they cannot form a continuum, since it is essential that it be divisible into parts which are distinct and spatially separate (231b4–6).

On the other hand, if the indivisibles are distinct and separate in place they cannot be successive, which is another condition that parts of a continuum must fulfil. Once again, Aristotle's argument (231b7) depends on the definition of two things as being in succession when nothing of the same kind comes between them. Now two points that are spatially separate cannot be successive because more points can always be taken on the line between them. Similarly, two moments in time are not successive because one can always find other moments in the stretch of time between them. So Aristotle insists that a continuum must be divisible into parts which are themselves divisible, since such parts can be in succession and in contact.

But, as Wieland (1962) 284–6 notes, Aristotle's concept of the continuum is not so much compositional as relational; i.e. the emphasis is on how the parts are related. Although continuity is a characteristic of the whole, it belongs to it only insofar as it is divisible into parts which are combined in a certain way. Hence Aristotle's most frequent shorthand definition of a continuum is that it is divided into parts which are themselves infinitely divisible; cf. *Phys.* 231b15–16. Against both the Atomists and the Platonists, he insists that the continuum cannot be reduced into parts which are not themselves continua. For future reference, we may call this Aristotle's irreducibility thesis about the continuum.

A corresponding thesis about the isomorphism of magnitudes, motion and time is used to resolve some of Zeno's paradoxes. Briefly stated, the isomorphism thesis is that either all of these are composed of indivisibles (and hence are divisible into indivisibles) or they are all continua; i.e. divisible into parts that are themselves indefinitely divisible. In *Physics* VI.1 Aristotle defends the latter option in a rather complex argument which focuses primarily on motion, since that also involves magnitude and time. For the connection with Zeno's paradox of the arrow, the most illuminating part of this argument is the claim that if the path of motion consists of indivisibles then motion is possible only in jerks (κινήματα). Thus in every indivisible part of that path, the moving arrow is at rest, since one can only say that it has moved (κεκινῆσθαι) but never that it is moving (κινεῖσθαι); cf. 231b29–30. But this yields the paradox that either the body is both moving and at rest or it is in both contiguous parts at the same time.

For Aristotle such a paradox shows the untenability of the first option in the isomorphism thesis; i.e. that magnitudes, motion and time are all divisible into indivisible parts. In the final analysis what makes it untenable is that it yields implications which are at odds with what we observe about motion in the sensible world. Furthermore, language reflects a clear phenomenal distinction between bodies that are in motion and those at rest, while it also appears that bodies move smoothly along a given path rather than in jerks. For this reason, Aristotle treats bodies that move in a staccato fashion as having a series of different motions each with its own beginning and end.

Therefore he rejects Zeno's analysis of the motion of the arrow into indivisible jerks not merely for mathematical but primarily for

physical reasons. His proposed solution to the paradox of the arrow is to accept the second option in the isomorphism thesis; i.e. that motion, magnitude and time are all indefinitely divisible. In other words, they are all continua and so by definition they cannot be constituted from indivisible parts. In *Physics* VI.2 he gives a long and complicated argument in support of this kind of isomorphism for magnitude, motion, and time. Among other things, it implies that all motions can be faster or slower (with no minimum or maximum), and that time and distance must be indefinitely divisible into similar parts (otherwise there might be a minimum time or distance). Since time and distance are inversely related in faster and slower motions, it is necessary that all three be continua if one is to avoid Zeno's paradoxes of motion. For instance, Aristotle asserts (233a22) that Zeno falsely assumes it to be impossible for a thing to come into individual contact with infinite things in a finite time. In fact, as he points out, there are two ways in which continua may be called infinite; i.e. either in respect of division (κατὰ διαίρεσιν) or in their extremities (τοῖς ἐσχάτοις). So, while it is impossible for some body to traverse a quantitative infinite in a finite time, it can traverse what is infinite by division, since time itself is infinite in the same way. Therefore, the body is really traversing a finite magnitude in a finite time, which corresponds quite well with our sensory experience of motion. But Aristotle's solution to Zeno's paradoxes obviously presupposes his own discussion of the different ways in which the infinite can exist, and so we must examine his views on the infinite.

II. *The question about infinity*

Within his general cosmological discussion in *De Caelo* I, Aristotle raises a specific question about whether or not there exists an infinite body in the universe. The question is prompted both by the fact that some philosophical predecessors posited infinite bodies, and by his own conviction about the central importance of the whole issue for cosmology:

> This is a point whose settlement one way or the other makes no small difference, in fact all the difference, to our investigation of the truth. It is this, one might say, which has been, and may be expected to be, the origin of all the contradictions between those who make pronounce-

> ments in natural science, since a small initial deviation from the truth multiplies itself ten-thousandfold as the argument proceeds.[12]

Obviously Aristotle sees the problem of infinity in Greek cosmological thinking as marking a crossroads where those who posit an infinite universe diverge from those who regard it as finite. Not only does infinity have the power of a principle but it also has wide implications for the whole universe, so that a vast difference for one's cosmology results from assuming the existence of an infinite body.

In discussing the question, Aristotle considers first the simple bodies because if these prove to be finite both in number and size then so also must the composite bodies which they constitute. He takes the previous arguments based on simple motions to have established that the simple bodies are finite in number, so his subsequent (271b24 ff.) strategy is to investigate whether it is possible for one of these bodies to be infinite in size. Beginning with *aither* as the primary simple body, he argues that any such body which revolves in a circle must itself be finite. His *reductio per impossibile* argument starts from the assumption that it is impossible to traverse an infinite distance. If a revolving body were to be infinite in size, then so would the straight lines radiating from the center, as well as the intervening spaces; hence an infinite body cannot rotate in a circle because its radii cannot cover the infinite distance between them. But Aristotle thinks (272a3–7) such a result would be at odds with our perception of a revolving universe, and with the logical argument which links that motion with a real body. In fact, the general strategy of all his arguments against an infinite simple body is to show that its postulation would lead to impossible results which are also contradicted by sensory evidence. Hence these arguments have physical and cosmological consequences, even though they involve mathematical principles and illustrations.

For instance, he offers a general argument against an infinite body based on the impossibility of there being an infinite weight. Strictly speaking, of course, the argument from weight applies primarily to earth (and secondarily to water and air), as Aristotle himself acknowledges by referring to a parallel argument against infinite lightness. While these appear to be purely physical arguments, they are also based on mathematical principles drawn from the theory of proportion.

[12] *De Caelo* I.5, 271b5–10: tr. Guthrie (1939).

For instance, the proof draws heavily on the method of reciprocal subtraction that was used by mathematicians to determine whether two quantities are commensurate or not.[13]

Using this method to establish proportions between quantity and weight, Aristotle shows that the assumption of an infinite body leads to many absurd conclusions, such as that the weights of finite and of infinite quantities will be the same. The basic axiom upon which he relies throughout for showing the absurdity of such conclusions is that there is no ratio between the finite and the infinite. Assuming some proportion between weight and speed, therefore, Aristotle can argue (273b27 ff.) that an infinite weight is impossible because it would have to move faster than any finite weight, and at the same time it could not move at all because there is no ratio between finite and infinite weights. On the basis of such applied mathematical arguments, he concludes that the body of the world cannot be infinite. But that leaves open the possibility of other types of infinity in the world, and this question is addressed in *Physics* III.4–8.

II.1. *The case for the infinite*

In Presocratic philosophy, the indefinite (ἄπειρον) first plays a leading role in the cosmology of Anaximander, who seems to have conceived of it as an inexhaustible material source for conflicting opposites that are regulated by a cosmic principle of justice. Similarly, the Pythagoreans posited the unlimited (ἄπειρον) as a principle along with the cosmic principle of the limit (πέρας). In addition, the Atomists seemed to have posited an infinite void as part of their cosmological thesis about an infinite plurality of worlds. Although his cosmos is finite, Plato introduces the unlimited both as a cosmological principle (i.e. Receptacle) and as an ontological principle (in the *Philebus).*

Therefore Aristotle has good precedents for treating the infinite as part of a physical and cosmological inquiry, although he also mentions (*Phys.* 204a34 ff.) a more universal (καθόλου) inquiry about the possibility of the infinite in mathematical objects and in intelligible things without magnitude. Yet he narrows his subject matter down to sensible things, and explains that his method of inquiry involves questions peculiar to physics; e.g. whether or not there is an infinite body in the direction of increase. Such a question requires one to

[13] Cf. Knorr (1975 & 1986), Szabo (1978), & Fowler (1987).

decide whether the physical universe is infinite in extension or not, and this is also a cosmological issue that Aristotle decides in favor of a finite universe.

Thus, while it is easy to see why Aristotle adopts a physical approach to the problem of the infinite, it is more difficult to understand why its formulation is ontological in character. The crucial insight here is that Aristotle's formulation and solution of most problems is determined by what his predecessors have said about it. Therefore, I take very seriously his dialectical approach which usually begins with a review of opinions, continues with an elenctic scrutiny of these opinions in order to reach a definition or principle that, finally, saves all of the relevant phenomena.[14] Aristotle's treatise on the infinite (*Physics* III.4–8) follows such a dialectical procedure, even though he is pursuing a physical inquiry.

The first indication of his procedure comes near the beginning of the treatise where he argues (203a1–4) that the infinite is an appropriate topic for physics because all those who are worthy of notice as physicists have posited the infinite as a principle of things. The difference between them, according to Aristotle, is that they give the infinite a different ontological status. For instance, he says that the Pythagoreans and Plato make the infinite an independent substance rather than an attribute of some other subject, as the physicists tend to do. But the Pythagoreans confine the infinite to sensible things, even in the case of the infinite outside the heaven; whereas Plato rejects any such external body, while placing the infinite both in sensible things and in the Forms. Still Plato and the Pythagoreans are united in accepting the infinite as a substantial principle, especially in mathematics where it is identified with indefinite magnitude. According to Aristotle, Plato posited the Great and the Small as two infinites, while the Receptacle also functions as an indefinite material principle.

We see the point of Aristotle's elaborate review of opinions when he summarizes five major reasons for belief in the existence of the infinite, since these constitute phenomena to be saved by any solution. First, there is the nature of time which appears to be infinite in extent, stretching backwards and forwards from the present moment in an unlimited manner. Second, there is the unlimited divisibility of magnitudes, especially as this shows itself in mathematics. Thirdly,

[14] See *De Caelo* IV.1, 308a4–7 for a pithy summary of this dialectical procedure.

there is the physical assumption that if generation and corruption are to continue indefinitely, as they appear to do, then there must be an infinite source of material. Fourthly, there is the more logical consideration that, since the limited is always limited by something else, there must be something without limit to provide a limit for the things that are limited. Finally, the most relevant consideration is that not only number but also mathematical magnitudes and what is outside the heaven are all supposed to be infinite because they never give out in thought (τῇ νοήσει). Again it is noteworthy that mathematical considerations are given such importance in a physical inquiry.

After concluding the negative dialectical stage of his treatise in which he surveyed and refuted most of the leading opinions on the infinite, Aristotle begins (206a9 ff.) his positive account by recalling some of the above reasons for thinking that the infinite must exist in some sense. If it does not exist then there will be a beginning and end of time, and a magnitude will not be divisible into magnitudes, and number will not be infinite. Since persuasive arguments can be made both for and against the infinite, one needs an arbitrator (διαιτητής) to distinguish the senses in which it exists from the senses in which it cannot exist. This forensic metaphor is favored by Aristotle in aporetic contexts where a compromise must be found between competing claims on both sides of an issue. In playing the role of philosophical arbitrator between the conflicting views of his predecessors, he usually introduces qualifications or distinctions that break the impasse but yet retain the grain of truth in previous opinions.

This appears to be his strategy here when he introduces two sets of distinctions which bear on the problem of the infinite. The first is a general distinction between potential and actual modes of being, while the second is specific to the infinite; namely, that it can exist by addition or by division (since there are no indivisible lines). With the help of these distinctions, he can clarify what has been ruled out in previous arguments; namely, that any magnitude is actually infinite by addition. The major objection to this, we recall, is that the universe is finite, so that it is not possible to continue adding magnitudes of a constant size *ad infinitum* because these would eventually exceed the finite size of the universe, and this is considered impossible.

Thus there remains only the option that the infinite is potential, though that is not a simple solution because the term 'potency' has at least as many meanings as 'actuality.' Indeed Aristotle warns against

the naive assumption that the infinite is potential in the same way as the material of ordinary artifacts, since the infinite cannot be actualized in the same way that a statue can be completed from bronze. In view of his preliminary definition of the infinite as that which cannot be gone through, we may assume that here he means that the infinite can never be a completed whole like the bronze statue. This seems to be the point of his comparison with the mode of being of a day or of the Olympic games, which are always coming into existence one after the other. In the case of such things the distinction between potential and actual is different from that of an artifact, since there can be a day and there is a day, but proverbially there will always be another day. The analogy with the Olympic games also indicates how the infinite reveals itself in time.

On the basis of these analogies, Aristotle gives (206a27 ff.) a general account of the mode of being of the infinite in terms of the continual taking (τῷ λαμβανεσθαι) of one thing after another, with each thing that is taken being finite but always different. What is significant about this account is its emphasis on the activity of some agent by means of which the potential infinite is realized. Since Aristotle holds (*Met.* 1049a11 ff.) that every genuine potentiality in nature must be realized at some time or another if nothing prevents it, the question is whether the same holds for the potential infinite. He has already declared that it is not actualized as something complete like a statue or an animal, but perhaps like artifacts it is actualized by an external agent. In this regard he says (206a25–27) that the infinite shows itself in different ways; i.e. in time, in the generations of men, and in the divisions of magnitudes. Subsequently (206b1–3), he explains the difference as follows: in spatial magnitudes, what is taken (τοῦ ληφθέντος) persists, while in the succession of time and of men, each instance passes away but the supply never fails. Again the operative language suggests that in each case some potentiality is being realized in a finite way by an external agent.

This suggestion seems to be supported by the way Aristotle explains how the infinite by addition is the same as the infinite by division, since they both come about at the same time in an inverse fashion. What he has in mind here is that if one divides a line in a given ratio (say 1:2) then one is simultaneously adding parts with the same ratio within the line, so that one never exceeds the length of the line. And even if one externally adds parts in the same ratio, one will never exceed a finite magnitude; whereas if one continually adds

parts of the same size, one will eventually exceed the finite universe. The guiding rule in the latter case is that every finite magnitude is exhausted by the addition of any determinate quantity, however small.

With the help of these distinctions, Aristotle specifies (206b12) that the infinite exists only potentially and by reduction, while it is actualized only in the way that the day or the Olympic games are realized; i.e. one at a time but never all together. In a very significant parallel, he says that the infinite is potential in the same way as matter (ὡς ἡ ὕλη) and not as something definite (ὡς τὸ πεπερασμένον) like an independent thing. The parallel with matter suggests that the infinite is something that is by nature indefinite and in constant need of determination by some form. This is confirmed for the legitimate potential infinite by addition, which is the inverse of the infinite by division, since it means that it is always possible to take (λαμβάνειν) something outside without exceeding every determinate magnitude. By contrast, it is illegitimate to assume a potential infinite by the addition of equal parts, since this would exceed the magnitude of the universe. Here Aristotle is rejecting the view of some natural philosophers that there is an actually infinite body such as air outside the universe, while also insisting that such an infinite is not even potential because what is possible must be realized in the case of eternal things.

Thus, with a note of triumph, Aristotle declares the infinite to be the complete contrary of what people usually say about it; i.e. that it has nothing outside it. By contrast, he insists, the infinite is that which *always* has something outside it (οὗ ἀεί τι ἔξω ἐστί, 207a1). Similarly, a bezel ring is called infinite because it is always possible to take a part which is outside a given part. But the parallel is misleading in a way, since the parts must always be different in the case of the infinite. Thus, more precisely stated (207a7–9), something is infinite if, taking it quantity by quantity, we can always take something outside. By contrast, what has nothing outside of it is something complete and whole like an individual man. This definition of a whole as that from which nothing is lacking shows clearly why Aristotle insists that the infinite can never be a completed whole, since it is that which always has something outside or that which lacks a limit.

From my perspective, Aristotle's most interesting conclusion (207b27) is that his account does not deprive the mathematicians of anything they need for their science. In the light of Euclid's parallel postulate, one might have thought that Aristotle is doing precisely that by denying

an actual infinite in the direction of increase on the grounds that the universe is finite. But he claims that mathematicians do not need the infinite and that they do not use it because they postulate only that a finite straight line may be produced as far as they would wish. Clearly Aristotle bases his argument on the nature of the continuum which allows any large magnitude to be reproduced in the same ratio at a smaller size. Therefore, he claims, for the purposes of their proofs it makes no difference to mathematicians whether an actually infinite magnitude exists or not. What he is trying to establish here is that his own finite cosmology does not contradict anything in the mathematical sciences, since he wishes to validate these sciences for the physical world through the principle of continuity. But this also means that for Aristotle the infinite is more of a physical than a mathematical problem.

This becomes quite clear when he finally reviews the considerations previously listed in support of the infinite as something separate and determinate. For instance, Aristotle rejects the physical argument for the infinite as an inexhaustible source for generation because it is possible for the destruction of one thing to give rise to the generation of something else in a finite material universe. With regard to the claim that the infinite surrounds the finite universe, Aristotle distinguishes between touching and being limited as relative and non-relative attributes, respectively. The upshot of this distinction is that, while a finite universe would need another body (e.g. the infinite) to touch it, it does not need anything else to limit it. Therefore, according to Aristotle's cosmology, the universe is finite and limited only by its own external boundary.

Finally, with regard to the argument for an infinite universe which uses the fact that one can always think a greater magnitude (e.g. one can imagine sticking one's hand beyond its boundary), Aristotle says (208a15) that it is absurd to rely on thinking. While this is often taken as a general rejection of any role for thinking in connection with the infinite, the immediate context suggests that it is a very specific claim having to do with a case in which we imagine something that Aristotle holds to be physically impossible; i.e. going beyond the limit of the finite universe. In such a case, he says, it is absurd to rely on thinking because one could always imagine that someone is bigger than the size he is by magnifying him *ad infinitum,* but that would not make him bigger than he is. Similarly, one can imagine a body bigger than the universe but it does not follow that

there is such a body. According to *Categories* 12, the direction of causal influence is from the physical world to our thinking, and not *vice versa*. This epistemological 'realism' on the part of Aristotle has wide-ranging implications not only for his science of physics but also for mathematics.

III. *The eternality of the universe*

After arguing for the uniqueness of a finite and all-encompassing world[15] in *De Caelo* I.8–9, Aristotle subsequently (I.10–12) takes up the question of whether the universe is ungenerated or whether it has been generated at some time and so would be destructible at some other time.[16] As this was a controversial question in Greek cosmology, Aristotle begins with a survey of opinions from previous thinkers:

> But first let us run over the theories of others, since to expound one theory is to raise the difficulties involved in its contrary. At the same time also the arguments which are to follow will inspire more confidence if the pleas of those who dispute them have been heard first. It will not look so much as if we are procuring judgment by default. And indeed it is arbiters, not litigants, who are wanted for the obtaining of an adequate recognition of the truth.[17]

The epigrammatic first sentence of the passage is so condensed as to make its precise meaning unclear. In Aristotelian usage, the term 'aporia' usually implies that his purpose in reviewing opinions is to discover difficulties which will be resolved by his own solution.[18] This is consistent with the close link here (279b7–8) between difficulties (ἀπορίαι) and proofs (ἀποδείξεις) as joint subjects of the verb εἰσίν, whichever way we interpret the two occurrences of ἐναντίων in this

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[15] As Furley (1987a) points out continually, it is never legitimate to translate the Greek κόσμος into English as 'universe,' so here I use the neutral 'world.' But, since Aristotle holds the visible cosmos to be unique, it is legitimate in his case to use the words 'cosmos' and 'universe' interchangably, as I do throughout this chapter.

[16] Aristotle devotes a whole chapter (I. 11) to distinguishing the different senses of 'ungenerated' and 'generated,' as well as of 'destructible' and 'indestructible.'

[17] Cf. *De Caelo* I. 10, 279b6–12: tr. Guthrie (1939).

[18] With regard to an aporia the *Topics* says: "Similarly, too, it would be generally held that the equality of contrary reasonings is a cause of perplexity; for, when we are reasoning on both sides of a question and everything appears to have equal weight on either side, we are perplexed which of the two courses we are to adopt."—*Top.* VI.6, 145b17–20, tr. Forster (1960).

ambiguous sentence.[19] Therefore we must conclude that for Aristotle some kind of proof is given by collecting opinions, generating difficulties, and then resolving them.[20]

This description does not fit demonstrative proof but rather forensic proof, especially where Aristotle says that the arguments to follow will inspire greater confidence after a complete hearing has been given to the claims of the contending arguments, since this allays the suspicion that one side is trying to win the case in any underhand way. In the same forensic vein, he stresses that it is not litigants (ἀντιδίκοι) but arbitrators (διαιτηταί) who are needed to judge the truth of the arguments that will follow. Obviously he thinks that a survey of common opinions about any issue is conducive to that impartiality of mind proper to an arbitrator who tries to settle a dispute between plaintiffs prior to a court hearing. Hence, although the decision may not be demonstrably true, it still inspires confidence because both sides are given an impartial hearing.

Having reviewed the opinions of his predecessors about the duration of the world, Aristotle finds that they all agree about it being a generated thing but differ over the possibility of its corruption. (a) Some say that it is everlasting, even though it was generated; (b) others claim that it is perishable like the things constituted by nature. (c) Still others claim that at one time it is being generated, while at another time it is perishing, and that this continues always in such a way.

After sketching the opinions of predecessors, Aristotle tries to show that each position is untenable. The view that the world is generated but eternal, for instance, seems to be impossible because these are mutually exclusive characteristics. As a basis for this argument, Aristotle appeals (279b18–20) to the maxim that it is only reasonable to posit

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[19] Ken Quandt (1978) finds ambiguity in the referent of ἐναντίων, since it could refer to either (a) opposing opinions among his predecessors, or (b) opinions opposed to Aristotle's own. If it means (a) then a review will simply tell us that disagreements exist among previous thinkers. Quandt thinks that such a yield would not justify the labor of a comprehensive review, so he leans towards the second meaning. But I will argue in my fourth chapter that an initial review has the dual purpose of covering the logical possibilities and of generating an impasse through opposition—both of which would fit the language here.

[20] Simplicius (*in Cael.* 292.20–4) takes Aristotle to be saying that proofs for one set of opinions constitute puzzles for an opposing set of opinions; e.g. the arguments for the cosmos being generable constitute puzzles for those who hold it to be ungenerated. So whoever wishes to resolve the difficulties must examine the opposing opinions and assess the arguments in support of them, just as a judge might do.

what we see holding generally or universally. Some ancient commentators thought that Aristotle was directing his criticism at the talk about generating a cosmos in Plato's *Timaeus* (33B ff.), which might be understood metaphorically in terms of a contrast between the temporal duration of the sensible cosmos and the eternality of the supersensible world which contains its causal principles.[21] Aristotle seizes on the temporal meaning of the words 'to have a beginning' (ἀρχὴν ἔχειν), and constructs his argument in two parts to cover the logical possibilities; namely, either the world had a beginning or it did not. (a) If the present state of anything (e.g. the world or its elements) had no beginning and for all time prior to this it has been impossible for it to be otherwise than it is, then it is impossible for that thing to change. For, if it could change, this would be due to some cause which already was present in some way. But this would mean that what could not be otherwise (i.e. what is eternal) could have been otherwise, and this is a contradiction. Thus it is clearly impossible for something to change which has no beginning.

Aristotle now takes up the second possibility. (b) If one supposes that the world has been formed from elements which at one time were otherwise than they are now: (i) and if they had always been in that state and could not change, then obviously the world could never have been generated; (ii) but since it has been generated, it is clear that these elements must be capable of change and are not fixed forever in any present state. Hence their present combination will be dissolved, just as previous combinations were, and this process can go on indefinitely. If this is the case then the world is destructible, for either it has been or it might be other than it is.

In concluding his refutation of the view that the world has everlasting duration, even though it had a beginning, Aristotle rejects the attempt to defend this position by drawing an analogy with the method of geometers. Cherniss[22] conjectures that the passage reflects a debate among the pupils of Plato, and that Aristotle is responding here

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[21] Cf. Alexander apud Simplicius (*in Cael.* 293.14–15 & 296.5–6). Against the Aristotelian interpretation, Simplicius (296.16 ff.) reads the *Timaeus* as showing that the sensible world is meant to be generated, whereas the intelligible world is not. Further, with respect to the intelligible world, he treats all temporal talk as metaphorical, since Plato makes clear that time is generated along with the cosmos.

[22] Cherniss (1944) 421–2 interprets the subsequent Platonic tradition as showing that γενητόν as applied to the universe could be taken to mean that it is 'always becoming,' as distinct from the Forms which 'always are'; cf. *Tim.* 27D–28A. Thus Xenocrates, Speusippus and Crantor took the cosmogony of the Timaeus to be

to a polemical attack from Xenocrates. A lively debate about Plato's *Timaeus* is reflected in the fact that the metaphorical interpretation offered by some Platonists is flatly rejected as untrue by Aristotle (279b34). As he sees it, his opponents already have conceded that there is an apparent contradiction involved in holding that the world is indestructible (ἄφθαρτον) and generated (γενόμενον) at the same time. Thus their defense consists in claiming that the contradiction is only apparent because what is said about the generation of the world is analogous to the diagrams drawn by mathematicians. Just as the geometer 'generates' diagrams for the sake of instruction (διδασκαλίας χάριν), so also the Platonists talk about the 'generation' of the cosmos for the purpose of understanding it better.

Aristotle refuses to accept the analogy on the grounds that the two situations are not parallel. In the case of geometrical construction, when all the elements have been put together, the resulting figure does not differ in kind from them.[23] Yet for the Platonists the result does differ in kind from the original elements; i.e. ordered things (τεταγμένα) are generated from disordered things (ἐξ ἀτάκτων). But the same thing cannot be both ordered and disordered, since these two states must be separated by a process involving time as a concomitant of change in the physical world. In the case of mathematical diagrams, however, there is strictly speaking no such temporal process because it is completely incidental to his science that it actually takes the geometer some time to construct the figures which he uses for instructional purposes.[24] Therefore Aristotle denies that there are any grounds for the analogy between mathematical 'generation' and the physical generation of the cosmos.[25] And behind this denial lies his distinction between physics and mathematics, which undermines

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θεωρίας ἕνεκα, since the universe is without a temporal beginning but is γενητόν insofar as it is dependent on an external cause.

[23] Dancy (1991) 80–1 takes Aristotle to be arguing against the analogy drawn by the Platonists because in a typical creation story one starts with disorder and ends with order; whereas in a geometrical construction the initial stages are preserved in the result. While the latter does not involve a temporal process, the transition from disorder to order is inescapably temporal.

[24] Aristotle here seems to accept the Platonic view about the eternal and immutable nature of mathematical objects, though his specific point is that the sense of 'generation' in mathematical construction does not involve a temporal process, since this is one way of distinguishing mathematical from physical objects; cf. *Metaphysics* VI. 1.

[25] Simplicius (*in Cael.* 305.12–4) expresses surprise at Aristotle's failure to see that Plato's purpose was to analyse the sensible cosmos by analogy with hypothetical

the whole project of the *Timaeus*. In referring (*Cael.* 280a29) to this Platonic work, he attributes to it the view that the world has been generated but will last for all future time.

Finally, he concedes that the views of all these thinkers about the cosmos have only been discussed on physical grounds (φυσικῶς) based on the nature of the heavens, and so he promises (280a34) to clarify the question further by means of a more general (καθόλου) and all-embracing set of inquiries. It is not clear whether he ever fulfils this promise nor what sort of inquiry he may have in mind.[26] Since I am primarily interested in Aristotle's criticism of the *Timaeus,* however, I will follow a textual lead into Book III of the *De Caelo* which begins with an explicit reference back to a completed inquiry about 'the first heaven' (τοῦ πρώτου οὐρανοῦ) and its parts. Admittedly, such terminology was not used already, but Aristotle does draw freely upon principles and conclusions established in the previous two books.

IV. *Generation and corruption in the sublunary world*

Having already dealt with the immortal heavens, in Book III.1 Aristotle turns to the sublunary realm which is subject to generation and corruption, and he brings the four elements under two major genera; namely, the heavy (water and earth) and the light (fire and air).[27] Contrary to sense experience, some thinkers have challenged the reality of change even in this sublunary realm, and so Aristotle feels compelled to begin (*Cael.* 298b13) with the question of whether or not change exists.

With respect to the reality of change, however, he finds a wide range of disagreement; e.g. there are some thinkers who completely

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reasoning in geometry. But I doubt whether Aristotle missed this point, given that he made extensive use of such hypothetical reasoning in his own analyses of physical processes.

[26] Guthrie (1939) 102 thinks that Aristotle may be referring here to a more general treatment of generation and corruption. But that would also be a physical inquiry and it would hardly merit the epithet καθόλου if that is taken as being opposed to φυσικῶς. Alexander of Aphrodisias (*in Cael.* 312,18) takes the latter to be a reference back to *De Caelo* I. 3–4, where it was proved that there is no contrary to the physical heavens and hence no generation and corruption of the cosmos. Elders (1965) 157 suggests that καθόλου may be a reference to a more dialectical examination of the evidence or to an inquiry in first philosophy, but he gives us no hint as to where such an inquiry might be actually carried out by Aristotle.

[27] Cf. Simpl. *in Cael.* 20.10 & 555, 6–12

abolished generation and corruption because they claim that 'nothing of what is' (οὐθεν . . . τῶν ὄντων) either comes into being or perishes but only appears to us to do so (*Cael.* 298b15–18). However, after including the views of Parmenides and Melissus in a preliminary survey for his own physical inquiry about generation and corruption, Aristotle then denies that either of these thinkers is speaking physically (φυσικῶς), although he concedes they argue well (καλῶς). His explanation (298b19–21) is that it is not the task of physical inquiry but of another and prior study (προτέρας) to ask about the existence of beings that are ungenerated and wholly immovable. While he does not name this prior inquiry in *De Caelo*, his reply to the Eleatics in *Physics* I mentions both dialectic and philosophy.[28]

Along with denying them legitimacy as physicists, Aristotle offers a revealing analysis of what he takes to be the basic error of the Eleatic thinkers. According to him (298b22–24) they did not think that any beings existed besides the substance of sensible things (παρὰ τὴν τῶν αἰσθητῶν οὐσίαν), even though they were the first to realize that some such natures were necessary for knowledge or wisdom to be possible; so they simply transferred the descriptions of scientific objects to sensible things. In effect, he attributes to them a prototype of the argument 'from the sciences' which the Platonists later used to prove that Forms must exist. But the point of Aristotle's remarks is that the Eleatics were wrong in thinking that sensible things could be made the objects of science by imposing on them the conditions for scientific knowledge. Due to their failure to grasp this simple point, the Eleatics denied the reality of change in order to make the sensible world a permanent and eternal object of science.[29]

In concluding his summary of *doxa* about change, Aristotle focusses on the views of those who make all bodies generable as being composed out of planes and corruptible as being resolved into planes again.[30] Since he postpones indefinitely his critical treatment of the

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[28] Cf. *Phys.* I, 184b25 ff., 185a5 ff., 185a17 ff.

[29] I find this analysis to be particularly revealing of Aristotle's tendency to understand all of his predecessors within the terms of reference of his own problematic, which was decisively shaped by his long sojourn in Plato's Academy. Cherniss (1935 & 1944) complains bitterly about this tendency to distort the historical record, almost as if he were upbraiding a fellow member of the American Philological Society rather than a creative philosopher who lived in the 4th century B.C. But Aristotle himself is a good illustration of the essential element of 'prejudice' (in Gadamer's positive sense) that is involved in all interpretation of predecessors.

[30] συντιθέντες καὶ διαλύοντες εἰς ἐπίπεδα καὶ ἐξ ἐπιπέδων, 298b35.

other views, Aristotle's principal target here is clearly the 'generation' of bodies from planes in Plato's *Timaeus* (53C–55C), against which he directs two major criticisms. Given the mathematical character of this construction, his first objection (299a3–4) makes the surprising claim that it has implications which conflict with mathematics. According to Aristotle, it is obvious that the conclusions following from Plato's theory are unmathematical, though he does not give any grounds for his confident claim. However, we may find some hints in his warning (299a5–6) against undermining the foundations of a science, unless one can replace them with something 'more trustworthy' (πιστοτέροις).

If we consider Plato to be the superior mathematician, we may find it difficult to accept that the constructions in the *Timaeus* could be said to shake the foundations of mathematics.[31] Since Aristotle does not enlighten us further here, we must turn to a parallel passage where he speaks about the shaking of foundations. At *De Caelo* I.5 (271b5 ff.), for instance, he warns us to be very careful about fundamental principles because a small deviation in the 'beginning' (ἀρχή) multiplies error as the argument proceeds. For instance, he says (271b10–12), suppose someone were to claim that there exists a minimum magnitude (ἐλάχιστον μέγεθος) then that person with his minimum would move the greatest things in mathematics. In this context it might appear that he is giving a purely hypothetical case but Aristotle elsewhere (*Met.* 992a20–22) reports that instead of points Plato posited indivisible lines, which would conflict with the definition of a mathematical continuum.

The report is relevant to Aristotle's second major objection at *De Caelo* III.1, which continues his criticism of the Platonic 'generation' of bodies from planes as follows. The theory that solids are composed of planes implies (299a6–9), by the same reasoning, that planes are composed of lines and that lines are composed of points. Yet this implies that the parts of a line are 'indivisible lines' (ἀδιαίρετα μήκη), though these are non-existent entities, as Aristotle claims to have shown already in his 'accounts concerning motion.'[32]

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[31] Elders (1965) thinks that Aristotle's objection is that the shapes we find in nature cannot be constructed out of planes or that geometrical figures cannot be reduced to numbers. While this may be true, it does not identify the precise objection which Aristotle makes at the mathematical level.

[32] Guthrie (1939) and Elders (1965) both point out that this refers to *Physics* VI.1. In addition, Bos (1973) 69 assumes that this argument in *De Caelo* III.1 depends on Aristotle's discussion of the nature of the continuum in Book VI of the *Physics*.

Apart from the mathematical difficulties, one should not overlook the impossible implications for physical bodies which follow from positing 'uncuttable lines' (ἀτόμους γραμμάς). He explains that:

> The mathematical impossibilities will be physical impossibilities too, but this proposition cannot be simply converted, since the method of mathematics is to abstract, but of natural science is to add together all determining characteristics.[33]

This passage introduces a distinction between mathematics and physics for the purpose of criticising Plato's 'generation' of the elemental bodies in the *Timaeus,* yet that distinction is fudged somewhat by Aristotle's admission that mathematical implications apply in physics though not *vice versa.* In order to make sense of this, we shall have to be more circumspect than scholars who already 'know' that the distinction between mathematics and physics is made in terms of abstraction.[34] Taken literally, the passage says only that mathematical objects (τὰ μαθηματικά) are things spoken about as a result of subtraction (τὰ ἐξ ἀφαιρέσεως λέγεσθαι), while physical objects (τὰ φυσικά) are things spoken about as a result of addition (ἐκ προσθέσεως).

But how does this clarify Aristotle's explanation as to why some physical impossibilites are not also mathematical? Clearly there must be some areas of discontinuity between mathematics and physics, although a basic continuity remains so that mathematical consequences are applicable to physical bodies. It seems that Aristotle is talking about some schematic construction such that simpler elements are successively integrated into more complex structures by addition. Hence what holds for the simple elements in such structures holds also for the complex whole but not *vice versa.* For instance, the attributes of a line will also belong to the boundary of a plane but some of the attributes of a plane will not belong to a line. In fact, the Pythagorean schema of point, line, plane, and solid appears to provide a leading example of what Aristotle has in mind here. Thus one might say that the line is 'generated' from the point as a result of the addition (ἐκ προσθέσεως) of one dimension; cf. *APst.* 87a35–6.

[33] *De Caelo* 299a14–18: tr. Guthrie (1939).

[34] By way of commentary on the passage, Elders (1965) 275 says that ἀφαιρέσις denotes the process of abstraction which, he explains, enables the mathematician to abstract from the sensible properties of bodies and to study extension only. But such an interpretation is obviously informed by what Elders knows about abstraction from the whole Aristotelian tradition.

Similarly, the plane can be 'generated' from the line, and the solid from the plane, by adding one dimension in each case. When we have added the third dimension, however, the mathematical solid is complete and no further dimensions need be added to make a physical body. So the identification of mathematical and physical bodies is implicit in this Pythagorean schema.

Although Aristotle accepts that both kinds of body are 'complete' (τέλειον), insofar as each kind has three dimensions, yet he must find a way of differentiating them in order to secure his distinction between mathematics and physics in terms of their characteristic objects of inquiry. The crucial interpretative problem is how we are to understand the way in which he describes, and thereby distinguishes, the respective objects of mathematics and physics.[35]

Let us look again to the text for guidance. The passage may be read as distinguishing between mathematical and physical predicates (τὰ λέγεσθαι), as well as between the appropriate subjects to which such predicates belong. Thus mathematical predicates (τὰ μαθηματικά) are attributed 'as a result of subtraction' (ἐξ ἀφαιρέσεως), while physical predicates (τὰ φυσικά) are attributed 'as a result of addition' (ἐκ προσθέσεως). Since predicates are real things (ὄντα) that belong to real subjects, they can be part of a distinction between the sciences that is made in terms of their respective objects.

Such a distinction is implicit in the subsequent passage (299a18 ff.) where Aristotle says that there are many things which cannot belong to indivisibles (τοῖς ἀδιαιρέτοις οὐχ . . . ὑπάρχειν) but which must belong to physical bodies (τοῖς φυσικοῖς). This elaborates on his previous claim that many impossibilities follow from assuming that there are indivisible lines. For instance, he says (299a19–21) that it is impossible for any divisible attributes to belong in an indivisible subject (ἐν ἀδιαιρέτῳ . . . ὑπάρχειν); whereas physical attributes (τὰ πάθη) are all divisible in at least two ways, i.e. with respect to form (κατ' εἶδος) or accidentally (κατὰ συμβεβηκός). An attribute like color might be divisible with respect to form into white and black, for example;

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[35] In this particular respect, Guthrie's translation is misleading in so far as it suggests that the distinction is made in terms of differences between the respective *methods* of these sciences. Along with having no textual foundation, such a suggestion conflicts with Aristotle's standard practice of distinguishing sciences from one another in terms of their characteristic objects. Furthermore, this practice is very much in evidence when he makes the distinction between mathematics and physics in the passage cited above.

while the same attribute might be accidentally divisible because the proper subject to which it belongs is surface, which itself is continuous and thereby essentially divisible. From this perspective, therefore, we can better understand Aristotle's general conclusion that all simple attributes (ἁπλᾶ τῶν παθημάτων) are divisible when considered in these ways. Obviously, he thinks that he has established a framework within which to show the impossibilities arising from the attempt to 'generate' bodies out of planes.

V. *The problem of weight*

Having established this framework, Aristotle exploits (299a24 ff.) difficulties arising out of the Platonic theory in the *Timaeus* that all bodies are constructed from planes. One crucial difficulty concerns weight, since this is a property of sensible bodies but not of mathematical planes:

> If each of two parts is weightless, the two together cannot have weight; but sensible bodies, if not all, at least some, such as earth and water, have weight—so our opponents themselves would admit. Yet if the point is without weight, so also are lines, and if lines, then surfaces as well, and hence bodies.[36]

The success of the objection depends on convicting his Platonic opponents of inconsistency. On the one hand, Aristotle claims, they admit that some bodies such as earth and water have weight. On the basis of this admission, he points out that the Platonic theory of generation from planes is inconsistent because, if the point is weightless, the line and the plane and the solid will all be without weight. But Aristotle's objection seems to depend on two implicit assumptions: (i) that the plane is made up of lines as its constituent parts and that these lines, in turn, are similarly composed of points; (ii) that such a model of 'generation' by composition is identical with that proposed in the *Timaeus* for planes and bodies. While the first assumption might be inferred from the Pythagorean schema of point-line-plane-solid, it is a moot question whether this schema underlies the theory of 'generation' in the *Timaeus*. Aristotle clearly presumes that it does and therefore, by way of objection, gives a number of arguments to show that a point cannot have weight.

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[36] *De Caelo* 299a26–30: tr. Guthrie (1939).

The first argument (299a32 ff.) appeals to weight as a continuum having various degrees of heaviness and lightness. On account of this intrinsic characteristic, Aristotle claims that it is possible for everything which is heavy to be heavier (βαρύτερον) than something else and, similarly, for something light to be lighter (κουφότερον) than another thing. Here the possibility of atomic weights is ruled out for similar reasons as things which are absolutely small. Thus, if one says that the point has weight, this means that it must be divisible (διαιρετόν) because 'to be heavy' implies 'to be greater in weight' than something else. But, on the other hand, the point is posited (by definition) as being indivisible, and so it clearly cannot have weight.[37]

In a later argument (299b15 ff.), Aristotle attacks the apparent assumption behind the construction of bodies from planes when he objects that weight cannot be composed of parts that are weightless.[38] He dismisses as fictitious all claims about the quantity or quality of such weightless parts which are to compose a body with weight. For Aristotle the crucial point is that, since it is by virtue of weight that one thing is heavier than another, it follows that each indivisible part would have weight. For example, he says, if four points are assumed to make up a heavy body, while another body composed of more than four points is heavier, then it can be shown by simple subtraction that each point must have weight. If the second body is composed of five points then, assuming that what makes one thing heavier than another must itself be heavy, this body will be heavier than the first by virtue of one point (μιᾷ στιγμῇ); cf. 299b21.[39] Therefore it is clear that each point would have weight.

In general, Plato's mathematical reductionism appears to be Aristotle's real target when he introduces objections based on the assumption that weight and lightness are absolute properties of the elements. This is confirmed at *De Caelo* III.1, where Aristotle directs an objection against a specific claim made in the *Timaeus* about the bodily attribute of weight:

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[37] ἡ στιγμὴ ἀδιαίρετον ὑπόκειται, 299b7. Elders (1965) 276 claims that this argument is based on the way of predication rather than on physical facts. But this may be a misleading distinction, since it is quite clear from Aristotle's *Categories* that predicates are things and so their logical implications have physical ramifications.

[38] Ἀλλὰ μὴν οὐδ' ἐκ μὴ ἐχόντων βάρος ἔσται βάρος, *Cael.* 299b15.

[39] David Konstan (1988) notes that the comparison of bodies as differing in size by a single point resembles the kind of argument used by Eudemus in his refutation of minimalism.

> Again, if the difference in the weight of bodies is determined by the number of their surfaces, as the *Timaeus* affirms, then clearly lines and points will also have weight, since all these stand in a definite ratio to one another, as we have already noted. If on the other hand it is not this which determines the difference but the fact that earth is heavy and fire light, then surfaces too must be light or heavy, and by the same reckoning lines and points; for the surface which makes earth must be heavier than that of fire.[40]

This objection clearly depends on a specific passage in the *Timaeus* (56A–B) where it is said that, among the four elements, fire is the lightest because it is composed of the fewest identical parts (ἐξ ὀλιγίστων ξυνεστὸς τῶν αὐτῶν μερῶν, 56B3).[41] Some commentators treat this as a mere passing remark within a discussion that is mainly concerned with the mobility and stability of the elements. But it would be rather perverse of Aristotle to fasten upon an isolated remark and ignore the more detailed account of weight in terms of resistance that is elaborated later in the dialogue; cf. *Tim.* 63B–E. So I accept Denis O'Brien's (1984, 184–5) suggestion that, because of his own views on sensibilia, Aristotle takes the definition of weight by number as the 'objective' fact which is fundamental for Plato's whole theory of weight. Since Aristotle himself regards weight as an objective aspect of a perceptible object, it is understandable that he should not take seriously any theory of sensibilia that held them to be actually constituted by the percipient; cf. *De Anima* III.2.

In addition, I suspect that Aristotle may have been silent about Plato's more relativistic account of weight because it would escape the exclusive dichotomy on which his own objection rests; i.e. weight depends either on the parts or on the whole of a body. His argument is that if differences in weight depend on numerical differences between identical parts that make up each body, then each of the parts must itself have some weight.[42] On the basis of a single remark in the *Timaeus,* therefore, Aristotle infers that the mathematical surfaces which compose a body have weight, and he goes even further

[40] *De Caelo* III.1, 299b33–300a7: tr. Guthrie (1939).

[41] Significantly, this passage is partially quoted by Aristotle at *De Caelo* IV.2, 308b4–7.

[42] Aristotle takes Plato to be espousing a kind of mathematical monism according to which the basic triangles provide the single matter out of which all the elements are compounded; cf. *De Caelo* 308b11–12 & 309b33–34. From this he infers (308b21 ff.) that whatever body has fewer similar parts is lighter, while a body with more similar parts is heavier. But this means that a quantity of air could be heavier than water, which conflicts with the phenomena.

by claiming that the line and the point must also have weight, since there is a proportion (ἀνάλογον) between all of these. Although he does not specify this proportion, he may have in mind the following: as the line is a part of the surface, so the point is a part of the line.[43] If that proportional relationship holds then lines and points must also have weight, since that which has weight (i.e. surface) cannot be composed of parts without weight, as Aristotle has previously (299a26–31) argued. But this has already been shown to be absurd because the point is indivisible by definition and so cannot have weight. Hence neither can lines nor surfaces have weight, especially if they are composed of points or lines as parts. So Plato's account of weight seems self-contradictory.[44]

On the other hand, Aristotle argues, if the difference in the weight of bodies depends on the fact that earth is heavy or that fire is light, then their respective surfaces must be either heavy or light. In this second part of the argument he appears to draw out some implications of the view (which he basically accepts) that weight is an absolute property of bodies rather than a relative attribute which emerges through a concatenation of weightless parts. Thus, since weight forms a continuum that is indefinitely divisible, it follows that the parts of a heavy body like earth must also have some weight. Therefore, on the assumption that surfaces, lines and points are constitutive parts of bodies, all of these will themselves possess the property of lightness or heaviness. This means that one must differentiate between the parts of different bodies because, as Aristotle puts it: "the surface that makes earth must be heavier than that of fire" (*Cael.* 300a7). The crucial point of the argument, with respect to the project of the *Timaeus,* is that one cannot reduce bodies to undifferentiated mathematical planes which *ex hypothesi* are neither heavy nor light.[45] Basically, Aristotle's refutation rests upon the commonsense axiom that a sum of nothings never yields anything.

Thus he concludes his set of objections against the Platonic theory about the 'generation' of bodies with the claim that it is incoherent:

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[43] But, since he does not think that a line is composed of points as material parts, it is unlikely that Aristotle himself would accept such a proportion; cf. *Physics* VI.1.

[44] It is possible that Plato may have held points to be related to lines as surfaces to solids, given that he is reported by Aristotle (*Met.* I.9, 992a20 ff.) to have regarded the point as a 'geometrical fiction' and to have called it rather the principle of a line, or in effect an uncuttable line.

[45] See also *De Caelo* IV.2, 308b36–309a2.

> In sum, either there is no magnitude at all on their arguments, or magnitude can be annihilated, once granted that as the point is to the line, so the line is to the surface and the surface to body; for all can be resolved into one another, and hence can be resolved into the one which is primary, so that it would be possible for there to exist nothing but points, and no body at all.[46]

Once again, Aristotle's summary of the conclusions takes the familiar form of a dichotomy with both options being considered absurd. Although it is obvious to sense experience that magnitude exists and that its most complete form is body, yet the Platonic theory implies either that there is no magnitude or that magnitude can be annihilated. The first implication depends on the fact that Aristotle has already (268a7–10) defined magnitude in terms of continuous divisibility. But the *Timaeus* account involves stopping the division of bodies at some ultimate triangles and, since continuous magnitude cannot be composed of indivisible parts, it implies that there exists no magnitude corresponding to Aristotle's definition. When this is combined with the difficulties about weight, one can see how the first implication is drawn from the account in the *Timaeus*.

The second implication is more obscure and requires elucidation in terms of its Pythagorean assumptions. For instance, the conclusion that magnitude 'could be destroyed' (δύνασθαι . . . ἀναιρεθῆναι) depends on the assumption that just as the point is to the line so is the line to the plane and the plane to the body. The other premisses needed for Aristotle's argument are implicit in his talk (300a10–11) about the resolution (ἀνάλυσις) of body into planes, of plane into lines, and of line into points. Since these are resolvable into one another, they could be resolved into 'the firsts' or the points which survive the dissolution of bodies.

To clarify the bearing of such a hypothetical case on the *Timaeus,* one must fall back on conjectures about the relevant Academic background. The dialectical basis of the argument seems to be a well-known Academic principle of priority; i.e. that those things are prior whose destruction leads to the destruction of the whole; cf. *Met.* V.8, 1017b16–20. This ontological principle was used to give greater substantiality to surfaces over bodies, to lines over surfaces and, ultimately, to numbers over everything else; cf. *Met.* III.5, 1002a6–8. So he may be referring to some such principle here because its ontological

[46] *De Caelo* III. 1, 300a7–12: tr. Guthrie (1939).

assumptions are consistent with the reductionist project of the *Timaeus*. In fact, when he deals with 'the prior and posterior' in *Metaphysics* V.11, he explicitly attributes to Plato a similar principle for deciding what is prior and posterior 'according to nature and substance' (κατὰ φύσιν καὶ οὐσίαν). Hence the metaphysical baggage[47] carried by this principle lends greater significance to its use at *De Caelo* III.1 in the summarizing argument directed against the *Timaeus*.

Along the same lines, Aristotle argues against Pythagoreans who try to construct the world out of numbers, since this is simply a more extreme form of what one may call mathematical reductionism:

> There are some, e.g. certain Pythagoreans, who construct nature out of numbers. But to construct the world of numbers leads them to the same difficulty, for natural bodies manifestly possess weight and lightness, whereas their monads in combination cannot either produce bodies or possess weight.[48]

In his translation Guthrie transposes the first and second sentences of the passage, thereby concealing the continuity with what has gone before. In fact, the passage begins by emphasizing that the same conclusion (i.e. the destruction of magnitude) follows for those who construct the heaven out of numbers. While 'natural bodies' (τὰ φυσικὰ σώματα) obviously have weight and lightness, the Pythagorean 'monads' neither have weight nor do they make a body (σῶμα) when they are put together (300a17–19). Here, in typical Aristotelian fashion, the plain evidence of the senses is contrasted with the implausibility of the Pythagorean supposition.[49]

Against such mathematical reductionism, therefore, Aristotle directs his most trenchant objections based on the irreducibility of physical properties like weight and lightness, since these properties are characteristic of physical rather than of mathematical bodies. By means of such sensible properties, Aristotle is able to make an initial distinction between physics and mathematics, even though he still admits that there is some overlap between their characteristic objects. For example, at *De Caelo* I.1 (268a23), he asserts that body is the only complete magnitude, but it is not clear whether he means mathematical or physical body. It is precisely this ambiguity which

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[47] Cf. Cleary (1988b) ch. 3.

[48] *De Caelo* 300a15–19: tr. Guthrie (1939).

[49] *De Caelo* IV.2, 309a26–27 also uses sensible phenomena as a last court of appeal in physics.

eventually forces Aristotle to ask deeper metaphysical questions about the foundations of mathematics.

VI. *The problem of movement and rest*

Since Greek mathematics deals almost exclusively with static numbers and figures, the problem of motion becomes doubly difficult for any cosmological account based on mathematics. With typical dialectical adroitness, Aristotle exploits this difficulty to drive a wedge between mathematics and physics, which is described as the science of motion in the *Physics*. But this division is not so obvious in *De Caelo* I (268a1–6) where the science of nature is said to deal with bodies and magnitudes, together with their attributes and motions and principles. Such a description of the subject-matter of physics does not immediately preclude the possibility of its fundamental principles being mathematical in character.[50] However, this is excluded in Book III when he assumes that the four elements, along with their irreducible qualities, are the basic constituents of natural bodies. Having examined his arguments for the irreducibility of physical properties like weight, let me now survey some arguments based on the observed motion of the four elements.

First he seeks to establish a basic premiss for all subsequent arguments; namely, that the elements necessarily have their own natural motions (300a20–27). Just as in his argument for the uniqueness of the world at *De Caelo* I.8–9, Aristotle's fundamental assumption here is that all motion is either natural or forced. Thus, from the observed motion of the elements, Aristotle concludes that this motion will be forced unless each body has its own proper motion. The necessity of this conclusion is primarily logical, since it depends on 'natural' and 'by force' being a mutually exclusive and exhaustive list of possible kinds of simple motion. This is implicitly confirmed by the identification of 'by force' with 'contrary to nature,' as this implies that unforced motion is the natural or proper motion of a simple body. With this dichotomy in place, the rest of the argument becomes more intelligible.

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[50] Even though Aristotle prohibits the reduction of body to surfaces as involving a transition to another genus, this does not rule out the suggestion of the *Timaeus* that the principles of stereometry might be fundamental for understanding physical bodies.

But Aristotle also assumes that natural motion is somehow prior to (e.g. presupposed by) forced motion when he claims (300a24–26) that, if there is some motion contrary to nature, there must also be a motion according to nature from which the former diverges. While natural motion might be prior merely in relation to us, he seems to treat it as prior *simpliciter*.[51] This point becomes important later for his argument against the Atomists who posited the random motion of atoms. Another important principle of priority seems to be involved in Aristotle's conclusion (300a26–28) that the natural motion of each body is unique because it is simple (ἁπλῶς), whereas forced motion is complex and manifold.

The subsequent argument (300a27–b2) that there is a natural resting-place for each simple body is a corollary of the previous argument for natural motion. Aristotle begins with the familiar dichotomy between natural and forced resting, which corresponds to natural and forced moving. His second step is to enunciate the general principle that wherever a thing was brought by force there it stays by force and, conversely, that wherever something goes by nature there it stays by nature. This principle presupposes the existence of special places and privileged points of view in the universe. For instance, when Aristotle says (300a30) that there appears (φαίνεται) to be something remaining at the center, he appeals to our experience of this earth as a stable center of the visible world; cf. *De Caelo* II.13.

So Aristotle begins his argument 'from rest' with an appeal to sense experience that provides him with his first solid step: (1) that the earth (as one simple body) remains permanently at the center of the universe. The next steps in the argument seem to depend once more upon the exhaustive dichotomy between natural and forced motion: (2) If it is natural for the earth to remain at the center, then clearly its motion to that place is also natural. (2a) On the other hand, if it is resting there by force, what prevents it from moving? (3) If one supposes that it is being prevented by something which itself is at rest, the question arises again for this latter thing (i.e. Is it resting naturally or by force?) and one has gone full circle. Hence (4) it is necessary either that 'the first rester' (τὸ πρῶτον ἠρεμοῦν)[52] be naturally at rest or that one goes on to infinity, which is impossible.

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[51] It could also be cited as an illustration of priority in definition, since forced motion seems to be defined with reference to natural motion; cf. *Met.* VIII.8, 1049b12 ff.

[52] Presumably Aristotle is here referring to the earth at rest in the center of the

Having set down the principles for the discussion, Aristotle gives a critical review of the *doxa* about the motion of the elements. First he takes up the views of the Atomists, even though they did not hold the Empedoclean doctrine of four elements:

> When therefore Leucippus and Democritus speak of the primary bodies as always moving in the infinite void, they ought to say with what motion they move and what is their natural motion. Each of the atoms may be forcibly moved by another, but each one must have some natural motion also, from which the enforced motion diverges. Moreover the original movement cannot act by force, but only naturally. We shall go on to infinity if there is to be no first thing which imparts motion naturally, but always a prior one which moves because itself set in motion by force.[53]

In the case of the Atomists, the so-called 'first bodies' (τὰ πρῶτα σώματα) are the atoms which move separately in the infinite void and collide in a random fashion.[54] Aristotle obviously thinks that this is not an adequate account of their movement, since it does not identify their natural motion. But the Atomists could refuse to accept Aristotle's dichotomy of natural and forced motion simply by insisting that the natural motion of the atoms also involves mechanical contact between the atoms.[55]

This highlights, once again, the set of implicit assumptions that lie behind Aristotle's criticism of his predecessors. With respect to the Atomists, he would probably side with Socrates in the *Phaedo* who complains that low-level mechanical causes are inadequate to explain purpose and order in the universe. This may be one reason why, in the present argument against the Atomists, he insists that 'the first moving' (ἡ πρώτη κινοῦσα) cannot move by force but only naturally. If there is no first thing that moves naturally, he argues (300b15–17), then we are led into an infinite regress because there is always a

universe, by contrast with the first heaven which is eternally moving at the circumference. Commentators have disputed whether or not the Prime Mover makes an appearance in the *De Caelo* but very few, as far as I am aware, have attached importance to the argument for the earth as a prime rester.

[53] *De Caelo* 300b9–16: tr. Guthrie (1939).

[54] Cf. Simplicius, *in Cael.* 242, 21 ff.

[55] Furley (1987 & 1989) suggests that the Atomists were still imprisoned in the old linear view of the universe according to which the parallel 'downward' motion of heavy bodies is natural, by contrast with the newer centrifocal view in which heavy bodies move towards the centre at different angles. But if that were the case then surely Aristotle's objection about the neglect of natural motion for atoms would show almost wilful blindness to what the Atomists were trying to say.

prior thing that moves on account of being set in motion by force. The infinite regress here is generated by Aristotle's demand that, for everything which is moved by force, one must find the natural motion of that thing or of another thing which is causing its motion. Ultimately, such a demand is motivated by the metaphysical assumption that natural rest and motion reflect the eternal order of the universe, whereas forced motion and rest do not.

From within such a metaphysical framework he can now launch his attack on Plato, even though they share many of the same cosmological assumptions. But Plato has an Achilles' heel which Aristotle attacks with considerable dialectical cunning:

> We get the very same difficulty if we accept the account of the *Timaeus,* that before the creation of the cosmos the elements were in disorderly motion. The motion must have been either enforced or natural. But if it was natural, careful consideration will show that there must have been a cosmos. For the first mover must move itself (since its movement must be natural), and things whose motion is without constraint must find rest in their proper places and therefore form the same arrangement as they do now, the heavy travelling to the centre, and the light away from the centre. But that is the disposition of the cosmos.[56]

This continues the previous argument against the priority of random (i.e. disorderly) over natural (i.e. orderly) motion, which is now directed explicitly at *Timaeus* 30A where it is said that the Demiurge took over all that was visible, which was previously not at rest but in a state of discordant and disorderly motion, and brought it into an ordered state because he judged it to be better in every way.

Although the claim about the superiority of the ordered state might be interpreted as Plato's commitment to the ontological priority of order over disorder, Aristotle's objection focuses exclusively upon the temporal priority of disorder in the *Timaeus* account. With respect to the disorderly motion of the elements which is held to precede the generation of the cosmos, he insists that this must be either forced or natural motion since that is the logical dichotomy that he has consistently applied to motion and rest. But he takes up only one side of the division because he has already argued that if the primary motion is said to be forced, this will lead to an infinite regress. Therefore he discusses only the possibility that the disorderly motion might be natural. If this is the case then there must already be a cosmos,

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[56] *De Caelo* 300b17–26: tr. Guthrie (1939).

since the very notion of 'cosmos' involves motion according to nature.

Yet the explanatory argument that Aristotle gives is rather obscure, since it depends on many previous arguments that are not explicitly mentioned. The first step in his explanation, for instance, is the claim that the first mover necessarily moves itself.[57] The necessity of this claim is not obvious unless we recall that, for Aristotle, being moved by something external constitutes forced motion and this leads to an infinite regress if it is attributed to the first mover. Therefore, since the infinite cannot be traversed and yet there is motion in the universe, it must be the case that the first mover moves itself and is yet a moved thing according to nature. But, the argument continues (300b23), the things that are moved without force also find rest in their proper places. Thus they will yield the same order as they have at present; i.e. the heavy things will move towards the center and the light things away from it. And, Aristotle concludes triumphantly, this is precisely the arrangement which presently constitutes the cosmos. To some extent his argument here depends on the similarity in meaning between τάξις and κόσμος, since both signify an ordering or arrangement of things. Furthermore, he assumes that disordered (or forced) motion must be preceded by ordered motion, on account of the logical relationship that holds between them. But all of this is still not enough to establish that the present cosmos is identical with the order which he insists must have preceded the disorder of the elements that is described in the *Timaeus*. In order for this to be established, he must have silently assumed that the cosmos is unique and eternal.

The point of this particular attack on the *Timaeus* is that the mathematical ordering by the Demiurge therein described cannot be the first and eternal order of the universe. This is a consequence of Aristotle's claim that the disorderly motion in the Receptacle (which is taken over and ordered by the Demiurge) itself presupposes an order (cosmos) in which the elements move naturally. If this prior order is unique and eternal then Aristotle can argue that mathematics alone does not account for motion (or change) in the universe. As evidence for this, he can refer to the *Timaeus* where Plato has to posit both an original moving chaos and a Demiurge to impose mathematical order on it.[58] Hence the mathematical explanation is

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[57] τό τε γὰρ πρῶτον κινοῦν ἀνάγκη κινεῖν αὑτό, *Cael.* 300b21–22. See also *Physics* VIII.5.

[58] See *Timaeus* 52E–53A where motion is said to separate out heavy from light

superfluous because it does not explain even the obvious motions of bodies; e.g. that heavy bodies move down, while light bodies move up. In a subsequent series of arguments in *De Caelo* III.2, which I will not rehearse here, he tries to show that the sublunary bodies owe their natural impulse to weight or lightness.[59] Contrary to the account given in the *Timaeus*, this implies that weight and lightness are irreducible properties of simple bodies which cannot be explained by mathematics alone.

VII. *The sublunary elements, their nature and generation*

Let us now consider a related problem about the sublunary elements, which is central to Aristotle's criticism of the *Timaeus* and its mathematical cosmology. At the beginning of *De Caelo* III.3, he raises questions about what bodies are subject to generation and why. Before answering these questions, however, Aristotle addresses other issues about the nature of the elements, their number, and their character. Such questions are both epistemological and ontological, since knowledge is always to be sought through what is primary, and the primary constituents of bodies are their elements (*Cael.* 302a11). Hence the prior questions about the elements are: (a) which of the constituting parts of bodies are elements? (b) and why? (c) how many elements are there? (d) and what is their character?

As a first step towards answering all these questions, Aristotle defines the nature of an element:

> Let us then define the element in bodies as that into which other bodies may be analysed, which is present in them either potentially or actually (which of the two is a matter for future debate), and which cannot itself be analysed into constituents differing in kind. Some such definition of an element is what all thinkers are aiming at throughout.[60]

Here Aristotle claims to be elucidating the concept of 'element'

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things, so that different kinds already have different regions, even before the generation of the cosmos. Yet Plato insists that the four elements had only 'traces' of their own natures, since they were in a chaotic condition in the absence of the divine being who is said (53B) to provide definite configuration by means of shapes and numbers.

[59] *De Caelo* IV.4 argues that fire and earth are absolutely light and heavy, respectively; while air and water are relatively so, since they are heavier than fire but lighter than earth.

[60] *De Caelo* III.3, 302a16–19: tr. Guthrie (1939).

implicit in current usage, as in *Metaphysics* V.3 where he gives a similar definition of στοιχεῖον. In both places the two major criteria used to identify an element are: (i) that it is a constituent (ἐνυπάρχον) of other bodies; (ii) that it is indivisible in kind (ἀδιαίρετον τῷ εἴδει). The second criterion here means that elemental bodies cannot be divided into bodies of another kind, though as extended magnitudes they may be indefinitely divisible in quantity. Even after such unlimited division, however, all their parts are still of the same kind, e.g. the parts of water will remain water; cf. *Met.* 1014a30–31.

Armed with his definition, Aristotle argues that some such elemental bodies must exist. For instance, he claims (*Cael.* 302a21–3) that fire and earth are potentially present in flesh and wood and other such sensible bodies because they can become apparent (φανερά) on being separated out (ἐκκρινόμενα) from them. Aristotle seems to have in mind such sensible phenomena as burning and rotting, which he would probably see as the dissolution of complex bodies into the simple elements that were potentially present in them. But the most difficult problem is how the basic elements are themselves generated, and he is convinced that none of his predecessors have solved it satisfactorily, least of all the Platonists.

After arguing in a later section (III.6, 304b23–305a13) that the elements are not eternal but generable, Aristotle considers all possible sources for their generation: (a) from something incorporeal or (b) from something corporeal. If the source is corporeal then it is either (bi) some body other than the elements or (bii) each other. After laying out the possibilities, he shows that all but the last are in fact impossible. The possibility that the elements might be generated from something without body implies that there could be a 'separated void' (ἀφωρισμένον κενόν), which Aristotle takes to be impossible; cf. *Physics* IV.6–9. Here he contents himself with showing how this impossible implication follows from the theory that the elements are generated from the incorporeal. The crucial link is the general principle that everything which is generated comes to be *in* something. Now the place in which the generation takes place must be either (i) without body or (ii) with body. But if the place already 'has body' then there will be two bodies in the same place at once, which is impossible. On the other hand, if the place is 'without body' then it is a separated void, which was also shown to be impossible. That effectively gets rid of the first possibility; i.e. (a) that the elements are generated from something incorporeal.

The second possibility has two parts, the first (i.e. 2a) of which becomes the target for elimination. The refutation begins with an appeal to a principle of priority: if the elements are generated from some other body, then that other body will be prior to the elements. Suppose, for the sake of argument, that there is such a prior body; one can ask whether it has weight or lightness. If it has either quality then Aristotle claims that this body must be identical with one of the elements, since they are determined in relation to heaviness and lightness. If, on the other hand, it has neither quality, the body will have no inclination in any direction so it will be an unmoved mathematical object (ἀκίνητον καὶ μαθηματικόν), which will not be in a place. Notice that this passage contains a clear distinction between mathematical and physical bodies in terms of occupying a place.[61] Such a criterion is important for the rest of his reductio argument and, as we shall see later, for his whole criticism of the *Timaeus*.

Through a process of elimination, therefore, Aristotle moves towards his own conclusion about the body from which the elements must be generated:

> If it rests in a place, it will be possible for it to move in that place, if by force, then unnaturally, or if not by force, then naturally. If then, it occupies a place somewhere, it will be one of the elements. If not, nothing can be generated from it, for that which comes to be and that from which it is generated must be together. Seeing therefore that the elements can neither be generated from the incorporeal nor from an extraneous body, it remains to suppose that they are generated from each other.[62]

Assuming that this is not a motionless mathematical body, Aristotle holds that it must be resting in some place and this means that it is

[61] In gathering the phenomena about place in *Physics* III. 4–6, Aristotle refers to the tendency of heavy objects to fall down and of light objects to rise up. But, along with these phenomena, he also appeals to his own theory of natural places which implies that 'up' and 'down' are absolute directions and not merely relative to us (πρὸς ἡμᾶς). If places and directions were not absolute but merely relative to us, then they would not remain the same as we changed our position. Aristotle says (208b22 ff.) that this is clearly the case for mathematical objects (τὰ μαθηματικά) which are not beings in place, even though with respect to position (κατὰ την θέσιν) they have a left and a right relative to us. But these relative positions are only so called by convention, since they do not have a natural basis. Here we find a clear distinction between mathematical and physical things being reported as one of the relevant phenomena about place. Natural places are differentiated not only by their position by also by their power (τῇ δυνάμει) to influence the motion of elemental bodies; cf. 208b21–22.

[62] *De Caelo* III.6, 305a27–32: tr. Guthrie (1939).

also possible for it to move to another place. If this requires force then its motion is contrary to nature but, if not, then its motion is according to nature. Whichever is the case, the body will still be one of the elements as long as it has its own proper place somewhere. As already noted, Aristotle assumed a limited number of proper places in the universe because there are a finite number of absolute directions; i.e. up and down. But these are exactly the assumptions introduced here along with the distinction between natural and forced motion. If one accepts this distinction then one must also accept a limited number of elements with their own proper places.

Thus, in a certain sense, Aristotle is guilty of begging the question, even though he does offer a powerful supporting argument here about the necessity of generation occurring in some place. His major point is that, in order for any generation to happen, it is necessary (ἀνάγκη) for the thing being generated (τὸ γινόμενον) and the thing from which it is being generated (τὸ ἐξ οὗ γίγνεται) to be together (ἅμα) in place and time; cf. *Cael.* 305a30–1. This appears to be his version of a necessary condition for causal interaction; namely, that the cause and the effect must be contiguous in space and time. However, this apparently innocuous condition for the generation of the elements undermines the mathematical cosmology of the *Timaeus*, since mathematical objects are not in any place, even though Plato supplies the Receptacle for generation. Yet here Aristotle only draws the conclusion that the elements must be generated from each other, since he has already eliminated the other two possibilities; namely, that they might be generated from some other body or from something without body. It is noteworthy that Aristotle does not here posit some 'prime matter' from which the elements might be generated, as a substitute for the Receptacle in the *Timaeus*.

VIII. *The mode of generation of the elements*

Having determined through a process of elimination that the elements can only be generated from each other, Aristotle next turns to the question about the manner in which this generation takes place. The possibilities seem to be specified, once more, in terms of the various opinions about this question: (i) the way described by Empedocles and Democritus, or (ii) by those who dissolve bodies into surfaces or (iii) some other way. In his usual manner, Aristotle

approaches the question by showing up the error in previous answers, while preserving whatever is truthlike in them; and his most extensive critique of the *Timaeus* is found within this dialectical discussion of the question about the mode of generation of the elements from each other. In fact, I think it is a mistake to isolate his criticism of Plato from its specific context, since Aristotle is thereby made to appear excessively polemical. Contrary to Taylor and Cherniss, I will try to show that many of his criticisms of the *Timaeus* are reasonable, given the general set of assumptions about natural change which Aristotle makes.[63]

As a result of all the objections he raises against the views of Empedocles and Democritus, Aristotle concludes that the transition from one element to another cannot happen by a process of separating out which leaves the basic elements unchanged. But the only alternative is that they are generated by some kind of actual transmutation into one another and Aristotle lists two different kinds that have been suggested: (a) by change of shape (τῇ μετασχηματίσει), just as either a sphere or a cube might come to be from the same piece of wax; (b) by the dissolution into surfaces (τῇ διαλύσει τῇ εἰς τὰ ἐπίπεδα), as some people claim. It is clear from what follows that the latter view is that of Plato's *Timaeus,* while the first involves some kind of atomism.[64] This is confirmed by his single objection to it:

> Change of shape carries with it the necessary corollary that there are indivisible bodies. (For) were all bodies divisible, a part of fire would not be fire, nor a part of earth earth, because the part of a pyramid is not always pyramidal nor of a cube cubical.[65]

We should notice immediately that this objection depends upon drawing out some logical implications from the theory of transformation by change of shape. We may find a clear parallel with a previous objection (304b2–12) to those who posited fire as the basic element and gave it a specific shape. If, on the one hand, they treat this

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[63] Although I had developed my own approach through an independent reading of Aristotle, I find myself in agreement with Denis O'Brien's (1984) general interpretation of the philosophical relationship between Plato and Aristotle, and especially on the question of weight.

[64] In the parallel discussion in *De Generatione et Corruptione* I.2, however, Aristotle says that the Atomists explained generation and corruption in terms of the association and dissociation of atoms, which themselves do not change shape. So the view being criticised here can hardly be that held by Atomists like Democritus, and indeed it may be the product of Aristotle's own logical analysis.

[65] *De Caelo* 305b32–306a1: tr. (with one addition) Guthrie (1939).

shape as indivisible then they are open to objections based upon Aristotle's definition of the continuum. But, on the other hand, if they make it divisible then those people who assign a figure to fire will be implying that a part of fire is not fire because a pyramid is not composed of pyramids.

In both passages he assumes that it would be unacceptable to have parts of the basic element which were not of the same shape as the element itself, and this assumption rests upon the definition of an element with which he began the whole discussion; cf. *Cael.* 302a16–19. For instance, if the shape assigned to fire is divisible into other different shapes, then the parts of fire are different in kind (ἕτερα τῷ εἴδει) from fire itself because (according to this theory) the shape defines the element. But that would mean that fire is not a basic element, and so these theorists are refuted by the (hidden) consequences of their own theory of generation through change of shape. The only way to save the theory is to make the basic shape (or element) indivisible but this logical option is itself open to the objections which were previously brought against indivisible bodies.[66] Thus the theory seems impossible whichever logical route one takes.

Let us now examine his series of objections against the second theory of transformation among the elements, which was proposed by Plato. In contrast to the previous theory, we should notice that many of these objections hinge on what was actually said and on how this is either inconsistent, absurd, or non-explanatory. For instance, his first objection goes as follows:

> But if (b) the process is one of analysis into surfaces, there is the absurdity of not allowing all the elements to be generated from each other. This they must and do uphold. But for one element alone to have no part in the change is neither logical nor apparent to sense: all should change into each other without discrimination. These philosophers find themselves, in a discussion about phenomena, making statements with which the phenomena conflict. This is because they have a wrong conception of primary principles, and try to bring everything into line with hard-and-fast theories. For surely the principles of sensible things are sensible, of eternal things eternal, and of perishable, perishable: put generally, a principle is of the same genus as what falls under it. Yet out of affection for their fixed ideas these men behave

[66] See parallel passage in *De Generatione et Corruptione* I.2, 315b25 where Aristotle says that, since it is contrary to reason to stop the analysis at planes, it is more reasonable to posit indivisible bodies, even though this also involves considerable difficulty.

> like speakers defending a thesis in debate: they stand on the truth of their premises against all the facts, not admitting that there are premises which ought to be criticized in the light of their consequences, and in particular of the final result of all. Now practical knowledge culminates in the work produced, natural philosophy in the facts as presented consistently and indubitably to sense-perception.[67]

This is an important passage because it sets the tone and identifies the target for subsequent objections. The tone is quite clearly polemical, while the target for most of the objections is the theory of the generation of the elements given in the *Timaeus.* For instance, according to this account (*Tim.* 54B–D), only three of the elements can be generated from one another because they are composed from and dissolved into the same basic triangle. Since earth is assigned a different constituent triangle, the theory says (56D) that earth lies outside the cycle of transformation.

But this is precisely what Aristotle finds unreasonable, and indeed, he thinks it absurd (ἄτοπον) that they do not allow all of the elements to be generated from each other. The logical overtones of his objection here suggest that, along with making an explicit appeal to sense experience, he is also implicitly appealing to some principle of sufficient reason. This is suggested, for instance, by the Greek syntax when he says that it is neither reasonable (οὔτε εὔλογον) nor is it apparent to sense experience (οὔτε φαίνεται κατὰ τὴν αἴσθησιν) that one of the elements should be excluded from the transformation. However, he does seem to rely principally on sensation for his assertion that, contrary to what the Platonists say, all the elements are alike transformed into each other. This amounts to a refusal on his part to accept the mathematical rationale for excluding earth from the cycle of transformation, since it fails to 'save the phenomena.'

In effect, Aristotle accuses the Platonists of making statements which are not consistent (μὴ ὁμολογούμενα) with the phenomena. If we take phenomena here to be the sensible appearances of things, then perhaps there is some basis for Aristotle's accusation.[68] As we have already seen in the *Timaeus,* Plato appears to change his mind about the apparent transformability of all the elements for reasons that

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[67] *De Caelo* III.7, 306a2–18: tr. Guthrie (1939).

[68] Gregory Vlastos (1975) 82 ff. argues to the contrary that there were no 'hard facts' which could have decided in Aristotle's favor against Plato, or even against the physiologists who completely denied the transformation of the elements into each other. The historical reason, according to Vlastos, is that in the field of terres-

are primarily mathematical. Hence the basic gripe about Plato's method should be that he always gives priority to mathematical over physical reasons.

Aristotle must have something like this in mind when he complains that the Platonists do not have a proper conception of first principles because they wish to bring everything into line with some of their own definite opinions.[69] In this context he appeals to his own methodological rule that principles must belong to the same genus as the subject-matter to which they are applied.[70] For example, the principles of sensible things should be sensible; those of eternal things eternal; while the principles of perishable things should be perishable. In general, he insists (306a9–12), the principles should be of the same kind as the subjects (ὁμογενεῖς τοῖς ὑποκειμένοις). Thus when he distinguishes the 'kinds' according to whether they are sensible, eternal or perishable, this is a way of introducing the distinction between mathematics and physics, without begging the question at issue. If he can justify this distinction then he has, in effect, denied legitimacy to a cosmology that subordinates physics to mathematics in the manner of the *Timaeus.*

Along these lines he criticises those who cling to their mathematical principles in spite of the phenomena, by comparing them to eristic debaters who defend their position at all costs. As far as Aristotle is concerned (306a15–16), it is a mistake to defend one's principles in this way against all consequences, as certain theses ought to be judged from their consequences (ἐκ τῶν ἀποβαινόντων) and, especially, from their end result (ἐκ τοῦ τέλους). In the case of a productive science the goal is the end-product (τὸ ἔργον) but physics is a theoretical science and therefore its goal is to explain the appearance which is always presented most properly to sense perception.[71] Aristotle here makes sense experience a touchstone for the adequacy of physical explanation in a way that would hardly be acceptable to Plato who considered sensible appearances to be potentially confusing, unless they were clarified by a rational (or mathematical) account. However, Aristotle

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trial physics, unlike astronomy, no corpus of satisfactory 'hard facts' was available to decide between two competing theories. Since each theory is safe from refutation by experience, it cannot be confirmed by it either for the very same reason.

[69] πάντα βούλεσθαι πρός τινας δόξας ὡρισμένας ἀνάγειν, *Cael.* 306a8–9.

[70] Cf. also *APr.* 46a3–27, *APst.* 71b19–25, *Met.* 1000a5–9, 1000b21–1001a3, 1060a27–36, 1075b13–4.

[71] τὸ φαινόμενον ἀεὶ κυρίως κατὰ τὴν αἴσθησιν, *Cael.* 306a16–17.

cannot mean that the Platonists had completely abandoned the explanatory ideal of saving the phenomena, since that was part of the heritage of Plato.[72] It is more likely that, for his own dialectical purposes here, Aristotle has confined the meaning of 'phenomena' to things that are apparent to the senses.[73]

An illustration of this dialectical technique is to be found in the subsequent objection which clearly presupposes his own conception and definition of an element:

> Their conclusion must mean that the earth has best right to be called an element, and is alone indestructible, if it is true that what cannot be dissolved is indestructible and an element; for the earth alone cannot be resolved into any other body.[74]

This particular objection is regarded by Taylor (1928, 404) as being little more than an expression of Aristotle's own prejudice that the four simple bodies are all on the same level. Cherniss[75] seems to agree with Taylor's diagnosis when he suggests that Aristotle's own doctrine forms the basis for his refutation of Plato's exclusion of earth from the cycle of transmutation, and that this doctrine itself is a consequence of the theory that each element is 'prime matter' informed by two of the four contrary qualities. But all his citations in support of this diagnosis are from *De Generatione et Corruptione*,[76] which suggests that Cherniss may have overlooked the fact that at *De Caelo* 305a27–32 Aristotle rules out the possibility that there might be another body prior to the elements from which they are generated. Thus his later theory of prime matter (if he ever held such a theory)[77] can hardly be a presupposition for his objection here. It is more likely that he assumes the previous definition of element in which he claims to have captured the notion of element that is shared by all parties to the debate. Since Plato also used the term 'element,' Aristotle presumes that they share a common concept, which provides him with a dialectical basis for his objection.

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[72] Cf. Simplicius, *in Cael.* 488.20–4.

[73] Cf. Nussbaum (1982b) 267–293. But see Irwin (1988) for a more empiricist reading.

[74] *De Caelo* III.7, 306a18–21: tr. Guthrie (1939).

[75] Cf. Cherniss (1944) 151 n91.

[76] Cf. *GC* 331a12–332a2, 332a27–33.

[77] William Charlton (1970) has argued that Aristotle never held any such theory, but he seems likely to remain in the minority among Aristotelian scholars. See also Gill (1989b) 42 ff.

From this perspective the objection looks quite different. While assuming the notion of an element as agreed, Aristotle tries to draw out some unforeseen consequences from Plato's theory of generation in the *Timaeus*. For example, earth is excluded from the cycle of transformation because it is compounded from a different basic triangle. But this means that it cannot be dissolved into any of the other elemental bodies, unlike water which can be dissolved into fire and air. Thus, according to the second criterion, earth is an element; i.e. it is not divisible into other bodies that are different in form; cf. *Cael.* 302a15–19. *Pace* Cherniss, it does not matter that earth itself is divisible into basic triangles because these are not other *bodies*. In any case, Plato uses the word στοιχεῖον ambiguously to refer both to these basic triangles and to the solids or geometrical bodies which they compound; cf. *Tim.* 56B6–7.

So Aristotle's explicit definition of element is the basis for the consequences which he draws from the *Timaeus* account. It is true that earth is singled out from the other elemental bodies, inasmuch as it cannot be dissolved into them. If we accept that dissolution is tantamount to destruction, then earth is indestructible in a certain sense. It can be dissolved into basic triangles but that does not seem to count against this special sense of 'indestructible' which also makes it an element; i.e. that it cannot be dissolved into other bodies.[78] Thus a hidden consequence of Plato's account is that earth is the only elemental body whose parts (i.e. basic triangles) are neither actually nor potentially different in kind; i.e. they are not and cannot be used to compound a different elemental body. So Aristotle can claim that earth has the best right to be called an element, even though he apparently neglects the first criterion given in his previous definition; i.e. that thing into which other bodies are divided and which belongs in them, either potentially or actually, is called an element.

His next objection seems to reflect the commonsense point of view of a student of nature who trusts sense perception rather than mathematical reasoning as a guide to generation in the sensible world:

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[78] But Aristotle neglects to mention that this is also true of the smallest pyramid identified as fire in the *Timaeus* account, since it is so small that it cannot be dissolved into any of the other elemental bodies. Still, the basic triangles that result from its dissolution can be used to compound air or water, unlike the basic triangles belonging to earth. In addition, the larger bodies of fire can be theoretically dissolved into smaller bodies of air and even of water.

> But even in the conception of the bodies which suffer dissolution there is an irrationality, namely the suspension of the triangles in space. This is something which occurs in the process of transition owing to the fact that each body is composed of a different number of triangles.[79]

The target of this objection is the passage in the *Timaeus* (56D–E) where Plato talks about earth being dissolved into its parts by the sharpness of fire. Since the basic triangle of the earth cube is incompatible with the other basic triangles, the parts (τὰ μέρη) of earth will drift about until they happen to meet with other like parts and fit together again to make earth. On the other hand, the parts of fire, air, and water are compatible, so that there is no reason in principle why a body like water (containing 120 basic triangles) should not be directly divisible into air (48 basic triangles) and fire (24 basic triangles).[80] Indeed, Plato explicitly says that when water is broken up it is capable of becoming one body of fire and two bodies of air.

From the practical geometrical point of view, however, it is difficult to see how a regular icosahedron (i.e. water) can be divided directly into two octahedra (i.e. two corpuscles of air) and one regular pyramid (i.e. one corpuscle of fire).[81] Therefore it would seem that the logic of Plato's model requires that the icosahedron be completely dissolved into its basic triangles, which then float about before they reconvene to form the other regular solids. Furthermore, if we follow the logic of the numbers themselves, it would appear that some triangles must be drifting about apart from any body because water requires two and a half times the number of triangles that constitute air.

I think it must be something like this that Aristotle has in mind when he cites the unequal number of compounding triangles as the reason why some triangles must float free in the process of transformation between the elements. But it is precisely this 'suspension' (παραιώρησις) of geometrical figures which he holds (306a21–22) to be implausible (οὔδ' εὔλογος). If the basic triangles are really geometrical planes, as the *Timaeus* seems to indicate, then it is hard to

[79] *De Caelo* III.7, 306a21–3: tr. Guthrie (1939).

[80] Simplicius (*in Cael.* 647.9 ff.) thinks that water is composed of 20 equilateral triangles, air of 8 such triangles, and fire (presumably) of 4. But he must be thinking of the minimum number of such triangles that it would take to construct an icosahedron, an octahedron and a pyramid. In the *Timaeus*, however, Plato does not settle for the minimum number because, as Cornford (1937) explains, he wants to correlate different appearances of each element with different grades of size.

[81] Cf. Euclid's *Elements* Bk. XIII, Prop. 16.

blame Aristotle for finding it unreasonable to have them floating about as if they could be detached from bodies. Of course, if their logical priority implies ontological priority over bodies, then the free-floating independence of mathematical planes would make a certain kind of metaphysical sense. But such a metaphysical 'dream' does not agree with common sense because we never *see* mathematical surfaces detached from bodies and floating about freely. From this physical point of view, it is unreasonable to suppose that bodies can be constructed from mathematical planes, since the latter do not occupy a place like a physical body.[82]

This is reinforced by a subsequent objection, where Aristotle takes aim at the very heart of the Platonic project by arguing that the theory is not even mathematically consistent:

> Besides this, they must assert that not every body is divisible, in contradiction to the most accurate of sciences, mathematics. For mathematics conceives even the intelligible as divisible, whereas these theorists, in their eagerness to preserve their hypothesis, deny the property to some of the sensible world. By assigning to each of the elements a geometrical figure, and claiming that this constitutes the essential difference between them, they are committed to making them indivisible; for a pyramid or sphere can be divided in such a way that there will be a remainder which is not a sphere or pyramid. Either, therefore, a part of fire is not fire, but there must be something prior to the element (prior, because every body is either an element or composed of elements), or every body cannot be divisible.[83]

It is difficult to ascertain whether this objection is based upon something that the Platonists actually said or upon some hidden logical implications which Aristotle draws from their theory. The emphasis upon ἀνάγκη in the passage seems to indicate logical necessity, but there is also a suggestion that the Platonists may have explicitly denied that all sensible bodies are divisible. Whichever the case may be, it is clear that this objection is very similar to the one brought forward previously (303a20–4) against those who are committed to 'uncuttable bodies' (ἄτομα σώματα). In both passages Aristotle claims that the respective theorists must come into conflict with the mathematical

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[82] On the contrary, *De Generatione et Corruptione* I.6, 323a1–3 says that place as well as contact belongs to mathematical objects, whether these exist in separation or in some other manner. But Aristotle must have relative place rather than absolute place in mind.

[83] *De Caelo* III.7, 306a26–b2: tr. Guthrie (1939).

sciences which are also described as the 'most accurate' (ἀκριβεστάταις) of all the sciences.

Once again, the target for the objection seems to be the theory that each one of the elements has a specific figure (σχῆμα) by means of which its substance (οὐσία) is distinguished from that of the others. Aristotle claims that this theory also implies that these figures are indivisible because the parts of a sphere or of a pyramid, for instance, are not always spheres or pyramids themselves. But, as Guthrie (1939, 317 note b), has pointed out, this objection can hardly be applied to Plato who insisted that the so-called elements are compounded from basic triangles, even though he identified each of them with a regular solid. Thus the objection has more force against Democritus whose fire atoms are held to be spherical in shape and also indivisible. Such a view is more vulnerable to the final dilemma; i.e. either a part of fire is not fire or not all bodies are divisible.

If one grasps the first horn of the dilemma and accepts that a part of fire is not fire, then Aristotle thinks that one is committed to saying that there is something prior to the element (*Cael.* 306b1). His reason for thinking this can be found in the exhaustive dichotomy between things which are elements and things which are composed from elements. If one accepts this dichotomy and, at the same time, one accepts that a part of fire is not fire then there is a contradiction in one's position. While claiming that fire is an element (and therefore prior), one also admits that there is a prior part of fire which is not fire; so that fire is a compound rather than an elementary body. But if one gives up the claim that there is a part of fire which is not fire then, according to Aristotle, one falls upon the second horn of the dilemma because one is forced to admit that not all bodies are divisible. Yet this is an equally impossible claim because it conflicts with a fundamental mathematical principle; namely, the indefinite divisibility of any continuous body.[84]

Cherniss (1944, 153) objects that Aristotle's appeal to mathematics is invalid (with respect to both Plato and the Atomists) since the hypothesis of physically indivisible planes does not deny unlimited mathematical divisibility. This may be true in general but Cherniss does not adduce any convincing evidence to show that either Plato

[84] Despite Aristotle's reference to this as a mathematical principle, in *Physics* VI his own analysis of the continuum as indefinitely divisible is based primarily on physical arguments about the continuity of motion, time and magnitude.

or the Atomists made such a distinction. On the contrary, with reference to this passage, Furley[85] has shown convincingly that the Atomists must have posited theoretically indivisible magnitudes if they were to make a satisfactory reply to the Eleatics. With regard to the earlier parallel passage, he also points out that when Aristotle holds the Democritean theory of indivisible magnitudes to be in conflict with mathematics, he cannot mean anything other than theoretically indivisible magnitudes. Similarly, he must have the same thing in mind here when he argues against the Platonic theory that the simple bodies are reducible to planes. The counterpoint in the Greek brings out the conflict between the two points of view, while underlining the distance between them. On the one hand, the mathematical sciences even assume what is intelligible (τὸ νοητόν) to be divisible, which implies that they assume what is sensible (τὸ αἰσθητόν) to be divisible as a matter of course. By contrast, these theorists even deny that all sensible bodies are divisible because they wish to save their own hypothesis.[86]

The tone of surprise here suggests that it is more incredible for someone to insist upon the indivisibility of a sensible body than of a conceptual magnitude. In order for the argument to succeed, however, it must also be the case that denying the divisibility of a sensible magnitude is just as much contrary to mathematics, if not moreso. Thus Cherniss is correct to notice a logical connection between the two kinds of divisibility but it is hardly accurate to formulate it as the thesis "that what is divisible in thought must be *a fortiori* actually divisible" (1944, 154). Furley is nearer the mark when he points out that Aristotle believed every physical body to be potentially divisible everywhere in exactly the same way as every mathematical body. Here one can refer to *De Caelo* I.1 where Aristotle defines body as that which is divisible in every way.[87] As I have already argued with reference to that passage, it is clear that body is defined as a continuous magnitude which is indefinitely divisible in all three dimensions. In other words, body is defined in mathematical terms and thus the Atomists and the Platonists undermine mathematics by

[85] Cf. Furley (1967) 88 ff.

[86] It is noteworthy that 'saving the hypothesis' (σώζειν τὴν ὑπόθεσιν) is used here to characterize the mistaken method of the Platonists, by implicit contrast with 'saving the phenomena' which would be the correct method of conducting a physical inquiry.

[87] σῶμα δὲ τὸ πάντῃ διαιρετόν, *Cael.* 268a7.

holding that some physical bodies are not divisible. This, I think, is the key to understanding Aristotle's objection.

A later objection is also aimed at the general theory that the geometrical shape of the element defines its essential nature:

> Secondly, the shape of all the simple bodies is observed to be determined by the place in which they are contained, particularly in the case of water and air. The shape of the element therefore cannot survive, or it would not be everywhere in contact with that which contains the whole mass. But if its shape is modified, it will no longer be water, since its shape was the determining factor. Clearly therefore the shapes of the elements are not defined. Indeed it seems as if nature itself here shows us the truth of a conclusion to which more abstract reasoning also points. Here, as in everything else, the underlying matter must be devoid of form or shape, for so, as is said in the *Timaeus,* the "receiver of all" will be best able to submit to modification. It is like this that we must conceive of the elements, as the matter of their compounds, and this is why they can change into each other, and lose their qualitative differences.[88]

Once again, in typical Aristotelian fashion, the objection is launched on the basis of sense experience when he says that elements like water and air appear (φαίνεται) to take the shape of their container. In opposing those who arbitrarily assign figures (σχηματίζειν) to the simple bodies, he makes the rather pointed remark that these bodies are assigned figures (σχηματιζόμενα) by the place that contains them (τῷ περιέχοντι τόπῳ). In saying this, he may be presupposing his own concept of place as "the primary motionless boundary of that which contains."[89]

It seems to me that the validity of the whole objection hangs upon the plausibility of the analogy between place and a containing vessel. If place is like a vessel then the elemental mass contained in it must conform to the shape of the containing vessel, otherwise there will be some empty space or void. This is what Aristotle means when he argues (*Cael.* 306b11–13) that the shape of the element cannot endure because, if it did, the whole elemental mass would not touch its container everywhere.

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[88] *De Caelo* III.8, 306b9–22: tr. Guthrie (1939).

[89] τὸ τοῦ περιέχοντος πέρας ἀκίνητον πρῶτον, *Phys.* 212a20. Henry Mendell (1987) finds in *Categories* 6 an earlier conception of place as the volume or extension of a body that is occupying (κατέχει), as distinct from the concept of place as the inner limit of a containing (περιέχεται) body. While this earlier concept may be closer to that of the Receptacle, I think the language here suggests that it is the later concept of the *Physics* which is being presupposed.

Hence it is clear that Aristotle's own conception of place informs this objection against the theory which defines an element in terms of its geometrical shape. Since it is an essential condition for its being a body that an element must be in some place, it cannot retain its original shape without an empty place (or void) resulting, except in the unlikely event that the shape of the elementary body and the shape of the place to which it moves happen to coincide exactly. For this to happen consistently, Aristotle might argue, the element must conform to the shape of the place to which it moves and in which it is situated. But if an element has its shape changed (μεταρρυθμισθήσεται) then, according to the theory in question, it will no longer be water (for example) since it was originally differentiated by means of its shape (τῷ σχήματι). In this manner Aristotle brings out what he considers to be another inconsistency in the theory which assigns fixed shapes to the elementary bodies. He concludes (306b14–16) that the elements cannot have determinate shapes, not merely because of the dialectical refutation but also because he thinks that nature itself (ἡ φύσις αὐτή) seems to point us towards the truth of this conclusion.

In thus appealing to nature itself, Aristotle seems to presuppose a consensus about the concept among all parties to the debate.[90] So he refers to the Receptacle (τὸ πανδεχές) in the *Timaeus,* as if to say that even Plato himself recognized the true character of a material substratum, in spite of his mistaken theory of the elements.[91] In every case, he argues, the substratum ought to be without form or shape (ἀειδὲς καὶ ἄμορφον) like the Receptacle, especially if it is to be similarly capable of being ordered. Here Aristotle may be referring to that part of the *Timaeus* where Timaeus insists (50D–E) that that in which (ἐν ᾧ) the elements are stamped must itself be an entity without form (ἄμορφον ὄν) if it is to be capable of receiving all sorts of forms. Previously, by means of an analogy with gold, he had declared (50C3) that the Receptacle is set down by nature as a moulding-stuff for everything. Thus there are some grounds for Aristotle's assumption that Plato would agree with him about the character of a material substratum. However, there is no good reason to believe

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[90] At *De Caelo* I.1, 268a11–28 we find a similar appeal to nature as a witness to the Pythagorean consensus about the perfection of the number 3, especially as it shows itself in the three dimensions.

[91] In *De Generatione et Corruptione* I.7 Aristotle also introduces the notion of a ma-

that Plato would also agree with Aristotle's conception of the elements, given the elaborate mathematical constructions made for them in the *Timaeus*.

IX. *The locomotion of the elements*

Aristotle concludes his critique of the *Timaeus* with some objections directed against the attempt to explain the sensible properties and movements of the elements in terms of their characteristic shapes. Once again, he lumps the Atomists with the Platonists presumably because he sees both groups as trying to reduce qualitative to quantitative attributes:

> A serious objection is that the geometrical figures do not even suit the properties, powers, and movements of the bodies which they had especially in mind when they allotted them as they did. For instance, because fire is the most mobile, and has the faculty of heating and burning, the one school made it spherical, and the other pyramidal. These figures they considered the most mobile, because they have the fewest points of contact and are least stable, and the best able to heat and burn, because the one is all angle, and the other has the sharpest angles. For the burning and heating properties, they claim, lie in their angles.[92]

By presenting the conflicting opinions of the two groups together, Aristotle has already made a *prima facie* case for the 'disharmony' (ἀσύμφωνια) between the shapes assigned to the elemental bodies and the sensible properties which they are supposed to explain. Perhaps he chose the example of fire to engineer this conflict between the two schools of thought, since a previous passage (*Cael.* 303a12) reports that the Atomists did not assign a specific shape to any other element, even though they postulated the same reductionist principle for them. Since fire is the most mobile (εὐκίνητον) of all the elements and has the power of heating and burning, one group assigned it the shape of a sphere, whereas the other group made it a pyramid; cf. *Cael.* 306b33–34. So much for the consistency and coherence of quantitative reductionism! At least, this is what Aristotle seems to be suggesting with his deliberate contrast between the two groups of

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terial substratum, which is acted upon by contraries, as the hidden nature of things with which all thinkers were somehow in touch.

[92] *De Caelo* III.8, 306b30–307a3: tr. Guthrie (1939).

thinkers, since the inconsistency is produced partly through dialectical counterpoint.[93]

This strategy should be kept in mind also when we consider the reasons given for attributing different shapes to fire. Since fire is the most mobile of the elements, they assigned to it the most mobile figures which they judged to be either the sphere or the pyramid because these shapes have the fewest points of contact (ἐλαχίστων ἅπτεσθαι) and the least stability (ἥκιστα βεβηκέναι); cf. *Cael.* 306b34–307a1. In the case of the Platonists, at least, we can check the accuracy of this account against the *Timaeus* (56A–B) where it is argued that fire must be the most mobile (εὐκινητότατον) of the elemental bodies because it has the fewest bases (ὀλιγίστας βάσεις). But it is difficult to understand what Aristotle could mean by saying that the pyramid has 'the fewest points of contact' (ἐλαχίστων ἅπτεσθαι). If this means contact with other pyramids, then it is neither true nor consistent with the Platonic intention of packing these pyramids together to make a mass of fire which excludes the void.

This sort of reason would be easier to understand in the case of the sphere because, when spheres are packed together, they have fewer points of contact than any other regular solids. But here we are completely at Aristotle's mercy, since we do not have any independent access to the writings of the Atomists which might tell us why they chose the sphere as the specific shape of fire atoms.[94] In a different context, Aristotle reports that Democritus held the spherical to be the most mobile of shapes and hence designated it as the shape of mind and of fire.[95] What this passage suggests is that Democritus chose the sphere as the most appropriate shape for both mind and fire because of the peculiar ability of each to break out of their material constraints. Since his model for the stability of material bodies is the contact or 'hooking' of atoms with each other, it makes sense that

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[93] Compare *De Generatione et Corruptione* I.8 where Aristotle also contrasts the explanations given by Plato and the Atomists, despite their shared assumption that the elements are indivisible. For instance, Leucippus holds the indivisibles to be solids, while Plato makes them planes; the one posits an indeterminate number of figures, the other a determinate number; the one assumes that the void exists, whereas the other denies its existence.

[94] Even the fragments preserved by Simplicius in his commentary on the *Physics* seem to be extracted from Aristotle's book on Democritus; so we do not get outside his sphere of influence, as it were.

[95] τῶν δὲ σχημάτων εὐκινητότατον τὸ σφαιροειδὲς λέγει τοιοῦτον δ' εἶναι τόν νοῦν καὶ τὸ πῦρ, *DA* I.2, 405a11.

the least stable bodies (mind and fire) should have atoms with the least capacity for contact with each other or with other atoms.[96] Thus Aristotle may be citing one of the reasons given by the Atomists for giving a specific shape to fire but this would not also apply to the pyramid.[97]

Keeping all this in view, let us try to untangle the second set of reasons which he reports as having been given by these predecessors to explain the capacity of fire for heating and burning. Once again, the reasoning attributed to the Platonists is more intelligible and easier to check. For instance, in the *Timaeus* the pyramid is described as being naturally the most mobile of the perfect solids because it is in every way the most cutting and the sharpest of all.[98] This is at least consistent with Aristotle's report that the pyramid was assigned to fire because it is the sharpest-angled (τὸ ὀξυγωνιώτατον) of all the figures. From this we can reconstruct the Platonic account (*Tim.* 57A) of the phenomena of heating and burning in terms of the cutting activity induced by the pyramidal shape of fire.

It is much more difficult, however, to reconstruct the parallel account which might be given by the Atomists. According to Aristotle's report, the spherical atom of fire is capable of heating and burning 'because the whole is an angle' (διότι τὸ μὲν ὅλον ἐστὶ γωνία). But what are we to make of this? Perhaps we can look for guidance to a later passage where Aristotle says that Democritus regarded the sphere as being a kind of angle which cuts due to its mobility.[99] Although this does not dispel the strangeness of the concept for those who are accustomed to the Euclidean definition of angle as the intersection of two straight lines, still we know from some titles of his lost works that Democritus was interested in the so-called 'horn-angle' that is generated by the tangent to a circle or a sphere; so perhaps he regarded the whole circle or sphere as the inverse of a horn-angle.

Having clarified some of the reasons which he attributes to his opponents, we may better understand Aristotle's own objections. He begins with an objection to their accounts of the motion of fire:

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[96] There is even a hint that the soul atom (and possibly also that of fire) might be the only truly indivisible atom; cf. Konstan (1988) 1–33.

[97] As against this one should note that Theophrastus (*De Sensibus* 65 = DK 68A135) reports an account of bitter taste given by Democritus in terms of angular atoms that are small and thin, so that their sharpness makes them able to dodge in and out more easily from any place.

[98] τμητικώτατόν τε καὶ ὀξύτατον ὂν πάντῃ πάντων, *Tim.* 56B1–2.

[99] ἡ σφαῖρα ὡς γωνία τις οὖσα τέμνει ὡς εὐκίνητον, *Cael.* 307a17–18.

> In the first place, however, both schools are wrong on the question of motion. Even if these are the most mobile of figures, yet they are not mobile with the motion of fire: the motion of fire is in a straight line upwards, whereas the motion to which these figures are suited is a circular one, namely what we call rolling.[100]

The general gist of this objection is that the characteristic motion of fire is not explained satisfactorily by attributing to it a definitive shape, whether this be a sphere or a pyramid. It seems, however, that Aristotle is thinking only of a spherical shape when he objects that this would make fire most mobile in a circular rolling motion. By contrast, he holds that the natural motion of fire is rectilinear (κατ' εὐθεῖαν) and upward (ἄνω). Commentators like Taylor have been quick to pounce on this as an invalid objection because it presupposes a whole theory of natural places that Plato does not accept.

But Plato does explicitly admit (*Tim.* 63B) that fire has a special place in the universe to which it moves and in which it masses. So, even though he denies that there are absolute directions like 'up' and 'down,' he tries to account for the massing of fire in one particular region rather than another. Still there seems to be no clear connection between this fact and the geometrical shape which is assigned to fire. In fact, there are some hints in the dialogue that the massing of fire is a pre-cosmic result of the random shaking of the Receptacle. Against the Atomists, on the other hand, Aristotle objects that a spherical shape would make fire naturally mobile in a circular fashion which he calls 'rolling' (κύλισις). Presumably, he intends to make a similar objection to the pyramidal shape which the Platonists attributed to fire, though the text does not specify what sort of motion would be appropriate for such a figure. But Aristotle claims that, while the pyramidal shape would make fire mobile in a certain way, this is not the 'ease of motion' (εὐκίνητα) that is observed to be characteristic of fire. So again he would probably conclude that the Platonic account does not save the phenomena because it is guided by rigid mathematical assumptions.

His second objection is more explicitly directed against the Platonic account of rest or stability in terms of a particular shape:

> Secondly, if the reason for calling earth a cube is its stability and immobility, then, since earth is not immobile wherever it happens to be, but only in its proper place, and moves away from anywhere else

[100] *De Caelo* III.8, 307a4–8: tr. Guthrie (1939).

> unless prevented, and the same is true of fire and the other elements, it clearly follows that fire and each of the elements are spherical or pyramidal when at a distance from their proper places, and cubical when they reach them.[101]

It is quite clear here that Aristotle is presupposing his own doctrine of natural or proper places as a basis for this objection. His point is that if earth is naturally at rest in a particular place (for him this is a fact of observation) then the Platonic account implies that it has a cubical shape only in that place. This may appear to be a strange implication for Aristotle to draw from the *Timaeus* account but if we look at the relevant passage (*Tim.* 55D ff.), we shall find a certain plausibility to it. There Plato assigns the cubic form to earth because it is the most immobile (ἀκινητοτάτη) of the four Kinds and the most malleable body (πλαστικωτάτη). Undoubtedly there is an attempt to explain the sensible characteristics of earth in terms of its invisible mathematical structure when Timaeus says (55E) that such a body necessarily has the most stable bases; i.e. half-square triangles with equal sides.

Therefore, Aristotle is not misrepresenting the Platonic account when he assumes that the reason for calling earth a cube is that it stands fast and remains in one place. Of course, one could protest that he is loading the dice here since the mathematical structure of earth should remain constant wherever it is located.[102] But that is precisely the point at issue because the mathematical shape is intended to explain the sensible behavior of the body in question, and Aristotle claims that, when earth is removed from its proper place at the center of the universe, it is no longer immobile but tends to move back towards its place of rest, unless it is prevented from doing so. Hence the logic of the Platonic explanation in terms of shape dictates that we assign to it a different mathematical figure to account for its new-found mobility. This is what Aristotle appears to mean when he says 'it is obvious that' (δῆλον ὅτι) fire or earth or any one of the elements which is not in its proper place will have the shape of a sphere or a pyramid. Furthermore, since the Platonists use the cubical shape to explain stability and rest, they would be

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[101] *De Caelo* III.8, 307a8–13: tr. Guthrie (1939).

[102] If fact, Plato seems to assume that the shape of each element remains constant, except when it is broken up either by smaller or bigger shapes, so that its parts move to another place and become different elements; cf. *Tim.* 58C.

logically committed to saying that each element has this shape when it is at rest in its proper place.

The next objection is aimed at the attempt of both groups of predecessors to explain the sensible quality of heat in terms of mathematical shape:

> Thirdly, if the power of fire to heat and burn lies in its angles, all the elements will have this power, though perhaps in different degrees; for they all have angles, e.g. the octahedron and the dodecahedron. The sphere itself is regarded by Democritus as an angle, which pierces owing to its mobility. The difference therefore will be one of degree and this is palpably false. Even mathematical bodies will have to heat and burn, for they too have angles, and include indivisible spheres and pyramids—especially if indivisible magnitudes are a reality, as these philosophers affirm. If one class of bodies has the property and another not, they should have stated where the difference lay, not simply asserted that they do.[103]

The main thrust of this objection seems to be that whoever reduces quality to quantity is committed to making the difference between qualities simply a difference of degree. It is based on an implication which he draws from the theories of his opponents who explain the capacity of fire to heat and burn in terms of the angular properties of the shapes assigned to fire. Since the figures assigned to the other elements also have angles, Aristotle argues that the Platonists should in all consistency attribute some degree of heating and burning capacity to air (i.e. the octahedron) and water (i.e. the dodecahedron), though obviously they do not.

His strategy here is to use Democritus as an illustration, so as to complete his *reductio* argument against the whole mathematical approach. In spite of being the least angular of all figures, the sphere has a burning quality because it is seen as a whole angle that cuts due to its mobility. It is more probable, however, that Democritus made this quality a function of the mobility of a sphere rather than of its angular properties. But Aristotle seems to be pressing Democritus into service in order to attack the angular hypothesis of the Platonists. His point seems to be that, if Democritus gave the sphere a burning quality due to its angularity (the inverse of a horn-angle), then surely the Platonists are committed to giving a similar quality to the octahedron and dodecahedron, which are certainly more angular than

[103] *De Caelo* III.8, 307a13–24: tr. Guthrie (1939).

the sphere.[104] But, since they do not do so, the implication is that their theory is inconsistent. Furthermore, since the difference in angularity between the four perfect solids is merely a difference of degree, the same should be true for their respective heating and burning qualities. But Aristotle declares this latter consequence to be manifestly false, presumably on the basis of sense experience. For him, therefore, the explanation of the sensible quality of heat in terms of angularity is not adequate to save the phenomena.

Like Proclus,[105] most commentators have protested that Aristotle is misrepresenting Plato's explanation (*Tim.* 61D–62A) of heat in terms of the way in which fire acts upon our bodies by dividing and cutting. Sense perception registers the property of heat as a sharp sensation but, in order to explain this property, Timaeus appeals to the mathematical figure that was assigned to fire. With reference to the pyramid, he cites (*Tim.* 61E1–6) the fineness of its sides, the sharpness of its angles, the smallness of its parts and the speed of its locomotion to explain the sensible properties of fire; i.e. that it is a violent and keen entity which sharply cuts everything it encounters. As Cherniss (1944, 158) points out, this whole passage shows that Plato did not consider heat to be a result of angularity as such. However, I think the reductionist tendencies of the explanation itself enable Aristotle to push him in this direction. For instance, the mobility of the pyramid (i.e. fire) can be reduced to the fact that it has the sharpest angles among all the perfect solids; cf. *Tim.* 56B. Perhaps here Aristotle found sufficient grounds for his own dialectical assumption that the burning and heating properties of fire can also be reduced to angularity.

In any case, the second part of his objection still holds; namely, that neither the Platonists nor the Atomists have properly distinguished between physical and mathematical bodies. As a result, it follows from their theories that mathematical bodies (τὰ μαθηματικὰ σώματα) must burn and heat other things, since these also have angles; cf. *Cael.* 307a20–21. Cherniss concludes from Aristotle's final remarks that he did not take this argument seriously himself, but this can

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[104] My conjecture would be more viable if Democritus regarded the sphere as a polygon with a great number of sides, which gave the appearance of having a smooth surface just like the side of a cone which is really composed of a multitude of invisible steps made by the thin laminae of unequal size which are held to constitute the cone.

[105] Proclus apud Simplicius, *in Cael.* 663.18–664.24

hardly be a legitimate inference from what he actually says about the failure of his opponents to distinguish between the two different kinds of bodies. After all, this has been one of his major complaints against the Platonists and he will surely treat it seriously here, especially since he has found an important differentiating feature of physical bodies. His opponents would have to agree that a physical body like fire actually heats and burns, whereas it is difficult to see how a mathematical body can do so. Of course, the reductionist thesis implies that the sensible properties of fire are mere appearances, while its real structure and activity are invisible. But Aristotle will not accept this implication, and so we reach an impasse in the debate. If sense experience is accepted as the touchstone of physical reality, however, he will be in a stronger position than his opponents.

This is true for a later objection directed against the *Timaeus* account of fire, which gives it a shape designed to cut:

> Fifthly, it is absurd to assign to fire its shape for the purpose of division only. Fire to all appearance brings together and unifies rather than divides. It divides things of different kinds, but unites things of the same kind. Moreover uniting is essential to it—it is a property of fire to weld together and unify—but division is accidental: in uniting the homogeneous it expels foreign bodies. Its shape therefore ought to have been devised with an eye to both, or by preference to its unifying power.[106]

Contrary to its initial appearance, this objection starts from a concession to the Platonic point of view which was made in a previous objection. There (307a27–28) he seems to concede that it is reasonable (κατὰ λόγον) for the cutting and dividing activity of fire to follow from a certain shape. Now he objects that it is laughable (γελοῖον) to assign fire a shape which will only explain its dividing activity. By contrast, he insists (307b1–3), the essential activity of fire is to unify homogeneous bodies, as a result of which it accidentally divides bodies which are literally 'not of the same tribe' (τὰ μὴ ὁμόφυλα). Both Taylor and Cherniss find this to be an unargued and arbitrary assertion on the part of Aristotle, which therefore does not carry any weight as a basis for criticism.

It seems to me, however, that this judgment is unbalanced because it ignores the appeal to the phenomena which motivates Aristotle's objection. For instance, he is clearly referring to the welding

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[106] *De Caelo* III.8, 307a32–b6: tr. Guthrie (1939).

action of fire when he says that it seems (δοκεῖ) to bring together and fuse things more often than to separate them. It is an undeniable fact of experience that fire fuses certain materials, whereas it differentiates others. So Aristotle can legitimately object that the pyramidal shape assigned to fire by the Platonists does not account for its fusing and uniting actions. However, he goes beyond criticism when he asserts that this is the primary action of fire, while dividing and separating are merely secondary. This is another way of formulating his distinction between putting together, as the essential (καθ' αὑτό) activity of fire, and separating as an accidental (κατὰ συμβεβηκός) activity. I think that this distinction obviously presupposes his own theory of the elements and their powers.

For a brief summary of this theory one may use a parallel passage in *De Generatione et Corruptione* (II.2, 329b24–32) where Aristotle is dealing with the principles of sensible body. By equating 'sensible' with 'tangible,' he narrows down his search to the tangible contrarieties (329b7–11). The list is as follows: hot/cold, dry/moist, heavy/light, hard/soft, viscous/brittle, rough/smooth, coarse/fine. From this list he eliminates heavy and light as candidates for primary and characteristic contraries of the elements, since these are neither powers of acting nor susceptibilities to action. But the so-called elements (τὰ στοιχεῖα) must be reciprocally active and susceptible to action, since they combine and are transformed into one another (329b20–4). Thus the prime candidates for elemental contraries are hot/cold and dry/moist because the first of each pair is an active power and the second is a passive capacity. It is within this context that we come upon the remarks which interest us about the primary activity of fire or, more precisely, of heat which is the active power that characterizes fire. Aristotle says that 'hot' just *is* the uniting of things of the same kind.[107] In other words, according to his theory, the fusing of homogeneous bodies is the characteristic and defining activity of heat. This becomes even clearer when he insists that the separating activity (τὸ διακρίνειν) which some people attribute to fire is really the fusing of things of the same kind (συγκρίνειν ἐστί τὰ ὁμόφυλα) because this results in the separation of things of a different kind (ἐξαιρεῖν τὰ ἀλλότρια); cf. *GC* 329b26–8.

We should notice the striking parallels between the language of both passages which argue that fusion rather than separation is the

[107] θερμόν γὰρ ἐστί τὸ συγκρίνον τὰ ὁμόγενη, *GC* 329b26.

primary and essential activity of fire. In both cases the argument is the same against those who make separation the defining activity of fire and assign it a characteristic shape on this basis. The dissolution of compound bodies induced by fire is argued to be the result of the combination of homogeneous bodies, which is held to be the primary activity. Cherniss (1944, 159) remarks sarcastically that this argument is on a level with Aristotle's attempt to prove that the 'Strife' of Empedocles is really the cause of 'natural' movement.[108] There is a grain of truth to this remark insofar as it implies that Aristotle tends to introduce his own theories as a basis for criticising his predecessors. In this case, however, he could claim that his theory about the structure of fire saves the phenomena more adequately than Plato's theory. For instance he can explain why fire joins some bodies together while separating others but the pyramidal shape attributed to fire in the *Timaeus* explains only its cutting activity. This is what I take to be Aristotle's point at the end of this objection when he says that a shape should have been chosen to explain both activities of fire but, especially, its capacity to fuse bodies. Although his own prejudices are apparent, he is making a valid objection to the Platonic theory.

Aristotle's own theory of the elemental contraries is very much in evidence as a presupposition for his final objection to the Platonists:

> Sixthly, since hot and cold have opposite effects, it is impossible to assign a shape to the cold, for it would have to be an opposite, but a shape has no opposite. For this reason they have all left out coldness. But they ought to have defined everything by shape or nothing. Some do make an attempt to describe its power, but only to contradict themselves. That which has large parts, they say, is cold, because it clogs and cannot pass through the pores. Well, then, clearly the hot will be that which can pass through, and this cannot but be what has small parts. In effect, therefore, the difference between hot and cold has become a matter of size, not of shape. If, further, the pyramids were of different sizes, the large pyramid would not be fire, and their shape would not cause burning but rather the reverse.[109]

The general thrust of the objection is that these mathematical reductionists have not, and indeed cannot, account for all the sensible contraries in terms of shape. For instance, hot and cold are contrary

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[108] Cf. *GC* 333b26–33.

[109] *De Caelo* III.8, 307b6–18: tr. Guthrie (1939).

powers and so it is impossible to assign any figure to the cold because there are no opposite figures.

According to Aristotle's analysis, this is the reason why these theorists neglect to assign any figure to coldness, even though they should have defined everything or nothing by shape, if they are to remain consistent. Some people do try to talk about its power, without actually assigning it a special figure, but they end up contradicting themselves. Here Aristotle is probably referring to the passage in the *Timaeus* (62A–B) where Plato gives a descriptive explanation of the phenomenon of cold. There it is described as the opposite (τὸ ἐναντίον) of heat but Timaeus makes no attempt to assign it a particular shape. The rest of Aristotle's account also fits generally with what we find in the dialogue. For instance, he attributes to some people the claim that what consists of large parts (τὸ μεγαλομερές) is cold because it compresses rather than moves through the passages. From this he concludes that the hot will be whatever can move through such passages and, in virtue of this, it will be what always has small parts (τὸ λεπτομερές). Hence, he argues, it follows that the hot and the cold are differentiated by means of smallness and largeness rather than by their shapes.

Having wrung from the theory of his opponents the implication that the difference between hot and cold is a matter of size rather than shape, he tacks on a further codicil to his objection. Since Plato postulated (*Tim.* 57C–D) that fire pyramids are of different sizes, he seems to be committed to the contradictory position that the larger pyramids are not fire nor would their shapes cause burning but rather the opposite. Plato, of course, would not be aware of such a contradiction because he insists that the pyramidal shape, whatever its size, is definitive for fire. Although he does admit (58C–D) that there are many kinds of fire, such as flame and light and even the smouldering kind of fire left behind in the embers, these different kinds do not seem to be systematically linked with different sizes of pyramids. Therefore Aristotle's claim that the larger pyramids would not be fire appears to be carried over from his previous conclusion that the difference between hot and cold is due to size rather than shape. But, since this was an implication ostensibly drawn from the *Timaeus* account of cold, he feels entitled to palm off this contradictory claim on Plato. This is the sort of sleight-of-hand which prompts modern commentators like Cherniss to cast doubt on the legitimacy of his whole critique.

Despite such dubious moves, however, it is clear from Aristotle's general conclusion to this section that all his objections are designed to undermine mathematical reductionism:

> From what has been said it is clear that it is not shape which differentiates the elements from one another. In fact, the most essential differences between bodies are differences in properties and functions and powers, for these are what we speak of as pertaining to every natural object. These therefore must claim our attention first, in order that from a consideration of them we may come to grasp the differences between element and element.[110]

Since he has offered many objections to the theories of his opponents who assign specific shapes to the elemental bodies, it is natural for him to conclude as obvious that such shapes do not really differentiate the elements from one another. For instance, if there are no opposites among figures (as Aristotle claims) then it will not be possible to explain the contrary powers of the elements (e.g. hot and cold) in terms of shape. For Aristotle, therefore, the most decisive differences (κυριώταται διαφοραί) between natural bodies are their respective 'sufferings' (πάθη), actions (ἔργα), and powers (δυνάμεις). In support of this claim, he appeals to our ordinary way of speaking about natural things (τῶν φύσει). This is an indication of Aristotle's insistence upon the phenomenal aspect as crucial for understanding the nature of physical things. It also amounts to a refusal on his part to accept the reduction of qualitative to quantitative differences, which reductive tendency he sees as being central to the projects of the Platonists and the Atomists in their different ways. He would claim, by contrast, that qualitative rather than quantitative differentiae provide us with a clearer guide to the essential natures of physical things.

This is what I take him to mean in the present passage when he says that we must first (πρῶτον) deal with the affections, actions, and powers of bodies so that, by studying these, we may grasp the true differentiae of the elements from one another. This could serve as an outline of his procedure in *De Generatione et Corruptione* where he sets out his own definitive theory of the elements. It could also be used as an introduction to the treatise on weight and lightness in *De Caelo* IV, even though these attributes of natural bodies are not the essential and defining characteristics of the elements.

The phenomena of heaviness and lightness, as we have already

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[110] *De Caelo* III.8, 307b19–26: tr. Guthrie (1939).

seen, pose a serious problem for the reductionist models of the Platonists and Atomists. If their reductionist theories of matter fail to save the phenomena connected with weight, then Aristotle will be half-way towards showing that the simple bodies have ultimate qualitative differences. Thus, as Cherniss rightly points out, this fourth book is crucial for Aristotle's system of the physical world. My terms of reference, however, preclude any detailed look at this treatise on weight and lightness, and I have already considered in a general way Aristotle's objections to the reductionist theories of his predecessors about these physical attributes. Therefore, I want to conclude this chapter with a brief look at some characteristic criticisms of Plato's *Timaeus* which are made in other works besides *De Caelo*. This will enable me to focus once more upon Aristotle's objections to the mathematical cosmology contained in that dialogue.

X. *Physics and mathematics*

As one might expect, *De Generatione et Corruptione* contains many critical remarks about the *Timaeus* but I will concentrate on a few that are relevant to the distinction between mathematics and physics. In the second chapter of Book I, for instance, he complains (315a30) that Plato concerned himself only with generation and corruption insofar as they belong to things (ὁπῶς ὑπάρχει τοῖς πράγμασι). This is clarified somewhat by his subsequent complaint that Plato does not deal with all generation but only with that of the elements (τῶν στοιχειῶν). There Aristotle claims (315a30–32) that Plato does not say how flesh and bone and other such compounds are generated; and that he does not examine the conditions under which alteration and growth belong to things. Along with appearing to contradict the first complaint, this charge seems to be simply mistaken about the *Timaeus* because in that dialogue (cf. 73B ff.), Plato does deal with the generation of flesh and bone from the marrow which itself is formed by divine agency from elementary triangles.

In his commentary on this passage, Joachim (1922, 70) suggests that Aristotle may be complaining about the lack of precision in Plato's account of this generation or even about its fanciful nature. But, if that were the case, Aristotle would be more likely to talk about the inadequacy of mythical accounts as explanation. What he actually says is that Plato does not deal at all with the generation of com-

pound bodies like flesh and bone but only with the elements. This is made even stranger, as Cherniss (1944, 124n80) points out, by the fact that elsewhere Aristotle takes great pains in attacking the section of the *Timaeus* which explains the generation of compounds. I think that Cherniss is basically correct in suggesting that the answer to the puzzle must be sought in a passage from *De Caelo* (306b22–9) where Aristotle begins an objection to the construction of the elements from planes with a rhetorical question about how the generation of flesh and bone could be possible on such a theory. He concludes there that if such theories are examined carefully they can be seen to banish all generation from the world. It may be this implication which Aristotle has in mind here when he complains that Plato does not say how (πῶς) flesh and other compound bodies are to be generated. As I have already argued, this is a demand for a physical rather than a mathematical explanation of all generation in the natural world.

This interpretation is supported by the rest of this section (I.2, 315b6 ff.) from *De Generatione et Corruptione* where he goes on to praise Democritus for having paid due attention to the physical phenomena. For instance, he says that none of the others gave more than a superficial account of growth, whereas Leucippus and Democritus gave a detailed theory. Later in the section (315b27 ff.), he compares their theory of indivisible bodies rather favorably with the Platonic theory of indivisible planes which he considers to be completely unreasonable. According to Aristotle there is something to be said for the notion of indivisible bodies, even though it runs into mathematical difficulties due to conflicts with the definition of the continuum. The Atomists, at least, can explain some physical phenomena like alteration and generation in terms of 'turning' (τροπή) and 'interconnection' (διαθιγή) and 'rhythm' (ῥυσμός), which Aristotle calls 'differences of figures' (ταῖς τῶν σχηματῶν διαφόραις). Democritus, for instance, is said (316a1–2) to explain color in terms of the turning of figures.

By contrast, the Platonists who divide everything into planes cannot even attempt to generate a sensible quality (πάθος) from them because they yield only mathematical bodies (στερέα). It is within the context of such a comparison that Aristotle makes the following methodological remark:

> The reason why we have not the power to comprehend the admitted facts is our lack of experience. Hence those who have lived in a more

> intimate communion with the phenomena of nature are better able to lay down such principles as can be connected together and cover a wide field; those, on the other hand, who indulge in long discussions without taking the facts into account are more easily detected as men of narrow views. One can see, too, from this the great difference which exists between those whose researches are based on the phenomenon of nature and those who inquire by a dialectical method. For on the subject of atomic magnitudes one school maintains their existence on the ground that otherwise the "ideal triangle" will be many, while Democritus would appear to have been convinced by arguments germane to the subject and founded on the study of nature. What we mean will be clear as we proceed.[111]

The final remarks here show Aristotle's general attitude to the Platonic project by comparison with the alternative reductionist program of the Atomists. For him, the Platonists are too logical (λογικῶς) in their approach to nature.[112] Due to their fixation on logical arguments, they overlook the physical facts and so they prematurely declare their views after considering only a few things.

This statement reflects the whole tenor of Aristotle's attacks upon the Platonic project throughout the *De Caelo*. With regard to knowledge of nature, his diagnosis of the major weakness of the Platonists is lack of experience (ἀπειρία). Here he says that such a lack of experience is the reason for a diminished ability to keep in view the agreed facts (τὰ ὁμολογούμενα). Since these acknowledged facts are among the phenomena to be saved by any scientific theory, anyone who neglects them is almost bound to fail as a student of nature. By contrast, those who have lived more among natural things (ἐν τοῖς φυσικοῖς) become increasingly capable of hypothesizing such principles as will enable them to string together most of the phenomena into a coherent theory. In other words, those who wish to make a creative contribution to any science must first dwell among the facts, as modern philosophers of science would say. From this point of view, Aristotle's critique of the Platonists is legitimate because they

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[111] *De Generatione et Corruptione* I.2, 316a5–14: tr. Forster (1955).

[112] On this point, at least, the translation of Forster (1955) is inferior to that of Williams (1982) who renders the contrast in terms of a *physical* versus a *logical* mode of inquiry. Since Aristotle's own physical treatises make ample use of dialectical modes of inquiry, it is rather misleading to frame the contrast in terms of physics versus dialectic or even in terms of empirical versus dialectical, as Irwin (1988) does. The contrast is between a proper scientific inquiry which confines itself to the appropriate subject-genus, and a general dialectical inquiry which crosses over the boundaries of subject-genera; cf. Cleary (1994a).

appeared to him to be ignoring many physical facts by dogmatically pursuing a mathematical approach to nature.

It is interesting to notice that he characterizes this approach as 'logical' (λογικῶς), in contrast to the 'physical' (φυσικῶς) approach of the Atomists.[113] He also illustrates the difference between these approaches in terms of the kind of reasons which each would accept for positing atomic magnitudes. On the one hand, the Platonists are reported to have argued that there must be atomic magnitudes because otherwise 'The Triangle Itself' (αὐτὸ τὸ τριγώνον) will be more than one. This could be described either as a dialectical or as a mathematical argument, depending on its precise details.[114] But, on the other hand, Democritus is reported (316a11–13) to have been persuaded of the existence of indivisible magnitudes by proper physical arguments. Aristotle does not immediately give us an example of such argumentation but promises to clarify it further. Perhaps he has in mind the argument subsequently (316b29 ff) rehearsed to the effect that the process of dividing a body part by part is not a 'breaking up' (θρύψις) that can continue indefinitely nor can a body be simultaneously divided at every point but only so far.

In effect, this amounts to the claim that every perceptible body must contain indivisible magnitudes which are invisible, especially if generation and corruption are to happen by means of 'association' (συγκρίσει) and 'dissociation' (διακρίσει), respectively. This is the argument which Aristotle reports as convincing some people (presumably the Atomists) that there must be atomic magnitudes. But it is not easy to see what is peculiarly physical (as distinct from dialectical) about this argument, unless it be the appeal to some limit in the physical division of bodies. When Aristotle critically analyses (317a1 ff.) the argument, he declares it to contain a hidden paralogism because it assumes that a magnitude which has a point anywhere and everywhere (ὁπῃοῦν καὶ πάντῃ) can be dissolved by division into nothing.

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[113] For a good discussion of this contrast see 'Introduction' in Charlton (1970).

[114] *Lin. Insec.* 968a9–14 argues for the indivisibility of Line Itself on the grounds that it is first among the things which share its name. If it were divisible, however, its parts would be prior by nature to the whole, and this would presumably contradict the priority of the Line Itself as a whole. The same dialectical argument is said to apply to Square Itself, Triangle Itself, Surface Itself, and Body in general. Later (969a17–21) this argument is criticised as being weak and as potentially undermining the whole theory of Forms. Another argument (968b5–22) for indivisible lines is based on the mathematical conditions for commensurability; i.e. that some unit length can be found to measure the whole line.

This dialectical assumption is very reminiscent of Zeno's arguments,[115] and if the Atomists were responding to Zeno (which is at least historically possible) then they might be insisting that the 'beings' which compound physical bodies must have some finite magnitude but one not further divisible into things with no size (e.g. points), because nothing comes from nothing. On the other hand, these indivisible magnitudes cannot be visible because all perceptible bodies can be either physically or theoretically divided into parts. But it is not obvious that such an argument is exclusively physical in character, any more than is the argument reported by Aristotle.

Finally, let me briefly consider Aristotle's attack upon Plato's notion of the Receptacle that played a subsidiary role in the *Timaeus* account of the generation of the elements. As one might expect, Aristotle interprets this notion in terms of his own material substratum (ὑποκειμένη ὕλη) and hence his criticisms must be understood in this light. For instance, in *De Generatione et Corruptione* II.1, he says that previous thinkers have disagreed about the matter which underlies sensible bodies, even though there is a consensus about the necessity of such bodies for the formation of compound natural substances. Some thinkers postulate a single material, whereas others think that there are two, three, and four basic materials. After this short survey of doxa, Aristotle adopts a critical stance towards the theories of his predecessors about this primary material. But first he establishes a consensus about the notion of primary principles or elements; i.e. those things from which changes originate, whether by 'association' or 'dissociation' or by some other process.

Within this framework of shared agreement, Aristotle argues that those thinkers who posit a single primary material which is corporeal and separate (σωματική καὶ χωριστή) are mistaken. Their mistake is to think that this material body can be separated from every sensible contrary, such as the light and the heavy, the hot and the cold. Later in the chapter, Aristotle outlines (329a24 ff.) his own view that there is some basic matter (τινὰ ὕλην) for sensible bodies, but that it is not separated (οὐ χωριστήν) from the sensible contraries which characterize the so-called elements (τὰ καλούμενα στοιχεῖα) that are themselves generated from this primary matter.

Now this is the context in which he criticises Plato's Receptacle as a candidate for such a primary matter:

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[115] Cf. Fr. 1 & 2 (KR) = Simplicius, *in Phys.* 140.34 & 139.9.

> As it is described in Plato's *Timaeus*, it has no definiteness at all; for Plato did not state clearly whether his 'Receptacle' is separate [or separable] from the elements. Nor does he use it at all, when he says that it is a sort of subject which underlies the elements and is prior to them, like gold in things which are made of gold; and even this manner of expressing the situation is not well put, for although in a thing altered the underlying subject—gold, in the comparison—remains, a thing generated or destroyed cannot be named after the material from which it has been generated or destroyed [e.g., when water changes to air, air is not called "watery"]. Yet Plato says that to call a [golden] thing "gold" is by far the truest way of describing it; and, although the corporeal elements are solid, in making an analysis of them he ends up with planes. But it is impossible for the "nurse" and the primary matter to be planes.[116]

Aristotle seems to be making an implicit comparison with his own position, when he complains that what is written in the *Timaeus* is not definite enough with regard to primary matter. For instance, it does not state clearly whether or not the Receptacle is separated from the elements (εἰ χωρίζεται τῶν στοιχείων), whereas both Aristotle and the material monists have definite but opposite positions on the question. Perhaps this was a question which Plato never raised but, in his characteristic fashion, Aristotle demands an answer just the same. Secondly, he complains that Plato does not make use of the Receptacle in his account of the generation of the elements, even though he allegedly calls it a substratum (ὑποκείμενόν τι) that is prior to them. Not surprisingly, the *Timaeus* contains no such terminology, since it is obviously the result of Aristotle's thinking about the Receptacle in terms of his own material substratum.

Consequently, according to Cherniss (1944, 147n89), Aristotle misinterprets the real purpose of Plato's gold analogy which is to show that the nature of the recipient is always the same (i.e. gold is always gold), while the phenomena (like the shifting shapes of the gold) have no permanent nature. But Cherniss does not seem to consider that it may have been precisely this function of the Receptacle which encouraged Aristotle to identify it with his own substratum for change, while still criticising the formulation of the gold simile in the *Timaeus* on the basis of his own distinction between ἀλλοίωσις and γένεσις. The gist of his argument is that things which are generated and corrupted cannot be called by the same name as that

[116] *De Generatione et Corruptione* II.1, 329a13–24: tr. [with additions by] Apostle (1982).

from which (ἐξ οὗ) they have been generated, whereas this may still be possible in the simple case of alteration.[117] Yet Plato behaves as if there were no distinction between the different kinds of change when he has Timaeus say that "by far the safest reply, in point of truth, would be that it is gold."[118] By quoting this statement almost verbatim, Aristotle implies that it fails to make the distinction between generation simpliciter and alteration, where it would be permissible to retain the name of the substratum. Therefore, when Cherniss complains that Aristotle is mistreating the gold analogy, it is not obvious that his complaint is justified.

The passage quoted is also of interest because it is the only place where Aristotle explicitly considers the relationship between the planes and the Receptacle. He objects that such (mathematical) planes cannot be identical with 'the Nurse' (τὴν τιθήνην) which is expressly identified as 'first matter' (τὴν ὕλην τὴν πρώτην). I think that Cherniss is correct in finding this objection to be of the greatest significance for Aristotle's interpretation of the *Timaeus* but perhaps not exactly for the reasons which he puts forward. He suggests that Aristotle's identification of the Receptacle with prime matter blinds him to the role which the former actually plays in that dialogue.[119] Thus, anticipating his own theory of prime matter and finding only the theory of component planes, he assumes that this exhausts the nature of corporeal elements. In effect, he supposes that Plato, after introducing the notion of prime matter, abandoned it for the theory of planes.

Conclusion

In general, Cherniss's picture of Aristotle as an interpreter of the *Timaeus* is not very attractive, since he is represented as being wilfully blinded by his own preconceptions to such an extent that he makes 'elementary' mistakes. But this picture is implausible, given that Aristotle himself was a great thinker and that he was much better situated than we are for interpreting Plato. Therefore, it seems

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[117] In *Metaphysics* VII.7 & IX.7, Aristotle approves of the use of an adjective (wooden, bronzen) rather than a noun (wood, bronze) for matter, so as to avoid giving the impression that it is that 'out of which' (ἐξ οὗ) the generation comes to be.

[118] μακρῷ πρὸς ἀλήθειαν ἀσφαλέστατον εἰπεῖν ὅτι χρυσός, *Tim.* 50B1–2.

[119] Cherniss (1944) 149n89.

to me that modern interpreters like Taylor (who himself has 'platonic' sympathies) show a great deal of hubris by their dismissive attitude towards Aristotle as an interpreter of Plato. Admittedly, he does not share our ideal of complete historical accuracy and his dialectical method leaves the impression that previous thinkers are pawns in the chess-game of his own problematic. But that does not mean that he wilfully misrepresents their theories nor that he fails to understand them because of his own predilections. More than any other Greek thinker, Aristotle consciously presents his theories as a synthesis of the opposing tendencies of his predecessors; e.g. the physiologists are balanced against the mathematicians. Admittedly, so as to induce a greater sense of aporia, he does tend to overemphasize the conflict between previous thinkers.

While this may sometimes mislead *us*, it does not prove that Aristotle misrepresented the views of his predecessors. In this particular instance, he says that Plato failed to use the Receptacle to embody mathematical forms, as one might have expected. It seems to me that this is quite a fair comment upon the *Timaeus* because, as we have already seen, there is no clear connection between the section on the Receptacle (50A–52C) and the development of the theory of planes (53C ff.). For example, Timaeus never clarifies whether or not the mathematical planes are limits imposed upon the disorderly 'stuff' of the Receptacle by the Demiurge, and we are left to draw our own conclusions. For instance, it is possible to interpret the theory of planes as a completely a priori mathematical construction, inspired by developments in stereometry within the Academy. This seems to be how Aristotle took it, since he complains frequently about the neglect of appropriate (i.e. physical) first principles for explaining physical phenomena. By contrast, modern commentators insist that the planes must be the constraining limits upon the 'plastic stuff' (ἐκμαγεῖον) of the Receptacle, otherwise the solids assigned to the elements would be like empty cardboard models. But in doing so they may be unconsciously interpreting the theory of planes in terms of matter and form, without any basis in the dialogue. Since Aristotle does not assume this, perhaps he is a better reader of the Platonic text.

For the purposes of understanding Aristotle's own developing position, however, it is not necessary to establish whether he is absolutely correct in any scholarly fashion about the theories of his predecessors. Such scholarly precision did not become fashionable until

later Greek commentators started the trend that led to the medieval scholastic tradition. In any case, the crucial point is that Aristotle was a creative philosophical thinker whose very method dictated that he should consider his predecessors as making better or worse attempts to get at the same objective truth about the world. From this perspective, even his 'scholarly' mistakes turn out to be revealing for the Procrustean character of his dialectical method.

Furthermore, by comparing Aristotle's interpretation with the original texts, we can tease out the different metaphysical assumptions that often lie hidden beneath his criticism of the theories of some previous thinker, as I have tried to do in the present chapter. In order to clarify the cosmological context for the development of Aristotle's views about mathematics, I have made a detailed examination of his criticism of mathematical reductionism in the cosmology of the *Timaeus*. I have also considered the Atomists here because his criticism of their reductionism is consistently paired with that of the Platonists in a manner which suggests that he regarded both reductionist projects as being mathematical at bottom, even though the Atomists are more physical in their approach. For him the failure of both of these projects is instructive, in the sense that they are neither mathematical enough nor physical enough. In other words, by failing to make a clear distinction between mathematics and physics, they are seduced into positing indivisible magnitudes as the explanatory cause for a qualitative continuum. This he considers to be the fundamental mistake of both kinds of reductionist theory; cf. *GC* I.2, 315b25–30.

CHAPTER THREE

PROBLEMS ABOUT MATHEMATICAL OBJECTS

One might expect questions about the foundations of mathematics to be cleared up by Aristotle's rejection of Plato's mathematical cosmology but the issue is not so straightforward, since Aristotle inherited many of his deepest convictions about science from the Platonic tradition. In the *Posterior Analytics,* for instance, he treats the mathematical sciences as paradigms of scientific knowledge, while also accepting the Platonic assumption that genuine knowledge must have a real intelligible object. These two commitments alone raise some questions about the ontological status of mathematical objects. Does mathematics study supersensible substances as Plato suggests? Or are its objects merely figments of the imagination, as Protagoras seems to have suggested? Whether Aristotle sides with Plato or Protagoras or finds some third option, he cannot ignore questions about the mode of being of mathematical objects.[1] On the contrary, I will argue, he saw it as a formidable problem for first philosophy, though here I will merely sketch the problem in terms of Aristotle's debt to the Platonic tradition.

I. *The category of quantity*

There is a general consensus among Aristotelian scholars that the *Categories* is an early work.[2] In fact, some Greek commentators accused Aristotle of merely elaborating on the Academic categories of the 'by itself' (καθ' αὑτό) and the 'in relation to something' (πρὸς τι).[3]

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[1] *De Caelo* 299a5–6 shows that Aristotle took very seriously the duty of providing alternative foundations for a science whose basic assumptions have been challenged or removed.

[2] On the place of *Categories* in the Corpus see Jaeger (1923), Ackrill (1963), de Rijk (1951 & 1952).

[3] Sextus Empiricus (*Adv. Math.* 10.258–80) attributes to 'the Pythagoreans' a threefold division of reality as follows: (1) absolutes, which exist of themselves, like man, horse, and the elements; (2) contraries like good and evil, just and unjust, which are mutually exclusive; (3) relatives like right and left, which are mutually dependent. But this may be a Hellenistic elaboration on Plato's categories.

Presumably, this division corresponded roughly to the distinction between Forms and their instantiations, since Forms are usually described as 'themselves by themselves' (αὐτὰ καθ' αὑτά), whereas their sensible instantiations are dependent on something else. In terms of this distinction, the question becomes whether mathematical objects are independent substances like the Forms or whether they are dependent attributes which are relative to some other substances.

Hippocrates Apostle claims that Aristotle posited mathematics as a science of quantities, and he assumes that this settles the ontological question because quantity is a secondary category that is dependent on substance as a primary substratum.[4] While this attractively simple solution contains a grain of truth, it leaves unresolved some issues in Aristotle's discussion of the category of quantity in the *Categories* and in *Metaphysics* V. Therefore, against the background of his physical treatises, we must try to make sense of the different treatments of that category in both texts. For instance, his discussion of the continuum in *Physics* V & VI helps us to understand the initial distinction in *Categories* 6 between discrete and continuous quantity, which is formulated in terms of the relationship of parts to each other.

Thus continuous quantities are those whose parts have position (θέσις) relative to each other, whereas the parts of discrete quantities have no relative position. Number and speech are his examples of discrete quantities, while continuous quantities are primarily exemplified in lines, planes and bodies, and secondarily in time and place. We see better what Aristotle means by relative position when he explains that the parts of number have no common boundary (κοινὸς ὅρος) at which they touch (συνάπτει) each other. Similarly, spoken language is a discrete quantity because it is measured by long and short syllables, which do not join together at a common boundary. By contrast, a line is continuous precisely because the point functions as a common boundary at which its parts join together. In the same way, he says, time is a continuous quantity because the present moment (νῦν) joins on to both past and future time. It seems that Aristotle is here thinking of the 'now' as being analogous to a point on a line, since both connect two parts of a continuous quantity. This rather simple view of time as a derivative continuous quantity

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[4] Although this is an over-simplification of Apostle's position (1952, 1978–9, 1991), it is roughly the line that he adopts to the problem of mathematical foundations according to Aristotle.

seems to belong to an earlier phase in Aristotle's thinking than the more complex view found in the *Physics*.

This applies even more so to place, which is also said to be a continuous quantity because the parts of a body occupy some place and they join together at a common boundary. From this he infers that the parts of place which are occupied by the various parts of the body themselves join together at the same boundary as the parts of the body. Thus Aristotle makes place a derivative continuous quantity that is dependent on the structure of the body which occupies place. For instance, just as the parts of a body join together at a plane, so also the corresponding parts of place will be made continuous by a plane. But this means that place is conceived of as three-dimensional in the *Categories,* whereas in the *Physics* (IV.4) it is treated as the two-dimensional surface of a surrounding body. In fact, as Henry Mendell (1987) points out, the *Categories* view of place is very similar to the 'interval' view rejected in the *Physics.*

Indeed it is unclear from the *Categories* whether Aristotle has yet made a clear distinction between mathematical and physical objects. For instance, he explains relative position (θέσις) as a characteristic of the parts of continuous quantities; i.e. the parts of a line have a position in relation to one another, since each of them is situated somewhere (κεῖται που) and so one could distinguish them and say where each is situated (κεῖται) in the plane and which one of the other parts it joins on to. By contrast, in the case of discrete quantities like number, he says that one could not observe (ἐπιβλέψαι) that its parts have relative position or are situated somewhere (κεῖται που) or that any of its parts join on to one another. Yet the essential condition for parts to have relative position is that they must endure, as is clear from his treatment of time and of speech. Although time is continuous and speech is discrete, their parts cannot have relative position as they do not endure; cf. *Cat.* 5a27–28, 35–36. Instead, the parts of time and the parts of speech may have an order (τάξις), just like the parts of number; e.g. we count two before three and so on. Indeed, it is this prior and posterior ordering of parts that becomes most important for Aristotle's view of time in the *Physics.*

The second criterion for the generic division of quantity is that a continuous quantity has parts with position (θέσις) in relation to one another. The parts of a line, for instance, have a definite position in relation to each other (e.g. to the left or right, above or below, etc.). As Aristotle puts it (*Cat.* 5a19), each of these parts is situated

somewhere (που) and you could distinguish them and say where each is situated in the plane (κεῖται ἐν τῷ ἐπιπέδῳ).[5] We should notice that these distinctions are applicable to plane geometry, moreso than to physical bodies. Since he talks about lines, planes and solids rather than about the boundaries of sensible substances, he appears to be thinking of them as independent entities. With respect to numbers, he says that their parts cannot be seen to have any (spatial) relation to one another nor are they situated anywhere. This statement also seems to imply a separation of number from bodies or other physical entities that may be counted. However, in the *Categories,* Aristotle makes no effort to clarify the ontological status of entities like numbers, lines, planes and solids, even while treating them in a quasi-substantial manner.

As Ackrill (1963) notices, one of the peculiar features of Aristotle's discussion of quantity there is that he lists the owners of quantitative properties as well as the properties themselves. This is strange because the logic of his categorial framework seems to require that he list only the quantitative properties of substances, since Aristotle surely does not mean to treat lines, planes, solids or numbers as substances. Yet, in *Metaphysics* V.13, such things as a line are said to be quantities 'in themselves' (καθ' αὑτά) and 'with respect to substance' (κατ' οὐσίαν). This is the case, Aristotle explains (1020a17–19), because the definition of their essence (τί ἐστι) uses the words 'a certain quantity' (ποσόν τι). So, just like Platonic Forms, they have their being in themselves (αὐτὰ καθ' αὑτά) rather than with respect to something else (πρός τι).

Aristotle's language here betrays the residual Platonism in his treatment of the category of quantity, since it reflects a tendency to treat lines, planes, solids and numbers as if they were independent substances with their own *per se* attributes (πάθη καθ' αὑτά) such as long/short, broad/narrow, many/few. The mathematical sciences also tend to regard these entities as independent subjects for the proper attributes that are proven to belong to them. As we shall see, the Platonic argument 'from the sciences' cites geometry as an example of a science which demands the existence of independent entities distinct from sensible particulars. Thus it seems that everything conspires against the categorial account of quantity that Aristotle wishes

[5] In his treatment of place in *Physics* IV, Aristotle notes that all directions are relative in geometry, whereas they are absolute in the physical world.

to give in terms of ontological dependency.

In the *Categories,* therefore, Aristotle seems to accept numbers, lines, planes and solids as quasi-independent entities which are the bearers of quantitative properties. Although this makes them analogous to the primary subjects of predication which are usually substances, still Aristotle does not clarify the precise relationship between the two sets of subjects. Perhaps he assumes this to have been generally established by the principle that all the other categories, including quantity, are dependent upon individual substances. But this would be more plausible if he had talked about quantitative attributes, as he did for quality in *Metaphysics* V.14.

In *Metaphysics* V.13, on the other hand, Aristotle distinguishes between discrete and continuous quantities according to the character of the units which constitute them. For instance, a certain quantity is a plurality (πλῆθος) if it is potentially divisible into parts that are not continuous (μὴ συνεχῆ); i.e. not themselves divisible into further parts. By contrast, a quantity is a magnitude (μέγεθος) if it is divisible into parts which are themselves divisible in the same way; i.e. continuously. Thus an essential characteristic of a magnitude is that it is measurable (μετρητόν), whereas a plurality is countable (ἀριθμητόν). Having established the distinction between discrete and continuous quantity in this way, Aristotle goes on (1020a11–12) to list the species of magnitude as length, breadth and depth. These are differentiated according to the three dimensions: length is continuous in one dimension, breadth in two dimensions, and depth in three dimensions.

This should be compared with the treatment of continuous quantity in the *Categories* where lines, surfaces and bodies are listed, along with time and place, as continuous quantities. The *Metaphysics* account contains the form/matter distinction as an added subtlety because line, plane and body are each said to be a limited (πεπερασμένον) quantity in their respective dimensions. In a similar fashion, of course, number is held to be a limited plurality. However, speech is dropped as a species of discrete quantity, just as place is missing from the list of continuous quantities. But perhaps their demotion can be understood in terms of the subsequent (1020a14–15) distinction between things which are called quantities in their own right (καθ' αὑτά) and those which are called quantities by accident (κατὰ συμβεβηκός).

This distinction seems to correspond roughly with the one made

in the *Categories* (5a38 ff.) between those things which are called quantities in the strict sense (κυρίως) and those which are called so by accident (κατὰ συμβεβηκός). The former are the only things that are called quantities in their own right (καθ' αὑτά), while the latter are called so derivatively. But the account of quantity in *Metaphysics* V adds a new level of sophistication by distinguishing (1020a17–19) within *per se* quantities between those which are quantities 'according to substance' (κατ' οὐσίαν) and those which are attributes or states of such a substance. An example of the former is the line, according to Aristotle, because the formula of its essence must contain the words 'a certain quantity.' The latter is illustrated by 'the many and the few,' 'the long and the short,' 'the broad and the narrow,' and 'the deep and the shallow.' Apart from these being *per se* attributes of number, line, plane and solid, their ontological status is unclear despite Aristotle's subsequent (1020a24–25) claim with reference to 'the large and the small' that all of these are quantitative attributes in their own right (τοῦ ποσοῦ πάθη καθ' αὑτά).

Therefore, it seems that the only basis for the distinction among *per se* quantities is that numbers, lines, planes and bodies are being treated *as if* they were independent subjects for attributes like many/few, long/short, broad/narrow, deep/shallow. In fact, this corresponds quite well with Aristotle's treatment of mathematics as a demonstrative science which proves that certain attributes belong essentially to given subjects, such as lines and planes in the case of plane geometry. But along with such an approach to mathematics goes a tendency to treat numbers, lines, planes and solids as if they were separate substances. Furthermore, the practice of mathematicians themselves reinforces this tendency, as we can see from Euclid's *Elements*. Judging from the accounts of quantity given both in the *Categories* and in *Metaphysics* V, therefore, it seems that Aristotle has not yet discovered how to avoid the Platonic tendency to treat mathematical objects as separate substances akin to the Forms.

II. *The mathematical ontology of Aristotle's Topics*

As an illustration of that tendency, let us examine a passage from *Topics* VI, which refers to mathematical definitions. In keeping with the character of the whole work, the primary purpose of Book VI is to outline different ways of refuting or rejecting definitions proposed

by one's opponent within the framework of a dialectical joust. Since a proposed definition fails if it has not stated the essence (τὸ τι ἦν εἶναι), Aristotle suggests some ways of determining whether or not an opponent has defined the essence of the definiendum:

> First of all, see if he has failed to make the definition through terms that are prior and more familiar. For a definition is rendered in order to come to know the term stated, and we come to know things by taking not any random terms, but such as are prior and more familiar, as is done in demonstrations (for so it is with all teaching and learning); accordingly, it is clear that a man who does not define through terms of this kind has not defined at all.[6]

The basic argument of this passage is that a genuine definition must be framed in terms that are both prior and more intelligible, otherwise the essence of the definiendum will not have been stated. If this were not the case, he goes on to argue (141a36), there would be several definitions of the same thing, and this would be unacceptable because it is generally held that each thing has only one essence. In support of a strict view of true definition, Aristotle claims that the purpose of definition cannot be achieved by means of any chance terms but only through those which are prior and more intelligible. Although he assumes without argument that such terms are best suited to making known the unique essence of the definiendum, his remarks about demonstrations, and about 'all teaching and learning' (πᾶσα διδασκαλία καὶ μάθησις), suggest that an argument may be found in the *Posterior Analytics.*

In that work Aristotle specifies (71b20 ff.) as general conditions for scientific knowledge that it must proceed from premisses that are true, primary (πρώτων), immediate, more intelligible than and prior to (γνωριμωτέρων καὶ προτέρων) the conclusions of which they are causes. Such knowledge will be attained only when all of these conditions are fulfilled and hence the starting-points (ἀρχαί) are appropriate (οἰκεῖαι) to the demonstrandum. Within the list of conditions itself, there is a broad division between those which are formulated absolutely and those framed in a relative fashion. Since priority appears on both sides, there is a *prima facie* distinction between priority in an absolute and in a relative sense. Yet Aristotle focuses on two senses of relative priority which are integrally connected with two distinct senses of 'more intelligible than':

[6] *Topics* VI.4, 141a26–31: tr. W.A. Pickard-Cambridge in Barnes ed. (1984).

> Things are prior and more familiar in two ways; for it is not the same to be prior by nature and prior in relation to us, nor to be more familiar and more familiar to us. I call prior and more familiar in relation to us what is nearer to perception, prior and more familiar simpliciter what is further away. What is most universal is furthest away, and the particulars are nearest; and these are opposite to each other.[7]

A few points about this passage are worth noting. First, Aristotle consistently treats relative priority (προτέρον) and greater intelligibility (γνωριμώτερον) as coordinate conditions for appropriate first premisses in syllogistic demonstrations. Second, he appears to make the same distinction for both between their two different senses; namely, what is prior and better known by nature (τῇ φύσει) as distinct from what is prior and better known to us (πρὸς ἡμᾶς). Third, the intimate connection between these conditions is further confirmed when Aristotle explains that objects which are closer to sense perception are both prior and more familiar to us. Those objects, by contrast, which are more removed from perception are both prior and more intelligible *simpliciter* (ἁπλῶς).

Thus, according to Aristotle, the most universal things (τὰ καθόλου μάλιστα), which are the most remote from sensation, are prior by nature and more intelligible absolutely. Particular things (τὰ καθ' ἕκαστα), on the other hand, are prior and more intelligible in relation to us because they are closer to sensation. But it is typical for a Platonist to claim that the kinds are more real than the particulars or, in other words, that universals are prior by nature and hence more intelligible than particulars in an absolute sense. So this whole passage in the *Posterior Analytics* is quite Platonic insofar as its epistemological and ontological assumptions seem to be mutually interdependent.

The same applies to the parallel passage in the *Topics* (141a26–32), which declares that whoever does not frame his definition in terms that are prior and more intelligible has not really given a definition. It is possible, of course, to give many different descriptions of the same thing but these cannot all be definitions because that would mean that it had many different essences, and this would conflict with the general assumption that there is some unique answer to the question: what is it? (τί ἐστί;).[8] Since the proper answer to this ques-

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[7] *Posterior Analytics* I.2, 71b33–72a5: tr. Barnes ed. (1984).

[8] As Nehemas (1987) points out, Socratic inquiry would lack direction without some such assumption.

tion is a definition, it is natural to assume that there is a single account of the true nature of each thing, as this is essential for Aristotle's conclusion (141b2–3) that whoever has not framed his definition by means of prior and more intelligible terms has not really given a definition.

Just as the different senses of 'more familiar' are distinguished in the *Posterior Analytics,* so also here in the *Topics* Aristotle introduces a similar distinction:

> The statement that a definition has not been made through more familiar terms may be understood in two ways, either supposing that its terms are without qualification less intelligible, or supposing that they are less intelligible to us; for either way is possible. Thus the prior without qualification is more familiar than the posterior, a point, for instance, than a line, a line than a plane, and a plane than a solid, just as a unit is more intelligible than a number; for it is prior to and a principle of all number. Likewise, also, a letter is more familiar than a syllable. Whereas to us it sometimes happens that the converse is the case; for a solid falls under perception most of all, and a plane more than a line, and line more than a point; for most people learn such things earlier; for any ordinary intelligence can grasp them, whereas the others require a precise and exceptional understanding.[9]

In terms of the general distinction between what is more familiar absolutely (ἁπλῶς) and what is more familiar to us (ἡμῖν), the notion of priority determines what it means for something to be more familiar absolutely. The explanatory examples presuppose some ontological schema of priority; i.e. the point is held to be more intelligible absolutely than the line because it is prior to it. Following the same schema, a line is more intelligible than a plane, while a plane is more intelligible than a solid. Although the kind of priority involved here is not identified, we can infer from the guiding example that it is the natural priority of the unit over number. The unit is said to be more intelligible than number because of being prior to (πρότερον) and the principle (ἀρχή) of all number. Since this is offered as a paradigmatic example of such priority, we are justified in applying the explanation to other similar examples. For instance, we can say that the point is more intelligible than the line, and the line more so than the plane, and the plane more so than the solid, since each is a principle of the other and hence prior to it by nature.

[9] *Topics* VI.4, 141b3–14: tr. Pickard-Cambridge in Barnes ed. (1984).

What Aristotle uses here by way of illustration is a standard Academic schema of natural priority.[10]

According to this well-known schema, for instance, the plane is naturally prior to the solid because planes both limit and mark off a body as a particular thing. Hence, when these limits are destroyed, the whole body is also destroyed.[11] As one can see from *Metaphysics* V.8, this was a formula used by some thinkers as a criterion for priority in substance. In III.5, for instance, Aristotle reports that a similar criterion was used by some (unnamed) thinkers to conclude that the body (τὸ σῶμα) is less substantial (ἧττον οὐσία) than the planes and lines which limit and define it. The rationale behind this conclusion is that these defining boundaries "are thought to be capable of existing without body, whereas the body cannot exist without them."[12] Both may be treated as parallel reports of the same way of thinking about substance which was a commonplace within the Academy. The logical implication of this way of thinking is that numbers are prior in substance to everything else because they are the defining limits without which these other things would be destroyed. This conclusion may also be drawn from the Academic schema of priority that is clearly presupposed in the *Topics* passage quoted above.

A parallel passage from *Posterior Analytics* I.27 provides some insight into the epistemological and ontological implications of this schema of priority. Here Aristotle compares different kinds of knowledge in terms of greater accuracy (ἀκριβεστέρα) and priority (προτέρα), just as Plato did (*Phil.* 57A–59C). In the *Posterior Analytics,* therefore, Aristotle seems to be 'platonizing' when he makes a number of different comparisons with reference to these specific characteristics. For instance, he says (87a31–33) that knowledge of the reason why (τὸ διότι) combined with knowledge of the fact (τὸ ὅτι) is more accurate than and prior to knowledge of the fact alone. From other passages (cf. *APst.* II.1, 89b23 ff.) it is clear that Aristotle considers knowledge of the fact to be more accessible to sensation and hence prior with respect to us. But this means that knowledge of the reason why is absolutely prior or prior by nature and, *as such,* it is more accurate

[10] There is considerable evidence in the Aristotelian corpus for the Platonic origins of this schema of priority and for its influence on Aristotle; cf. Cleary (1988b) ch. 1.

[11] ὧν ἀναιρουμένων ἀναιρεῖται τὸ ὅλον, *Met.* 1017b18–19.

[12] τὰ μὲν ἄνευ σώματος ἐνδέχεσθαι δοκεῖ εἶναι τὸ δὲ σῶμα ἄνευ τούτων ἀδύνατον, *Met.* 1002a6–7.

knowledge. It is this sense of natural priority, then, which should be understood to hold for the rest of his comparisons.

Whereas the first and second comparisons appear to depend on Aristotle's own views, the third seems to presuppose the Academic schema of priority used in the *Topics*. His illustration also involves a comparison between two 'pure' sciences, namely arithmetic and geometry. Arithmetic is said to be more accurate than and prior to geometry because it comes from fewer things (ἐξ ἐλαττόνων), while the latter comes from additional things (ἐκ προσθέσεως). Significantly, Aristotle explains what he means by using the example of the unit (μονάς) as compared to the point (στιγμή). His explanation also reveals the ontological framework behind the comparison when he describes the unit as a substance without position (οὐσία ἄθετος), while the point is said to be a substance with position (οὐσία θετός). Even if this definition of the point is Pythagorean in origin, it is still compatible with the Platonic schema of priority.[13] With reference to such a schema, it makes sense to say that the point is 'the result of addition' (ἐκ προσθέσεως) because position has been added to a unit. Since points are typically objects of geometrical study, it becomes intelligible for Aristotle to contrast geometry with arithmetic by saying that the former depends on additional principles, whereas the latter depends on fewer principles.

In a later chapter, I will discuss Aristotle's use of the terminology of abstraction, which is sometimes contrasted with addition. Here I simply note the ontological implications for mathematical objects that seem to follow from *Posterior Analytics* I.27. It is significant, for example, that Aristotle should use the word οὐσία with reference to mathematical objects like the unit and the point. Since the context is a discussion of the priority and accuracy of the sciences in relation to each other, he appears to be accepting the Platonic assumption that priority in knowledge involves priority in substance. From this it follows that the unit is prior in substance to the point, the point to the line, the line to the plane, and the plane to the solid. This system of non-reversible dependencies was probably reflected in Academic prescriptions for the proper order of mathematical definition.[14]

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[13] Nicomachus (*Th. Ar.* 74.10 = DK 44A12) ascribes to Philolaus a system that contains a development of being in numerical stages, yet Burkert (1962, 247) thinks that the schema for gradations in being is exclusively Platonic because it conflicts with Aristotle's reports about the Pythagoreans.

[14] Compare the order of definitions to be found in the first book of Euclid's

Such a system can also result from an application of the criterion for priority in substance that Aristotle explicitly attributes to Plato in *Metaphysics* V.11, where he outlines different senses of 'prior' and 'posterior,' including natural or substantial priority (κατὰ φύσιν καὶ οὐσίαν).[15] With regard to this particular meaning of 'prior' and 'posterior,' Plato is said to have used the criterion that those things are prior which can exist without others, while the other things cannot exist without them. Even without illustrations of this kind of priority, one can see how an application of the criterion of non-reciprocal dependence could yield the ontological priority of unit to point to line to plane and to solid, especially given the Platonic assumption that the order of reality follows the order of knowledge. Furthermore, the criterion used here is very similar to that used at *Metaphysics* III.5 to decide on the substantial priority of planes to the body which they determine both logically and ontologically.

This Academic background throws into relief some of the implications hidden in the above *Topics* passage. When Aristotle there distinguishes between what is more intelligible to us and what is more intelligible absolutely, his examples show that he has mathematics in mind. Possibly he made this distinction in the first place because he recognized that the mathematical sciences require a special kind of thinking. At the end of the passage quoted above, for example, he contrasts the ordinary understanding (τῆς τυχούσης διανοίας) with the precise and extraordinary (ἀκριβοῦς καὶ περιττῆς) understanding which is demanded by mathematics.[16] Since our ordinary grasp of things is dominated by sense perception, what is more familiar to us will sometimes turn out to be the complete reverse of what is more intelligible absolutely, especially in the case of mathematical objects. For instance, as Aristotle explains, the solid (τὸ στερεόν) is most obvious to sense

Elements; cf. Heath ed. (1956). First (Def. 1) we have the definition of a point, then (Def. 2) of a line and subsequently (Def. 5) of a surface. This logical progression from the more simple (and hence prior) to the more complex (and hence posterior) seems to have dominated the subsequent mathematical tradition. Yet these definitions themselves do not directly reflect the Academic schema of priority, though there are alternative definitions which do so. These alternatives may belong to the older Academic tradition, since they are reported by Aristotle in the *De Anima* (409a4–6) and also noted by Proclus in his commentary on *Elements* I (*in Eucl.* 97.8–13).

[15] For an extended discussion of this chapter see Cleary (1988b) ch. 3.

[16] It can hardly be accidental that Isocrates (*Antid.* 264–5) uses almost the same terminology when he concedes a limited value to mathematical sciences like geometry and astronomy as a way of sharpening the wits of students.

perception, while the plane (τὸ ἐπίπεδον) is more obvious than the line (ἡ γραμμή) and the line moreso than the point (τὸ σημεῖον).

Hence we can see that the ordering of things with respect to perception reverses the mathematical order of definition. Aristotle insists that the mathematical order is 'more scientific' (ἐπιστημονικώτερον) and this is hardly surprising, given that he treats the mathematical sciences as paradigms of demonstrative knowledge. Yet he also accepts an alternative order of definition:

> Absolutely, then, it is better to try to come to know what is posterior through what is prior, inasmuch as such a way of procedure is more scientific. Of course, in dealing with persons who cannot recognize things through terms of that kind, it may perhaps be necessary to frame the account through terms that are familiar to them. Among definitions of this kind are those of a point, a line, and a plane, all of which explain the prior by the posterior; for they say that a point is the limit of a line, a line of a plane, a plane of a solid. One must, however, not fail to observe that those who define in this way cannot show the essence of what they define, unless it so happens that the same thing is more familiar both to us and also without qualification, since a correct definition must define a thing through its genus and differentiae, and these belong to the order of things which are without qualification more familiar than, and prior to, the species. For annul the genus and differentia, and the species too is annulled, so that these are prior to the species.[17]

The emphatic position of 'absolutely' (ἁπλῶς) in the Greek establishes the point of view from which Aristotle insists that it is better (βέλτιον) to aim at knowledge of posterior things (τὰ ὕστερα) by means of what is prior (διὰ τῶν πρότερον). From previous passages we may assume that he is thinking of the mathematical method of defining what is posterior in terms of what is judged to be naturally prior, according to the relevant Academic criterion of priority.

By contrast with the mathematical approach to learning, however, there is an alternative approach that Aristotle concedes to be necessary in the case of people who cannot acquire knowledge through what is prior and more intelligible in the absolute sense. In the case of neophytes, for example, one must give an account in terms that are familiar to them; i.e. that appeal to sense perception. In such cases it may be necessary, for example, to describe a figure as 'the limit of a solid' (στεροῦ πέρας), just as Socrates does in the *Meno*

[17] *Topics* VI.4, 141b15–29: tr. Pickard-Cambridge in Barnes ed. (1984).

(76A) for the benefit of his sophistic interlocutor.[18] In a more historical vein, this whole passage in the *Meno* (75–77) provides some evidence for an alternative tradition of empirical mathematics; just as the above *Topics* passage testifies to the existence of alternative definitions for mathematical entities such as the solid, the plane, the line, and the point.[19]

However, Aristotle is obviously adopting the Academic line when he insists that those people who define in this way cannot show the essence (τὸ τί ἦν εἶναι) of the definiendum, unless it happens by chance that the same thing is more intelligible both to us and absolutely at the same time. From his formulation here and from what has gone before, we may conclude that Aristotle does not consider such a coincidence of the two senses of 'more familiar' to be the usual state of affairs with respect to mathematical entities. In a subsequent passage (142a9–12) he says that what is familiar absolutely (τὸ ἁπλῶς γνώριμον) is not identical with what is intelligible to everyone (τὸ πᾶσι γνώριμον) but rather with what is intelligible to those who are in a sound state of understanding. Similarly, he says, what is healthy absolutely is identical with what is healthy for those in good physical condition. To avoid any subjectivist misunderstandings of these statements, we should interpret them in the light of a previous discussion (141b34–142a8) which concluded that a true definition cannot be given in terms that are merely intelligible to an individual, since such terms may change over time according to the state of knowledge of

[18] Vlastos (1988) has suggested that this definition may have been included in some pre-Euclidean axiom system. He argues that it is being held up as an example of scientific definition by contrast with the previous definition of color, which follows the muddled Empedoclean model.

[19] For instance, the definition of a point as 'a unit having position' shows traces of the Pythagoreanism which we know from other sources (cf. Proclus, *in Eucl.* 95.21) to have influenced Xenocrates and Speusippus. The description of a line as a moving point might be taken as a product of the later Pythagorean tradition because it seems to be consciously designed to escape the paradoxes arising out of the earlier view that a line is composed of points. Unfortunately, Proclus does not supply us with any names when he talks about the people who define the line as 'the flux of a point' (ῥύσις σημείου). But he does put it forward as an alternative to Euclid's definition of the line, together with another definition which is essentially that of Aristotle who defines the line as 'magnitude extended in one direction' (μεγέθους... τὸ... ἐφ' ἓν συνεχὲς μῆκος, *Met.* V.13,1020a11–12). Proclus remarks that this definition perfectly displays the nature of the line but he also thinks that its generative cause is shown in the alternative definition of the line as the flowing of a point. However, according to Aristotle, Plato rejects the whole genus of points as a geometrical fiction and so refers to them as 'indivisible lines' (τὰς ἀτόμους γραμμάς); cf. *Met.* 992a20–3.

the person involved. Aristotle may here be implicitly opposing the views of sophists like Protagoras who took a subjectivistic approach to truth.[20]

Whatever his target, it is clear that Aristotle himself does not deny the existence of absolute standards of health and intelligibility, since he is actually presupposing such standards when he talks about those people who are in a sound state of physical and intellectual health. The point is that only a few people reach these standards and, as a result, they are the best available measures of what is healthy or more intelligible in an absolute sense.[21] Hence the whole discussion here implies that some inductive process of education is usually necessary to bring what is more intelligible to us into line with what is more intelligible absolutely. This is especially true for exact sciences like mathematics which require a mode of thought that is different from the ordinary.[22] It also appears to be true for the Platonic science of dialectic, judging by Aristotle's claim that a correct definition of a thing must be framed in terms of its genus and proper differentiae. These conditions for correct definition reflect the legacy of Plato which Aristotle inherits, though he denies that division is a method of demonstration.

We can detect traces of this inheritance where Aristotle explains why the genus and the differentiae are absolutely prior to and more intelligible than the species. In his explanation he appeals (141b28–29) to the rule that if the genus and the differentia are together destroyed (συναναιρεῖ) then the species is also destroyed. But this is a specific version of the general criterion for priority with respect to substance or nature, which Aristotle explicitly attributes to Plato in *Metaphysics* V. It is within such a Platonic perspective that we should understand Aristotle's emphatic assertion that the essence of the definiendum can only be made known through terms that are prior

[20] Diogenes Laertius seems to be quoting from the beginning of Protagoras' work, *The Truth:* "Man is the measure of all things, of things that are as to how they are, and of things that are not as to how they are not." (DKB1). Sextus Empiricus reports that it came at the beginning of a work which he calls *The Overthrowing Arguments,* which is possibly the same as *The Truth.*

[21] There is an analogous role for the good judge (φρόνιμος) in moral and political affairs where things happen only 'for the most part'; cf. *EN* 1142a11 ff.

[22] This would fit very well with the mathematical education outlined in the *Republic,* since its purpose might be described as turning the soul away from what is more familiar to us toward what is more intelligible absolutely. Both Plato and Aristotle acknowledge that what is learnt (μαθῆμα) is especially the science of mathematics (μαθηματική), as is confirmed by the Greek language itself.

and more intelligible in an absolute sense. The definition of a species by means of its genus and proper differentiae satisfies this condition because they are prior to and more intelligible than the species, according to the relevant Platonic criteria. Another reason why this is the correct mode of definition has already been given in *Topics* VI, where Aristotle says that the genus is usually thought to indicate the substance of the definiendum (τὴν τοῦ ὁριζομένου οὐσίαν). Hence, in order to give a true definition for any species, it is crucial to find its proper genus.

But such an unqualified acceptance of Platonic views on definition raises questions about Aristotle's own ontological stance throughout the *Topics* passage and the parallel passages from the *Posterior Analytics*. Indeed he appears to be uncritically adopting some basic elements of Plato's ontology; i.e., that the genus is naturally prior to and more intelligible than the species. From my point of view, however, what is most interesting about the *Topics* passage is Aristotle's acceptance of the natural priority of point to line to plane to solid. It seems that he is adopting the mathematical ontology of the Academy as a basis for the 'more scientific' procedure of definition, especially in refusing to accept that alternative definitions could show the essence of the definiendum.

Of course, one might object[23] that this is the most logical of Aristotle's four causes and that for him the essence of any definiendum is dictated by its formal definition. Therefore it does not necessarily follow that he is making an ontological commitment when he insists that the essence of a definiendum can be given only by a definition that is formally correct. Yet, in the *Topics* passage, Aristotle fails to enter any caveat which might indicate that he wished to distance himself from Platonism. For instance, there is no hint of a challenge to the crucial Platonic assumption that the order of reality follows the order of knowledge, taken absolutely; nor is this assumption undermined by the distinction between what is more intelligible to us and what is more intelligible absolutely. Hence the *Topics* reflects Platonic assumptions about mathematical objects.

[23] I owe this objection to Hans-Georg Gadamer.

III. *The break with Platonism*

Against this background, it is necessary to examine Aristotle's critical treatment of Plato's theory of Forms, so as to assess its implications for his own views on the ontological status of mathematical objects. I will focus mainly on some fragments from *On Forms,* where he rehearses ostensibly Platonic arguments for Forms based upon the existence of organized bodies of knowledge. But let me begin with a fragment from *On the Good,* which confirms my previous hypothesis about the Academic schema of priority and its ontological implications for Plato. This fragment is reported by Alexander (*in Metaph.* 55.20 ff.) and, in the context of Aristotle's reports[24] on Plato's so-called unwritten doctrines (ἄγραφα δόγματα), it gives some typical reasons for identifying Forms with numbers:

> Both Plato and the Pythagoreans assumed numbers to be the first principles of existing things, because they thought that it is that which is primary and incomposite that is a first principle, and that planes are prior to bodies (for that which is simpler than another and not destroyed with it is prior to it by nature), and on the same principle lines are prior to planes, and points (which the mathematicians call *semeia* but they called units) to lines, being completely incomposite and having nothing prior to them; but units are numbers; therefore numbers are the first of existing things. And since Forms or Ideas are prior to the things which according to Plato have their being in relation to them and derive their being from them (the existence of these he tried in several ways to establish), he called the Forms numbers. For if that which is one in kind is prior to the things that exist only in relation to it, and nothing is prior to number, the Forms are numbers. This is the reason why he called the first principles of number first principles of the Forms, and the One the first principle of all things.[25]

While one might quibble about Plato and the Pythagoreans being lumped together in this way, I will not discuss its historical accuracy because I am more interested in Aristotle's own understanding of Platonism. From that perspective one cannot overlook the importance that he attaches to the unwritten doctrine, especially since it is integrally connected with Plato's mathematical ontology.

Thus the above passage represents his understanding of the Platonic

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[24] Cf. *Phys.* 209b35–210a1, *DA* 404b8–30. Cherniss (1944) rather unfairly downgrades the value of such reports, which he regards as being either misinterpretations of Platonic texts or perhaps even concoctions on the part of Aristotle himself.

[25] Alexander, *in Metaph.* 55.20–56.5: tr. Ross (1952).

reasons for identifying Forms with numbers. In view of the *Topics* passage, it is not surprising that the argument hinges upon the natural priority of both numbers and Forms, since one finds the same criteria of priority being applied to reach these conclusions. Similarly, the same schema of priority emerges from their application to mathematical entities like point, line, plane and solids. It is interesting to notice the reasons which are given for calling this a schema of *natural* priority. In general, something is judged to be a first principle (ἀρχή) if it is primary (πρῶτον) and incomposite (ἀσύνθετον). But, as the passage points out, those things are primary by nature (τῇ φύσει) which are simpler (ἁπλούστερα) and which are not destroyed (μὴ συναναιρούμενα) with the others. The latter condition is undoubtedly a pithy version of the Platonic criterion of non-reciprocal dependence. The application of this criterion would yield the natural priority of planes to solids because, if the former are removed, then the latter are destroyed. However, it is not immediately clear what it means to say that planes are 'simpler' than solids, unless it is connected with the fact that they have fewer dimensions. This conjecture is supported by the case of points (or units) which are completely uncompounded entities and which are thus said to have nothing prior to them. Presumably this would make them completely simple and absolutely prior, according to the Platonic criteria.

A remark of more than passing historical interest in the above passage is that the mathematicians use the term σημεῖα for points (στιγμαί), whereas Plato and the Pythagoreans called them 'units' (μονάδα). This is at least consistent with other reports that the Pythagoreans treated both the point and the unit as substances which are merely differentiated by the fact that the former has position.[26] Furthermore, in Euclid's *Elements,* there is evidence for alternative mathematical usages.[27] Although Aristotle does not say whether any philosophical point hangs on such terminological differences, this passage suggests that by calling points 'units' the Pythagoreans can conclude that numbers are the first among all beings (πρῶτοι τῶν ὄντων). The critical link in this chain of reasoning is the description of a point as a unit, since this brings number into the schema of natural priority. Thus, units (and hence numbers) are held to be the first principles of all things because they are absolutely prior and completely incomposite.

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[26] Cf. Proclus, *in Eucl.* 95.21.
[27] Cf. Book. I, defs. 1, 3, & 4.

According to Burkert (1962, 240 ff.), such reasoning was typical of the Pythagoreans, and this passage from *On the Good* suggests that Plato accepted their conclusions and applied them to his own theory of Forms. In fact, it states explicitly that he called the Forms numbers because they are prior to those things which participate in them and have their being from them. There is an obvious lacuna in this argument which can hardly be covered over by the analogy between the priority of numbers and the (different) priority of the Forms.[28] Aristotle appears to be aware of this gap when he offers an explanation as to how the conclusion was reached. If one assumes that what has a single look (τὸ μονοειδές) is prior to those things that depend on it, then Forms must be numbers because there is nothing prior to numbers. This, says Aristotle, is the reason why Plato identified the first principles of number with those of the Forms and why he made the One (τὸ ἕν) the first principle of everything. Here Plato is represented as going beyond the analogy between the priority of Forms and the priority of Numbers so as to reach an ultimate first principle which is the same for each. This is reminiscent of the ascent to the Good in the *Republic*.

These passages from *On the Good* constitute an important source for the whole tradition about the so-called unwritten doctrines of Plato, which has been a bone of contention among modern scholars.[29] But it can hardly be denied that Aristotle was influenced in his thinking about mathematics by some oral tradition within the Academy. For instance, he continually refers to the One and the Indefinite Dyad as first principles in mathematics, which can take different forms for numbers, lines, planes, and solids. In a subsequent passage from *On the Good,* Aristotle reports that Plato used to call the unit and the dyad the first principles of number.[30] The reasoning behind his choice of first principles is also reported, thereby providing some insight into typically Academic arguments. First, a distinction is made among numbers between what is One and what is other than one, i.e. the Many and the Few (πολλά τε καὶ ὀλίγα). It is assumed, next,

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[28] Cherniss (1944) 193 ff. is extremely suspicious of Aristotle's attempt to find common ground between Plato and the Pythagoreans with respect to their first principles, given that elsewhere he emphasizes their differences when it suits his purposes; e.g. that Plato separated the numbers, whereas the Pythagoreans did not.

[29] Pro: cf. Krämer (1959 & 1964), Gaiser (1962), Findlay (1974). Contra: cf. Cherniss (1944 & 1945) & Brisson (1993).

[30] Alexander, *in Metaph.* 56.7–8 (Hayduck). See also Theophrastus, *Met.* 6a24–26, 6b11–15.

that whatever comes after the One in numbers can be postulated as a principle of the many and the few. This assumption may look rather arbitrary but, for the Platonists, it is supported by the fact that the Two (ἡ δυάς) which comes after the One contains in itself the dual character of manyness and fewness (τὸ πολὺ καὶ τὸ ὀλίγον). On the one hand, the double is many, whereas, on the other hand, the half is few. Since both characteristics belong to the Dyad, it is contrary to the One insofar as it is divisible whereas the latter is indivisible (ἀδιαίρετον). This pair of contrasting principles can be found under different guises in a number of Plato's dialogues, but especially in the *Timaeus* and the *Philebus*.

III.1. *Arguments for Platonic Forms*

In the above passage from *On the Good,* there is a parenthetical remark to the effect that Plato tried in many ways to show that there are such things as Forms. This may be taken as a reference to the many arguments for the existence of Forms which were either borrowed from Plato himself or reconstructed by Aristotle in his lost work, *On Forms.* Among the latter, I shall stick to those arguments that seem to have a direct bearing on mathematics, while keeping in mind the *Metaphysics* passage with reference to which Alexander transcribed arguments from *On Forms.* In *Metaphysics* I.9 Aristotle critically reviews the arguments of 'those who posited the Forms' as causes for sensible things. After naming each argument, he briefly alludes to some difficulty that reinforces his general belief that the arguments for the existence of Forms are not convincing. He says, for instance, that "according to the arguments from the sciences there will be Forms of all things of which there are sciences."[31] Although the context suggests that Aristotle's remark has critical intent, the point of his criticism remains unclear without the full arguments. Thanks to Alexander's scholarly instincts, however, we can elaborate on Aristotle's laconic remarks.

With reference to Aristotle's comment about the arguments 'from the sciences,' for example, Alexander remarks that the sciences are used in many ways for setting up the Forms.[32] But, instead of the

[31] κατά τε γὰρ τοὺς λόγους τοὺς ἐκ τῶν ἐπιστημῶν εἴδη ἔσται πάντων ὅσων ἐπιστῆμαί εἰσι, *Met.* 990b11.

[32] κατασκευή seems to be standard dialectical usage for a constructive argument; cf. *Top.* 102a15 ff., 109b26 ff., 110a15 ff. For parallel usage about constructive

Platonic dialogues, he refers to *On Forms* where Aristotle has outlined many of the arguments 'from the sciences.' In view of Alexander's strategy, one might well wonder just how much of their formulation is due to Aristotle himself.[33] One might, on the other hand, suppose that Aristotle is drawing upon the oral tradition of formal argumentation within the Academy, although we have little access to that tradition apart from his own reports. Within this tight hermeneutical circle, the best approach may be to accept these arguments as reported, while making explicit comparisons with the Platonic dialogues. Yet even if we find discrepancies, we cannot assume that Aristotle is misrepresenting Platonism, since this was a many-faceted philosophical position that went beyond what was expressed in the dialogues. In addition to these artistic works, therefore, some attention must be paid to the so-called unwritten doctrines and to subsequent Platonists like Speusippus and Xenocrates.

The arguments reported by Alexander seem to be quoted directly from *On Forms* as follows:

> (1) If every science does its work with reference to one self-identical thing, and not to any particular thing, there must be, corresponding to each science, something other than sensible things, which is eternal and is the pattern for the products of the science in question. Now that is just what the Idea is.
> (2) The things of which there are sciences must exist; now the sciences are concerned with things other than particular things; for the latter are indefinite and indeterminate, while the objects of the sciences are determinate; therefore there are things other than the particulars and these are the Forms.
> (3) If medicine is the science not of this particular instance of health, but just of health, there must be such a thing as health-itself, and if geometry is knowledge not of this equal and this commensurate, but of what is just equal and what is just commensurate, there must be equal-itself and a commensurate-itself, and these are the Forms.[34]

Although there are three distinct arguments, they follow a common pattern at the general and particular levels. For example, the first two arguments refer generally to all of the sciences while the third

arguments for the existence of Forms; cf. *Met.* 991b28, 1034a2–4, 1060a18, *Phys.* 216a22, *Cael.* 293a24, *NE* 1096a19.

[33] This seems to be an implicit assumption made by Gail Fine (1993) when she analyses and explicates the arguments from *On Forms* primarily with reference to an Aristotelian framework of concepts and terms; and only then checks them against the Platonic dialogues with predictable results.

[34] Alexander, *in Metaph.* 79.5–15: tr. Ross (1952).

argument mentions two specific sciences, namely medicine and geometry, which might be taken to exemplify the productive and theoretical sciences, respectively. In addition, the initial argument seems to set down conditions for the existence of Forms, whereas the last two arguments make strong existential assertions.[35] The hypothetical form of this first argument provides a suitable vehicle for specifying the conditions which an object of science must fulfil. For instance, there must be a self-identical unity (ἕν τι καὶ τὸ αὐτό) with reference to which the science in question obtains knowledge. But, as the second argument makes clear, none of the particular things perceived by the senses satisfy that unity condition. Therefore, since the sciences do actually exist, there must be some object different from particulars and apart from sensible things (παρὰ τὰ αἰσθητά) that is eternal (ἀΐδιον) and that serves as a pattern (παράδειγμα) for the products of each science. But a Form has such a character.[36]

Hence the first argument establishes that Forms satisfy all the conditions specified for something to be a basic object of science; i.e. it must be (i) unified and self-identical, along with being (ii) eternal and (iii) a paradigm for the products of science. The language used here also suggests the sort of parallel between the productive and theoretical sciences that was quite typical of Plato. Another characteristic Platonic assumption is that sensible things lack the unity and self-identity required for basic objects of scientific knowledge.[37] In fact, this assumption is supported by his general metaphysical view that the world of sensible appearances is merely an image of the real world of Forms. Thus the first argument reflects Plato's view that Forms, as objects of science, are prior to and explanatory of sensible particulars. Furthermore, the character of Forms as eternal paradigms is amply confirmed by the dialogues.[38]

The second argument 'from the sciences' clarifies what makes sensible particulars unsuitable as basic objects of knowledge. They are declared to be indefinite (ἄπειρα) and indeterminate (ἀόριστα), whereas objects of science must be determinate (ὡρισμένα). Borrowing from the first argument, the determinate character of scientific objects may

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[35] However, we cannot assume that the modern concept of existence always corresponds to the Greek concept of being; cf. Kahn (1973) & (1976) 323–34.

[36] Cf. Cleary (1987b) for my analysis of these arguments 'from the sciences.' See also Fine (1993) 66 ff. for a more extensive (and different) analysis.

[37] Cf. *Phd.* 72A ff., *Lach.* 198D–199B, *Rep.* 339A & 523A.

[38] Cf. *Phd.* 79A, *Tim.* 37E–38A, 52A, *Rep.* 472C, 592B, 596B.

be described in terms of their unity and self-identity. In this way one can see that the claims of the first argument are being developed in the second, even while the latter makes more categorical assertions. For example, its first premiss asserts the existence of those things about which there are sciences, and the second asserts that these scientific objects must be other than sensible particulars (παρὰ τὰ καθ' ἕκαστα). But the most interesting part of the argument is the lemma which states why particular things are unsuitable as objects of science.[39]

What seems to lie behind this explanation is Plato's claim that sensible things are subject to change, and so are unsuitable as objects of knowledge because they are not 'really real' (ὄντως ὄν).[40] Also relevant here is his virtual equation of what is completely intelligible (παντελῶς γνωστόν) with what is completely real (παντελῶς ὄν); cf. *Rep.* 477A. Plato's metaphysical distinction between that which always exists (τὸ ὂν ἀεί) and that which is generated (τὸ γιγνόμενον) is consistently linked with the claim that the first is grasped only by intelligence, and the latter by sense perception.[41] As a result, therefore, the account of the sensible world given in the *Timaeus* (29D) is characterized as a likely story (εἰκός μῦθος) rather than as the truth (ἀληθεία). In the *Republic,* for instance, Plato criticizes contemporary astronomers for their exclusive attention to the sensible things in the heavens. Even though these appear to be relatively unchanging, he insists (530A–B) that the heavenly bodies must deviate here and there from true order precisely because they are visible and corporeal.

From this Platonic perspective, one can explain the alpha-privative terms that are used in the second argument to describe sensible particulars. The term ἄπειρα probably means that particular sensible

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[39] Fine (1993) 70 ff. notes that the indeterminacy of particular things may be either quantitative or qualitative, and then cites passages from Aristotle rather than from Plato so as to argue that the indeterminacy of particulars is not due to change but to having features that are explanatorily irrelevant in a given context. On this basis she claims that particulars may also be known in a derivative sense, even though they are indeterminate, as long as they are viewed under an appropriate explanatory theory.

[40] There is a long-standing scholarly dispute as to whether Plato thinks that sensible things are unsuitable as scientific objects because they are subject to changes like generation or alteration (which are temporally successive) or because they suffer the simultaneous compresence of opposites. Fine (1993) 54–7 follows Irwin's (1977) interpretation of Plato's Heracliteanism in terms of the compresence of opposites, whereas I tend to agree with Bolton (1975) that temporally successive change is primarily intended when Plato talks about the realm of becoming as distinct from the realm of being.

[41] Cf. *Rep.* 510B, *Tim.* 27D–28A.

things are numerically unlimited and so they are unsuitable as objects of knowledge which must be self-identical unities. This line of reasoning is found in the *Philebus* (14D), for instance, where the one-many problem about sensible particulars is acknowledged to be commonplace. Plato clearly describes (15A–C) particular sensible things as having an indefinite number (ἀπείροις) of aspects, in contrast to the definite unity and self-identity of a Form. As he points out subsequently (16C–E), such numerical indefiniteness undermines the possibility of knowledge.

This may also help us to interpret the adjective ἀόριστα which is used in *On Forms* with reference to the sensible particulars. It is an apha-privative word derived from the verb ὁρίζειν, and this provides a linguistic clue as to why sensible particulars cannot be basic objects of science. For both Plato and Aristotle the possibility of scientific knowledge depends ultimately on definition (ὁρίσμος). But if sensible particulars are indefinite in the number of aspects which they display, then they will be indefinable and hence unknowable.

The third argument 'from the sciences' relapses into the hypothetical mode, although this does nothing to weaken its conclusions because the argument makes strong assertions about the existence of appropriate objects for the sciences of medicine and geometry, respectively. For logical purposes the third argument may be divided into separate parts corresponding to the productive and theoretical sciences, respectively. The gist of the first part is that, since medicine is not a science of any particular instance of health (τῆσδε τῆς ὑγιείας) but rather of health *simpliciter* (ἁπλῶς ὑγιείας), there will be such a thing as health-itself (αὐτοϋγίεια) which is the basic object of that science. The second part argues, similarly, that since geometry is not the science of this particular instance of equality (τοῦδε τοῦ ἴσου) nor of this instance of commensurability (τοῦδε τοῦ συμμέτρου) but rather of equality *simpliciter* (ἁπλῶς ἴσου) and of commensurability *simpliciter* (ἁπλῶς συμμέτρου), there will be some equal-itself (αὐτόισον) and some commensurate-itself (αὐτοσύμμετρον). But these are none other than Forms.

The key to both parts of the argument is the contrast which is drawn between particular instantiations and the Form itself. For example, it is argued that the productive science of medicine is not about any particular case of health but rather about health in an absolute sense (ἁπλῶς). Similarly, the theoretical science of geometry is held to be about absolute equality and not about instances of equality. Given the use of demonstrative adjectives here, I take the

argument to be saying that neither science is concerned with the particular as such.[42] Moreover, the general reasons for this have been given in the previous argument which stated that particulars are both indefinite and indeterminate.[43]

In each one of the arguments 'from the sciences,' the conclusion that there are Forms is represented as a distinct step. This is important for understanding Aristotle's strategy of denying that there are such Forms as Health-itself (αὐτοϋγίεια) or Equal-itself (αὐτόισον) or Commensurate-itself (αὐτοσύμμετρον). It is worth noting that, even though the Form of Equality is explicitly introduced with reference to mathematics in the *Phaedo,* it is never described as αὐτόισον but rather as αὐτὸ τὸ ἴσον or as αὐτὰ τὰ ἴσα. Since Aristotle's rebuttal does not hinge upon such differences in terminology, however, it is unclear whether his reformulation of the description of Forms has an elenctic purpose.[44] Perhaps the terminology is the legacy of post-Platonic formulations of the arguments within the Academy, since Alexander's report credits the arguments to 'Platonists' rather than to Plato.

According to Alexander, Aristotle's response to the arguments 'from the sciences' is as follows:

> Such arguments do not prove the point at issue, that there are Forms, but they do show that there are things other than sensible particulars. It does not follow, however, that if there are things other than particulars these are Forms; for besides particulars there are universals, which we maintain to be the objects of the sciences.[45]

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[42] By contrast, D.H. Frank (1984) 20–23 & 67 takes Aristotle to be pointing out that the sciences operate at a certain level of generality that is different from the particular *kinds* of health or equality that are observable through the senses. But I find this interpretation rather implausible in view of the usual meaning of τὰ καθ' ἕκαστα and the particularity of the demonstratives here. In the argument from thinking, for instance, the particulars are identified with perishable individuals like Socrates.

[43] Fine (1993) claims that the argument also covers particular kinds or types of F because these do not satisfy the unity condition (i.e. one and the same thing), nor are they determinately F nor unqualifiedly F. But that claim forces her to admit (287n51) that the distinction between universals and particulars is not exclusive, although Aristotle's formulation of the argument requires this for it to be valid.

[44] Aristotle is very fond of the objection that Forms are nothing but eternal sensibles (αἰσθητὰ ἀΐδια); cf. *Met.* 997b6–12, 1040b30–34, 1060a16–18, 1086b10–11. This objection would obviously be strengthened by the description of Forms in terms of the αὐτό- compound, whereas it would not be much advanced by such formulations as αὐτὸ τὸ ἴσον. Preference for the first formulation might lead one to suspect foul play but I find no grounds for suspicion here because Aristotle's objection has a different target.

[45] Alexander, *in Metaph.* 79.15–19: tr. Ross (1952).

In terms of our analysis, it is clear that Aristotle is objecting to the final step of each argument, which specifies as Forms those things that are held to exist apart from particulars (παρὰ τὰ καθ' ἕκαστα). Furthermore, it is obvious that nothing depends on the preposition παρά because it is also used with reference to the relationship between sensible particulars and the so-called common things (τὰ κοινά), which Aristotle proposes as alternative objects for the sciences. In other words, he accepts the argument that the objects of the sciences must be other than sensible particulars yet he rejects the Platonic term for such objects. If his dispute with the Platonists is to rise above the level of verbal quibbling, however, there must be some way of specifying the difference between Forms and τὰ κοινά, especially on the question of their relationships to sensible particulars.

Yet Alexander's report contains no such specification of differences between Platonic Forms and Aristotelian 'common things.' Of course, if one assumes that these things are identical with what Aristotle elsewhere calls universals (τὰ καθόλου), then one can draw upon other texts for these distinguishing marks.[46] While this assumption is accepted by most scholars,[47] I find it rather strange that he did not use τὰ καθόλου instead of τὰ κοινά because the former makes for a more natural linguistic contrast with τὰ καθ' ἕκαστα. Even if he had not yet developed the terminology, one would expect some clarification of the ontological differences between Forms and universals in relation to sensible particulars.

But, as one can see from Alexander's report, most of Aristotle's rebuttal is taken up with elenctic arguments that are designed to show up the inconsistencies in the Platonic position. There is a puzzle here about his own position which has escaped the notice of most commentators.[48] In brief, the difficulty is that Aristotle has not adequately clarified the ontological status of the so-called common things (τὰ κοινά) which he has substituted for Forms as the objects of the sciences. For instance, apart from the bald assertion that the sciences are about τὰ κοινά, there is no clear statement about the mode of being of such objects of knowledge; e.g. whether they are separate from sensible particulars or somehow dependent on them.

This lacuna is also noticeable in the case of another argument for

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[46] Cf. *Metaphysics* VII.13, *De Interpretatione* 7.
[47] Cf. Cooper (1973), Frank (1984b), & Fine (1980 & 1993).
[48] See Cleary (1987b) for an elaboration of this problem and a possible solution.

the existence of Forms that is analogous to the argument from the sciences; namely, the argument 'from thinking':

The argument from thinking (ἀπὸ τοῦ νοεῖν) that establishes Forms is of this sort:

> I. If, whenever we think of man or footed or animal, we think of (a) something that is (τῶν ὄντων τι) and (b) of none of the particulars (for the same thought (ἔννοια) remains even when they have perished), clearly there is something apart from sensible particulars (παρὰ τὰ καθ' ἕκαστα καὶ αἰσθητά), something which we think of both when these latter exist and when they do not; for surely we do not then think of something non-existent. But this is a Form (εἶδος) or Idea (ἰδέα).[49]

Alexander reports this argument together with Aristotle's rebuttal, where we might have expected him to present the universal as an alternative object of thinking. Instead we find him objecting that such an argument also establishes Forms of perishable particulars, such as Socrates and Plato, since we think of them and exercise our imagination (φαντασία) about them when they have perished.[50] Aristotle explains that there is some appearance (φάντασμα), even when these perishable particulars no longer exist. Furthermore, he objects, we can also think of things like a hippocentaur and a chimera which are not at all real. Thus he concludes that such an argument 'from thinking' does not prove that there are Forms.[51]

The general line of Aristotle's objection suggests that he is concentrating on that aspect of the argument which depends on the capacity of the mind to think of sensible particulars when they are absent or have perished. But surely that consideration is subsidiary to the main thrust of Plato's argument which depends on the fact that the mind can think of universals like 'man' or 'biped' or 'animal.' Since these are elements of a definition and therefore of scientific knowledge, Aristotle can hardly deny that they are real things in some sense because otherwise our knowledge would be without foundation in reality. The Platonic argument 'from thinking' therefore rests on the distinction between perishable particulars and generic or specific forms like animal or man, which are permanent objects of thought. Given this distinction, the argument validly concludes that there is

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[49] Alexander, *in Metaph.* 81.25–82.1: author's translation.

[50] Incidentally, the use of perishable particulars here as examples of καθ' ἕκαστα tends to confirm my narrow interpretation of this term, as against those of Fine and Frank.

[51] Cf. Alexander, *in Metaph.* 82.1–7.

some entity apart from sensible particulars which serves as an object for our thinking both when the particulars exist and when they do not exist.

However, it seems that Aristotle's pursuit of the phantasm in his objection is a wild-goose chase because an image is tied to a particular in ways that the universal is not. The response to this argument would have been the ideal opportunity to elucidate his own concept of the universal and its relation to sensible particulars, if Aristotle's theory of universals was to be found in *On Forms*. In fact, there is an alternative version of Alexander's commentary which adds that the argument 'from thinking' does prove that there is something else besides particulars; i.e. the universal which is in the particulars (τὸ καθόλου τὸ ἐν τοῖς καθέκαστα).[52] For what it is worth, my guess is that Alexander or some later commentator is filling in what was seen to be a lacuna in Aristotle's discussion.

The same lacuna may also be detected in Aristotle's attempt to refute the arguments 'from the sciences' in a dialectical fashion:

> Take, again, the argument that there must be Forms of the products of the arts, since every art refers its products to some standard, and the objects of the arts must exist, and must be different from particular things. This latter argument, besides failing, like the others, to prove the existence of Forms, will be seen to involve Forms of things of which the Platonists insist that there are no Forms. For if, because the medical art is knowledge, not of this particular instance of health but simply of health, there is such a thing as health-itself, there will be a similar object of each of the arts. For an art is concerned not with the particular, with the 'this,' but simply with that which is the object of the art; e.g. carpentry with bench simply, not with this particular bench, with bed simply, not with this bed; so too are sculpture, painting, building, and each of the other arts, related to their own objects. There will, therefore, be a Form of each of the objects of the arts—which the believers in the Forms do not want.[53]

In order to develop this dialectical refutation, Aristotle here proceeds as follows. First, he extends the argument 'from the sciences' to all of the crafts by outlining a similar argument that could be made from them. There is, in fact, a certain basis for this extension in the third argument from the sciences which uses the productive science of medicine. Second, he claims that such an argument from the crafts

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[52] Cf. Alexander, *in Metaph.* 82.6–7 (alt. rec. gr. discr.).
[53] Alexander, *in Metaph.* 79.19–80.6: tr. Ross (1952).

proves either (i) too little or (ii) too much. On the one hand, (i) it proves too little in the sense that it also fails to prove the existence of Forms, just as do the arguments 'from the sciences.' But, on the other hand, (ii) it would prove too much if it were to succeed, since it would establish Forms of things of which the Platonists do not wish to posit Forms. Now this latter claim appears rather strange, in view of the fact that many Platonic dialogues treat the arts and crafts as bodies of knowledge. It is even stranger to find Aristotle claiming that the Platonists do not want Forms of the objects of the crafts, given that the *Republic* (596A) uses the Forms of Bed and Table to illustrate the uniqueness of Forms.

Modern scholars have struggled with this interpretative puzzle, without reaching any completely satisfactory solution. In general, two types of solution have been given: (i) that either Plato or his followers changed the doctrine of Forms in such a way as to exclude Forms of artifacts; (ii) that Aristotle simply misinterprets Plato by assuming that the latter included all artifacts in his rejection of the possibility of Forms for the products of the useful arts. For instance, Plato did not reject Forms for the products of useful arts like housebuilding and carpentry; cf. *Rep.* 596B6–10, 597B2–598D6. While this may be plausible as an interpretation of Plato's text, it shows Aristotle in a very bad light as an interpreter of Plato. Daniel Frank[54] has tried to save Aristotle's reputation by arguing that, in fact, he is pressing Plato on the inconsistency of distinguishing between the imitative and the useful crafts. When the argument from the sciences is applied to the crafts, it tends to prove that there are Forms as objects for all *technai*. Yet the Platonists do not accept the existence of Forms for the objects of the imitative crafts because such objects are copies and not originals. On the other hand, however, they do not deny that the imitative crafts are *technai* and thus there is an inconsistency in their position.

As a solution to a long-standing puzzle, Frank's proposal is rather attractive since it confirms that Aristotle usually has some basis for his dialectical arguments. Yet it tells us practically nothing about his alternative objects of the sciences (i.e. the so-called common things) or about their ontological status compared with Forms. Perhaps we

[54] Cf. Frank (1984b). In a number of footnotes (#11–13), Frank has collected references to the recent scholarship on this question. For his more extended discussion of the same issue, see Frank (1984a).

should not expect such dialectical arguments to elucidate Aristotle's own position, except where he slips in some assumption that is not shared by his rivals. Indeed, the most successful elenctic arguments are those which take a set of assumptions made by a rival and then show that they lead to an inconsistency or a contradiction. This is precisely what Aristotle is trying to do here and, insofar as the Platonic argument tends to prove that there are Forms for all *technai*, his refutation is successful.[55] But, given the character of elenctic argumentation, it is difficult to establish his own position with any certainty. This leaves us with an outstanding problem about Aristotle's views on the foundations of the sciences, and especially on the ontological status of their objects.

IV. *The paradigm for demonstrative knowledge*

Another way of clarifying this problem with respect to mathematical objects is to explore some implications of Aristotle's use of mathematics as his paradigm of demonstrative knowledge. In *Posterior Analytics* I.11 there is a passage that distinguishes his own view from that of Plato:

> It is not necessary, in order to make demonstration possible, that there should be Forms or some One apart from the Many; but it is necessary that it should be true to state a single predicate of a plurality of subjects. Otherwise there will be no universal term and if there is no universal there will be no middle term and hence no demonstration. Therefore there must be something which is one and the same above the several particulars, and does not merely share a common name with them.[56]

If we compare this passage with *On Forms* for his response to the arguments 'from the sciences,' we can spot a number of important

[55] Fine (1993) 85 interprets Aristotle's objection as being that if the argument from the sciences proves that there are everlasting, non-sensible, separated, perfect paradigms, then this conflicts with Plato's own reluctance to posit such Forms for artifacts. In support of her interpretation, she cites passages from Aristotle (*Metaphysics* I.9 & XII.3) which imply that Plato rejects Forms for artifacts, and she claims that Plato does not talk about artifact Forms in the middle and late dialogues, except in that odd passage in *Republic* X. But, if she is correct that Aristotle's objection depends on ignoring the distinction between Socratic and Platonic Forms, then the problem that I have raised about the ontological status of universals is exacerbated rather than resolved.

[56] *APst.* I.11, 77a5–9: tr. Tredennick (1960).

differences in the Greek. Here he explicitly denies that the demonstrative sciences require some one thing (a Form) that is beside the Many (παρὰ τὰ πολλά), while insisting on the need for some one thing to be truly said of the many (ἕν κατὰ πολλῶν ἀληθὲς εἰπεῖν). In *On Forms,* by contrast, there is no such linguistic distinction when Aristotle concedes that the arguments 'from the sciences' do show that there are some things beside (παρά) sensible particulars, although these are not Platonic Forms but rather what he calls 'common things' (τὰ κοινά). The distinction there does not hang on the preposition παρά, which describes the relation between both kinds of entity and sensible particulars.

In the *Posterior Analytics* passage, by contrast, Aristotle is very careful to characterize the respective relationships in different linguistic terms. Why has this now become so important? Compounding the difficulty is his use of the term τὸ καθόλου to designate the common thing that is predicated truly of many particulars. This is the term we expected to find in *On Forms* where it would have made a natural linguistic contrast with τὸ καθ' ἕκαστον as the term for a sensible particular. Its notable absence there tends to support my conjecture that Aristotle had not yet developed his mature concept of a universal.[57] As discussed in *Metaphysics* VII.13, such a concept is directly connected with the question about the substance of things. This question does not appear on the horizon of interest in *On Forms* nor is it prominent in the *Posterior Analytics,* even though it is an inescapable question for a theory of science which requires knowledge of the essences of things.

Since this is a question about foundations, let me postpone its consideration in favor of a more pressing question for scientific knowledge; namely, the question about the conditions for true predication which dominates the above passage. Since scientific knowledge is general in character, it is necessary that certain common predicates should hold true of a multiplicity of particulars, otherwise one would be doomed to the endless task of enumerating all the special facts about these particulars. For scientific knowledge to be possible, therefore, it is necessary that there be some one thing which is true of many things. But, so far as it goes, this could be an argument either for Platonic Forms or Aristotelian universals. What makes the difference?

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[57] Cf. Cleary (1987b) but see also Brakas (1988) for a different account of the development of Aristotle's concept of the universal.

The conclusion of the above-quoted passage implies that the crucial difference lies in the distinction between homonymous and synonymous predicates.

In the case of sensible particulars which share the same name with a Form, we have an instance of homonymous predication presumably because of the ontological gap between Forms and particulars. This gap seems to be indicated by παρά in the Greek, though the implications of the word remain unclear. By contrast, the words κατά and ἐπί convey the sort of predication (of one thing over many) which, by implication, must be synonymous. The relevant distinction here seems to be the one at the beginning of the *Categories* between things predicated homonymously and synonymously. Those things are called homonymous which only have a name in common, while the account of their being is different.[58] For example, both a man and a picture are called 'animals' but only homonymously because the definition of the being of each will be different. By contrast, things are called synonymous when they share the same name and the same definition of their being.

But what light does this distinction throw on Aristotle's implicit claim that the Forms are homonymous with respect to the particular sensible things which share the same names with them? One way of interpreting this claim is that Platonic Forms are separated from particulars in such a way as to exclude the possibility that they function as the essences of these sensible particulars.[59] This interpretation is indirectly supported by another passage later in the *Posterior Analytics* (I.22, 83a33–35) where the Forms are dismissed as irrelevant nonsense. That passage also involves a discussion of the sort of predication which is necessary if demonstrative knowledge is to be possible. Aristotle has already (83a15–20) distinguished between essential and

[58] *Cat.* 1a1–2. It should be noted, however, that the Platonic meaning of homonymy centers on the claim that particulars are 'called by the same name' as a Form because they have in them the character defined by that Form; cf. *Phd.* 78E2, 102B2, 103B7; *Rep.* 596A7; *Parm.* 130E5. Since the Form is not predicated of the particulars which are named after it, Aristotle's dichotomy of synonymous and homonymous predication can be misleading when it is imposed on the relationship of participation. This is a priority relationship which might be more akin to the *pros hen* equivocity which Aristotle himself later accepts as a basis for a single science of metaphysics.

[59] There are, of course, a number of other ways of interpreting this claim and these are connected with the argument from relatives in *On Forms* which also lists three ways in which the same thing can be predicated non-homonymously of many things; cf. Owen (1957) & Fine (1982 & 1993).

accidental predication in terms of whether or not the predicate (τὸ κατηγορούμενον) always belongs to the subject. Furthermore, he identifies essential predication as the key to demonstrative proof. But predicates which denote the essence (οὐσία) are either wholly or partly identical with the subject of predication and so he puts (83a24–5) them under the category of substance. By contrast, predicates which do not denote the essence are accidents (συμβεβηκότα) and they may come under the categories of quality, quantity, relation, activity, passivity, place or time. While it is unclear from the text whether Aristotle classifies the Forms as accidental predicates or simply dismisses them as impossible predicates for sensible subjects, it is clear that he regards them as being irrelevant for scientific knowledge because demonstration requires essential predicates. And the reason why the Forms are not such predicates is that they fail to be identical, either wholly or in part, with sensible subjects of predication.

Having established the relevance of the implicit critique of Platonic Forms as objects of science, I want to show how this fits with the broader context of the passage in the *Posterior Analytics*. Ross takes the beginning of this chapter (I.11) to be unconnected with the previous chapter (I.10), which deals with the 'firsts' (ἀρχαί) of any demonstrative science. But I do not think that his reading withstands critical scrutiny. First, in a general way, any passage which has clear parallels with the Platonic argument 'from the sciences' cannot fail to be relevant to an Aristotelian passage on the foundations of the sciences. More specifically, I think that a clear connection can be established by analysing the passages themselves. For instance, in the concluding section of I.10, Aristotle clarifies the question about the truth or falsity of some existence claims made by the geometer in their hypotheses:

> Definitions are not hypotheses, because they make no assertion of existence or non-existence. Hypotheses have their place among propositions, whereas definitions only need to be understood; and this does not constitute a hypothesis, unless it is claimed that listening is a kind of hypothesis. Hypotheses consist of assumptions from which the conclusion follows in virtue of their being what they are. Thus the geometrician's hypotheses are not false, as some have maintained, saying that one should not make use of falsehood, and that the geometrician is guilty of falsehood in asserting that the line which he has drawn is a foot long, or straight, when it is not; the geometrician does not infer anything from the existence of the particular line which he himself has mentioned, but only from the facts which his diagrams illustrate.

> Further, all postulates and hypotheses are either universal or particular, whereas definitions are neither.[60]

The distinction between hypothesis and definition has already been made in an earlier chapter (I.2) where the assertion of the being or non-being of something (τὸ εἶναί τι ἢ μὴ εἶναι τι) appears to be the distinctive contribution of hypotheses. Indeed, some doubt has been cast upon the standard assumption that hypotheses are existential assertions in any modern sense.[61]

This is not an issue that I need to settle here because, for my purposes, it is sufficient to claim that hypotheses consist of assertions about the object being posited. This claim is supported by Aristotle's description of hypotheses as things which are of such a kind that, by virtue of being what they are, the conclusion comes about.[62] Furthermore, it can hardly be accidental that this formulation is echoed by the description of the second kind of cause listed in II.11; namely, the necessitating condition.[63] These also bear a striking resemblance to the phrases which Aristotle uses to describe syllogistic reasoning at the beginning of the *Prior Analytics* (24b18–23) and of the *Topics* (100a25). In both places he describes a syllogism in almost identical terms as an account (λόγος) in which, certain things having been laid down (τεθέντων τινῶν), something other than what is laid down necessarily follows by virtue of this being the case (τῷ ταῦτα εἶναι). Here the instrumental dative is used to express the agency of the premisses with respect to the conclusion, as Aristotle confirms in a *Prior Analytics* passage (24b20–21) where he explains the phrase τῷ ταῦτα εἶναι as meaning that the conclusion follows through the premisses (τὸ διὰ ταῦτα συμβαίνειν). Furthermore, by 'through the premisses,' he means that nothing beyond them is required to make the consequence necessary. In general, this is what constitutes a perfect syllogism; i.e. that it needs nothing further than what has been posited to make clear what necessarily follows. Such is the proper logical context for the second sense of cause (i.e. as necessitating conditions) and for the original meaning of hypothesis.

Returning to the *Posterior Analytics*, we now have a clearer idea of what is being implied when Aristotle insists that hypotheses belong

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60 *Posterior Analytics* I. 10, 76b35–77a3: tr. Tredennick (1960).
61 Cf. Gomez-Lobo (1980) 71–89 & Kahn (1976) 323–34.
62 ὅσων ὄντων τῷ ἐκεῖνα εἶναι γίγνεται τὸ συμπέρασμα, *APst.* 76b38–39.
63 τὸ τίνων ὄντων ἀνάγκη τοῦτ' εἶναι, *APst.* 94a22.

'among the premisses' (ἐν ταῖς προτάσεσιν), and so we can see why he raises the issue of whether or not the geometer's hypotheses are false. Geometry is, after all, one of the most scientific disciplines and it would be scandalous if (as Protagoras reportedly claimed) its practitioners made use of false hypotheses. Yet, on the face of it, this is what they appear to be doing when they assert that the lines they draw are straight or are exactly of a given length. Aristotle seems to have in mind such assumptions as 'Let this (AB) be a straight line,' which are very common in Euclid's *Elements*. The question of their falsity arises presumably because the geometer is pointing to a sensible diagram and predicating of it what it does not appear to have; e.g. perfect straightness.

Aristotle's response to this kind of Protagorean objection is that the geometer does not draw any conclusion by virtue of this (diagram) being a line (τῷ τήνδε εἶναι γραμμήν), which he had merely named a line (ἣν αὐτὸς ἔφθεγκται). On the contrary, it is the things made clear (τὰ δηλούμενα) through diagrams that are the real causes of the geometer's conclusions.[64] The point may be brought out as follows: the geometer does not draw any conclusion from the assumption that this particular figure is a line but rather from the fact that the things that are illustrated are lines. But this leaves unanswered the question about the ontological status of things which are made clear through diagrams. In addition, if hypotheses belong among the premisses, they must have the logical form of propositions with both a subject and predicate term. This poses something of a problem for the standard interpretation of hypotheses which takes them to be simple assertions of existence.[65]

In this particular case, there is some basis in the text for treating hypotheses as predicative statements. For instance, Aristotle is defending the geometer against the charge of making a false hypothesis when he posits that a drawn line (τὴν γεγραμμένην) is straight. The question of its falsity arises precisely because the geometer points to some sensible diagram and apparently predicates of it something that does not belong to it. This implies that his hypothesis has the logical

[64] The Greek is rather ambiguous here but I prefer to take διὰ τούτων as corresponding to the instrumental dative in the previous clause. We have already seen these two ways of expressing the agency of the premisses with respect to the conclusion through parallel constructions of the instrumental dative and the mediative genitive.

[65] Cf. S. Mansion (1976) & Upton (1978 & 1985).

form of a predicative proposition, since truth and falsity are most relevant to such statements; cf. *APst.* 72a8–11. That implication is indirectly borne out by the manner in which Aristotle defends the geometer by saying that no conclusion is drawn from this (diagram) being a line (τήνδε εἶναι γραμμήν). If this were an existential statement, we should expect to find τήν between τήνδε and γραμμήν because its omission is very rare in Greek.[66] So here we may have a predicative proposition with τήνδε as a subject term and γραμμήν as a predicate term. The demonstrative pronominal seems to be pointing out the subject of predication, though the relative clause (ἣν αὐτὸς ἔφθεγκται) indicates that this is not the real subject of predication. The verb ἔφθεγκται implies that the geometer has contingently attached the name γραμμή to the visible diagram, whereas the real referent is found elsewhere among the things illuminated through the diagram.

Thus Aristotle's defence of the geometers raises a question about the real objects of mathematics. It would appear that the truth of geometry, at least, requires the existence of subjects of predication other than sensible diagrams. Even if hypotheses are propositions rather than statements of existence, there must be some real subjects to carry the mathematical predicates, since Aristotle does not give propositions any ontological status of their own. The Platonic argument 'from the sciences,' as we saw, concludes with the postulation of Forms which are entirely different and separate from sensible things. So it is a natural progression in thought for Aristotle to go on at *Posterior Analytics* I.11 to declare that the existence of Forms is not necessary in order for demonstration to be possible. All that is necessary is that one be able to truly state (ἀληθὲς εἰπεῖν) a single predicate of a plurality of things (ἓν κατὰ πολλῶν) and this is made possible by the universal (τὸ καθόλου). Notice that the concern with true predication is carried over from the previous chapter and this indicates some continuity. Given his denial of Platonism, Aristotle must face the problem of identifying the real subjects of mathematical attributes and also of clarifying their ontological status.

We can approach this problem from a different direction by looking at a previous passage from the same section (I.10) of the *Posterior Analytics:*

[66] Kuhner & Gerth (1898) i, 628 cite just a handful of such omissions. I am grateful to Kenneth Quandt for drawing my attention to these linguistic points.

> For every demonstrative science has to do with three things: what it posits to be (these form the kind of which it considers the attributes (that belong to it) in itself); and what are called the common axioms, the primitives from which it demonstrates; and thirdly the attributes of which it assumes what each signifies. Nothing, however, prevents some sciences from overlooking some of these—e.g. from not supposing that its kind is, if it is evident that it is (for it is not equally clear that number is and that hot and cold are), and from not assuming what the attributes signify, if they are clear—just as in the case of the common items it does not assume what to take equals from equals signifies, because it is familiar. But none the less there are by nature these three things, that about which (the science) proves, what it proves, and the things from which (it proves).[67]

This passage gives a summary of the ingredients for any demonstrative science, as conceived by Aristotle. For instance, he insists that every such science is intrinsically concerned with three different things, even though some sciences need not deal explicitly with all of them.

First, every demonstrative science is typically dealing with a particular genus of objects, whose *per se* attributes (καθ' αὑτὰ παθήματα) it studies theoretically. In a previous section (I.7), Aristotle argued that there cannot be an arithmetical proof in geometry, for instance, since each science is differentiated and separated from others by virtue of the genus of objects which it studies. Now, in the above passage, he refers to this subject-genus of a science as 'what it posits to be' (ὅσα τε εἶναι τίθεται) and the Greek suggests that it is an 'existence' postulate. In a prior passage (76b3–7) he has said that the special principles (ἴδια) of a science include those things 'which are assumed to be' (ἃ λαμβάνεται εἶναι) and whose essential attributes the science studies. In both cases, the use of the verb εἶναι in a syntactically absolute fashion[68] appears to mean that it is the existence of some genus of things which is posited by each science. This is reinforced by the above-quoted passage, especially where Aristotle concedes that some of the sciences need not deal with all of the basic ingredients outlined. For example, he says, the genus might not be hypothesized to exist, 'if it is obvious that it is' (ἂν ᾖ φανερὸν ὅτι ἔστιν). By way of clarification, he then remarks that it is not equally clear that number exists as that hot and cold exist. Similarly, with respect to the essential attributes, it is not necessary to assume what they refer to

[67] *APst.* 76b12–23: tr. Barnes (1975).

[68] Cf. Kahn (1973). In his sixth chapter, Kahn lists 6 standard constructions with

(σημαίνει), if that is obvious.[69] These examples suggest that Aristotle differentiates physics from mathematics in terms of the sort of evidence required for their foundations.[70]

In the case of physics, it is clear enough that Aristotle would appeal to sense perception as evidence for the existence of the hot and the cold, just as he does for the existence of motion in *Physics* I. There he accuses Melissus and Parmenides of making false assumptions and, in explicit opposition to them, posits his own starting-point; i.e. that either all or some of the things existing by nature are in motion.[71] In addition, Aristotle claims (185a14) that this assumption is evident from sense experience. But the task is not so easy in the case of mathematics because its objects of study are not sensible figures, even though they may be illustrated through visible diagrams. Hence it is always possible for someone like Protagoras to question the existence of such 'ideal' objects. Plato can respond to this challenge by positing the Forms of mathematical objects (e.g. Triangle-itself, Square-itself), even while conceding that geometers use visible figures as aids in reasoning. But since Aristotle has rejected the separate existence of such Forms, he must give an alternative account of the ontological status of mathematical objects. His appeal to foundational hypotheses disguises the need for such an account, as it corresponds with the actual practice of mathematicians which Plato had already described as needing a dialectical grounding. Given his Platonic heritage, however, one may suppose that Aristotle felt the need for an alternative account, especially since he no longer accepted dialectic as a 'mistress' science that grounds all scientific knowledge in a vision of the Good.

In *Posterior Analytics* Book II, there is an important passage which throws some light on the problem that I have been teasing out of Aristotle's model for a demonstrative science. He begins by distin-

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the existential use of εἶναι which he has gleaned from Homeric and post-Homeric literature.

[69] Terence Irwin (1982) argues convincingly that Aristotle's notion of meaning is not primarily linguistic but rather has to do with real properties of things and how these relate to words.

[70] Leszl (1981) suggests a distinction between a broader and narrower meaning of 'hypothesis,' according to which the latter is connected with the traditional usage in dialectic and mathematics as it is found in the Hippocratic and Platonic writings.

[71] ἡμῖν δ' ὑποκείσθω τὰ φύσει ἢ πάντα ἢ ἔνια κινούμενα εἶναι· δῆλον δ' ἐκ τῆς ἐπαγωγῆς, *Phys.* 185a12–14.

guishing four kinds of question that yield four different kinds of knowledge. In order of listing, these are questions of: (i) the fact (τὸ ὅτι); (ii) the reason why (τὸ διότι); (iii) if it is (εἰ ἔστι); (iv) what it is (τί ἐστιν). Having listed these four types of inquiry (τὰ ζητούμενα), Aristotle illustrates and explains each of them individually. We inquire about 'the fact,' for instance, when we ask whether the sun is eclipsed or not; i.e. whether something is this or that (πότερον τόδε ἢ τόδε, 89b25). I think it is clear from the example given here that the question has to do with whether or not a certain attribute is predicated of a given subject. Thus, as Aristotle puts it, this line of inquiry stops when we have established whether or not this is the case. But knowing the fact prompts us to ask the reason why this is so; e.g. knowing that the sun is eclipsed or that the earth moves, we are led to ask why this is the case. What distinguishes this as a second line of inquiry, it seems, is that it cannot be satisfied by a direct appeal to sense experience, whereas the question of fact may be thus satisfied. The why-question, by contrast, sometimes demands an answer in terms of a hidden cause that will explain why some particular attribute belongs to a given subject.

After clarifying the first two types of question, Aristotle emphasizes that the second pair of questions have a different form:

> Now while (we seek) these things in this way, we seek some things in another fashion—e.g. if a centaur or a god is or is not (I mean if one is or not simpliciter and not if (one is) white or not). And knowing that it is, we seek what it is (e.g. so what is a god? or what is a man?).[72]

The first of these questions is usually understood as a question about existence simpliciter (ἁπλῶς), and this interpretation has some basis in the text. For instance, it is clearly distinguished from the question of fact which concerns the belonging of attributes to subjects; e.g. whether or not something is white (εἰ λευκὸς ἢ μή). By contrast, this question is said to be about whether something is or is not (εἰ ἔστιν ἢ μή). Thus it is quite natural to interpret it as a question about existence *simpliciter,* as distinct from a question about predication or the so-called question of fact. Indeed, some such interpretation has been almost universally accepted by ancient and modern commentators. It also appears to be confirmed in the text by the example which Aristotle chooses for illustrating the third type of question; i.e.

[72] *APst.* 89b31–35: tr. Barnes (1975).

whether or not a centaur or a god 'exists.'[73] In fact, the passage is cited by Kahn as a leading example of the use of the verb εἶναι in an existential sense, since it is used here in a syntactically absolute fashion. It is no accident that the question is about the existence of gods and centaurs, he claims, because it was in connection with such critical doubts that the existential use of εἶναι arose in post-Homeric literature. According to Kahn, such questions have the effect of isolating the existential presupposition of statements about the grammatical subject of the verb 'to be.'

However, both Kahn's analysis and the traditional interpretation of this third question have been challenged by Alfonso Gomez-Lobo (1980), who looks at these questions from a different historical perspective. He claims that the characteristic Homeric doubt as to whether someone is a god or a man retained a strong hold on the ancient Greek mind. Thus he argues that questions like whether or not there is a centaur or a god do not raise any existential issues but are simply questions about substantial predication. In other words, the question is whether some implicit subject (τόδε) is to be called a man or a god. The difficulty facing such an interpretation, of course, is that there is no trace of any designated subject of predication in the above passage from the *Posterior Analytics.*

Hence, to support his interpretation, Gomez-Lobo argues for the elliptical character of the εἰ ἔστι question. He claims that, in this case, the question is elliptical at the subject position in the following way: 'whether (y) is or is not a centaur or a god.' As substituends for y, he proposes demonstratives like τόδε which can be used to refer to something without providing the identification criteria that terms for substance normally furnish. Thus he challenges the standard distinction between the first and third questions in terms of predication and existence, respectively. Gomez-Lobo argues that they are both questions about predication, even though there is a fundamental difference between them. According to him, questions of fact are about whether or not a particular attribute is predicated of a substantial subject; whereas the so-called questions of existence are really about whether or not an unidentified subject can have a substantial attribute predicated of it; i.e. whether or not it can be identified as a substance.

It must be admitted that Gomez-Lobo's proposal is rather tempt-

[73] εἰ ἔστιν ἢ μὴ ἔστι κένταυρος ἢ θεός, *APst.* 89b33.

ing because it solves the difficulty of how Aristotle can put hypotheses 'among the premisses' (ἐν ταῖς προτάσεσιν), if they do not have the form of predicate propositions.[74] But, while it resolves one difficulty, his proposal raises another of the following kind. If the distinction between the τὸ ὅτι and the εἰ ἔστι questions is really the difference between accidental and substantial predication, Aristotle should be expected to clarify it somewhere in terms of predicates that are κατὰ συμβεβηκότα and καθ' αὑτά, respectively. But, so far as I can see, he never offers any such clarification. At *Posterior Analytics* II.2, 90a10–13, for instance, Aristotle contrasts being simpliciter (ἁπλῶς) with being this or that (τοδὶ ἢ τοδί), whether the latter is essential (καθ' αὑτό) or accidental (κατὰ συμβεβηκός). He also explains that by 'being simpliciter' he means being the subject (τὸ ὑποκείμενον) of predication; e.g. the moon, the earth, the sun, or the triangle. By 'qualified being' (εἶναι τι), on the other hand, he means any kind of predicate (essential or accidental) like being eclipsed, being equal or unequal.

I can find no evidence here that Aristotle wants to distinguish being *simpliciter* from qualified being in terms of the distinction between substantial and accidental predication. On the contrary, his examples suggest that he is distinguishing between being a subject (i.e. being *simpliciter)* and being a predicate (qualified being). Indeed, these different modes of being are correlated with different questions. For instance, the question of fact (τὸ ὅτι) is about predication; i.e. about whether or not the moon is eclipsed. This is what Aristotle calls a 'partial' (ἐπὶ μέρους) or qualified mode of being. The so-called question of existence (εἰ ἔστιν), on the other hand, is about the subject of predication; e.g. whether or not the moon exists. This is clearly a question about a different mode of being which he calls (90a1–5) simple or absolute (ἁπλῶς). Our task as interpreters is to clarify the implications of such a distinction between different modes of being.

The value of Gomez-Lobo's challenge, however, is that it warns us against interpreting existence in any modern sense. The same point is made in a different way by Kahn (1976) who argues that the notion of 'existence' is never thematized in Greek philosophy. In order to avoid misunderstanding, therefore, I will talk about 'modes of being'

[74] Another way of resolving this difficulty is proposed by Ferejohn (1991), who distinguishes between the deductive and 'framing' stages of demonstration. The latter covers existential hypotheses, axioms, and definitions, which need not be formulated as premises in a syllogism.

because it is more faithful to the text. Along these lines, I claim that the εἰ ἔστιν question is about the mode of being of subjects; whereas the τὸ ὅτι question is about the mode of being of predicates. This conforms better with the terms in which the distinction between the two questions is consistently made in *Posterior Analytics* II.1–2. But it also helps to explain the priority which is given to the εἰ ἔστιν question over the τί ἐστιν question. For Aristotle (89b34–35) this priority means that we must know there is some subject of inquiry before we can proceed with the inquiry; e.g. we must recognize something as divine before we can ask about *what is* the divine.

It is easy to confuse this with the modern question of existence, even though the example of the divine indicates that this is not a question of simple existence but rather a question of whether or not there is anything having the mode of being of divine substance. As Aristotle puts it later, if we do not know whether there is such a thing, it is impossible to know what it is.[75] For instance, one cannot know the essence (τί ἔστι) of a goat-stag, even though one may know what is meant (τί σημαίνει) by the description or the name. Within the dialectical context of II.7, this example of a non-entity is used (92b6) to illustrate Aristotle's claim that no one can know what is a non-being (τὸ μὴ ὄν). In the subsequent and more definitive context of II.8, however, he repeats (93a20–21) the same claim; i.e. that it is impossible to know what something is without knowing if it is (εἰ ἔστι). Hence I think it is safe to take this as being a characteristic Aristotelian claim. But the problem is to interpret such a claim without relying on the medieval distinction between existence and essence.[76]

One way of approaching this problem is to ask about the means by which the existence of the subject is established before the inquiry proceeds to the question of its essence. In II.7 Aristotle has already outlined some of the different possible ways in which one might try to show (δείξει) something; namely, by demonstration (ἀπόδειξις), by induction (ἐπάγωγη), and by means of perception (τῇ αἰσθήσει), although he denies that the substance (τὴν οὐσίαν) or the essence (τὸ τί ἐστιν) can be proved or 'shown' in any of these ways. In the course of that discussion, however, he appears to claim (92b1–2) that an inductive enumeration of particulars shows (δείκνυσιν)

[75] ἀδύνατον γὰρ εἰδέναι τί ἐστιν ἀγνοοῦντας εἰ ἔστιν, *APst.* 93a20.

[76] Cf. Aquinas, *De Entia et Essentia.* Owens (1968a) 29 points out that St Thomas was the first to posit a distinct faculty of judgment for grasping the existence of things, as distinct from their essence which is grasped through conceptualization.

whether something is or is not (ἢ ἔστιν ἢ οὐκ ἔστιν). Subsequently, however, he claims that it is through demonstration (δι' ἀποδείξεως) that everything must be shown to be (ὅτι ἔστιν), unless it is substance (εἰ μὴ οὐσία εἴη). By way of clarification, he goes on to say (92b12–13) that being (τὸ εἶναι) is not the substance of anything, since being is not a genus. Therefore, he concludes, there will be a demonstration that something exists (ὅτι ἔστιν) such as one finds in the mathematical sciences (αἱ ἐπιστῆμαι); e.g. the geometer assumes the meaning of 'triangle' but proves that a triangle exists.

Presumably Aristotle has in mind something like what we find in the first proposition of Euclid's *Elements:* 'On a given finite straight line to construct an equilateral triangle' (Bk. I, Prop. 1). Here the existence of a particular kind of triangle is proved by appealing to a number of postulates, definitions, and common notions, which have been laid down at the beginning of the book. In general, Euclid's propositions fall into six distinct parts: enunciation (πρότασις), setting-out (ἔκθεσις), definition (διορισμός), construction (κατασκευή), proof (ἀπόδειξις), and conclusion (συμπέρασμα).[77]

There are some variations in the wording of a Euclidean proposition, depending on whether it is a problem or a theorem. The first proposition, for instance, is a problem because its *protasis* sets out a task to be done and its conclusion declares the task to have been completed. With regard to the construction, scholars have been almost unanimous in treating it as a proof for existence.[78] Yet, in this first proposition, it is noteworthy that the construction depends entirely upon two previous postulates (1 & 3) which allow one to construct any straight line and any circle, respectively. Therefore, since the straight line and the circle are postulated, it is clear that not every figure in plane geometry can be proved to exist.

We should also notice that the postulates themselves appear to do no more than allow a certain construction; e.g. Euclid's first postulate goes as follows: 'Let the following be postulated: to draw a straight line from any point to any point.'[79] Similarly, the third postulate says: 'To describe a circle with any centre and distance.'[80] Although

[77] Proclus, *in Eucl.* 221,7–11. Cf. also Heath (1925) i, 129; Mueller (1981) 11.

[78] Some doubt has recently been cast on this interpretation of constructions in the Greek geometrical tradition; cf. Knorr (1983) 125–48.

[79] ᾐτήσθω ἀπὸ παντὸς σημείου ἐπὶ πᾶν σημεῖον εὐθεῖαν γραμμὴν ἀγαγεῖν, Heath (1925) i, 195.

[80] Καὶ παντὶ κέντρῳ καὶ διαστήματι κύκλον γράφεσθαι, Heath (1925) i, 199.

Gomez-Lobo (1977) has cast doubt on the exact correspondence between first principles in Euclid and in Aristotle, a general correlation between the former's postulates and the latter's hypotheses is sufficient to establish that, within the practice of the sciences, it was recognized that certain ultimate things must be assumed because they cannot be proved.

Thus we cannot take Aristotle seriously when he appears to claim that it is by demonstration that everything (ἅπαν) is proved to exist. Even his illustration does not show this, since a triangle is not one of the ultimate things in the Euclidean science of plane geometry. One way out of this difficulty, I think, is to view his remarks within their aporetic context of II.7, where he is more concerned with the question of whether or not one who is defining something can prove its substance or essence. The whole chapter, in fact, is given over to a series of objections against the possibility of proving a definition. As a result of these difficulties, Aristotle concludes (92b35) that definition and syllogism are not the same. In addition, he thinks it is clear that a definition neither demonstrates (ἀποδείκνυσιν) nor shows (δείκνυσιν) anything. Conversely, one cannot acquire knowledge of the essence either by definition or by demonstration. At this point, Aristotle seems to have gone too far with his negative conclusions because they imply that the essence can neither be demonstrated nor known in any way. It is obvious that he realizes this when, in the subsequent chapter (II.8), he turns back to review the validity of the previous objections.

I will not here analyse this difficult and controversial chapter,[81] as I simply want to focus on one distinction made there which is important for the problem about mathematical foundations; namely, the distinction between things which are self-caused in some way and those which are dependent on a cause other than themselves:

> As we have said above, to know what a thing is is the same as to know the cause of its existence; and the reason for this is that the thing has a definite cause, which is either identical with it or distinct from it, and which, if distinct, is either demonstrable or indemonstrable. Then if this cause is distinct and can be demonstrated, it must be a middle term, and be proved in the first figure; for (only) in this is the proved connection universal and affirmative.[82]

[81] On *Posterior Analytics* II.8 see Ackrill (1981), Barnes (1975), Bolton (1976) & Demoss/Devereux (1988).

[82] *Posterior Analytics* II.8, 93a4–9: tr. Tredennick (1960).

What Tredennick translates as 'the cause of its existence' (τὸ αἴτιον τοῦ εἰ ἔστιν) might also be rendered as 'the explanation of whether it is.'[83] The latter brings out more clearly the connection with the εἰ ἔστι question of the earlier chapters of Book II.

In fact, the above passage is integrally related to II.2 where Aristotle reduces all four kinds of inquiry to two basic questions about the middle term; i.e. whether there is a middle (εἰ ἔστι μέσον) and what is the middle (τί ἐστι τὸ μέσον). The explicit justification given there (90a6 ff.) for the reduction is that the middle term is the cause or reason (τὸ αἴτιον), which is what we are seeking in all inquiry. Both the logical and ontological senses of αἴτιον are relevant here because Aristotle subsequently explains that the middle term is the cause of something being, not this or that, but simply being.[84] In the light of the above passage from II.8, it is significant that the mode of being of substance is identified as being *simpliciter*, since this corresponds with the mode of being of something whose cause is identical with itself. By contrast, the thing whose cause is other than itself has a mode of being which is characterized both as being partial (ἐπὶ μέρους) and as being this or that (τοδὶ ἢ τοδί). From the examples given in II.2, it is clear that this mode of being is typical of both essential and accidental attributes which are dependent on some substance. The examples given of being simpliciter, on the other hand, are all of them subjects like the moon or the earth or the sun or the triangle. Whereas the first three examples seem to qualify as substances according to Aristotle's categories, there is an unresolved problem about the mode of being of mathematical objects such as a triangle.

In the practice of the mathematical sciences, such objects as lines and circles and triangles are treated *as if* they were independent substances by being made the subjects of essential and accidental attributes. In terms of Aristotle's distinction in *Posterior Analytics* II.8, they are being treated as if the cause of their being were identical with themselves. This implication is confirmed by another formulation of the same distinction in II.9 which summarizes the previous discussion:

[83] Following the reading εἰ in most manuscripts (AB²d) instead of τί in one MSS (Bn).

[84] τὸ γὰρ αἴτιον τοῦ εἶναι μὴ τοδὶ ἢ τοδὶ ἀλλ' ἁπλῶς τὴν οὐσίαν, *APst.* 90a10–11. Barnes (1975) proposes to excise τὴν οὐσίαν from this passage, even though it has good manuscript authority. Presumably he thinks that it is superfluous in this context, given that the distinction between being *simpliciter* and being this or that is clear enough without it.

> Some things have a cause distinct from themselves, and others have not. Thus it is clear that of essences too some are immediate; i.e. they are first principles, and both their existence and their definition have to be assumed or exhibited in some other way. (This is what an arithmetician does: he assumes both what a unit is, and that it exists.) As for things which have a middle term, i.e. something distinct from themselves which is a cause of their being, it is possible (as we have said) to exhibit their essence by demonstration, although we do not actually demonstrate it.[85]

The general structure of the passage is foreshadowed in the first sentence, since it lays out the contrast between things that have some cause other than themselves and those which do not. This distinction is further elucidated in terms of the contrast between those essences which are principles without a middle term (ἄμεσα καὶ ἀρχαί) and those which have a middle term (τῶν δ' ἐχόντων μέσον). It is clear that this contrast depends on taking the phrase 'having a middle term' in the narrower sense indicated by the additional clause (beginning with καὶ ὧν); i.e. having some cause of its existence other than the thing itself.

In order to explain this narrower sense of having a middle term, Aristotle himself refers back to the previous discussion in II.8. There, after a long and difficult discussion, he concluded (93b15–21) that demonstration can make clear but not prove the what-it-is (τὸ τί ἐστιν) of things that have a middle term in this narrower sense of having a cause other than themselves. His choice of examples in that discussion seems to indicate that he has in mind such things as attributes and events, whose mode of being is one of dependence upon substance. Thunder and eclipse are the examples used there to illustrate the priority of the if-it-is (εἰ ἔστι) over the what-it-is (τί ἐστιν) question.[86] Thus, in the case of attributes, the fact (τὸ ὅτι) that they exist can be demonstrated through a middle term, which is identical with the subject to which they belong and on which they depend for their subsistence. On the other hand, their what-it-is cannot be demonstrated by means of this middle term but it can be made clear (δῆλον) in demonstration.

This is Aristotle's resolution of the difficulties listed in II.7, which are connected with the question of how one who is defining something can prove its essence. Behind this rather uneasy resolution, however, stands his firm belief that it is the same knowledge dispo-

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[85] *APst.* II.9, 93b21–28: tr. Tredennick (1960).

[86] Demoss and Devereux (1988) argue that having even a nominal definition of

sition (ἕξις) which grasps the that-it-is and the what-it-is of things like attributes or events which can be demonstrated through a middle term. By contrast, as we can see from *Metaphysics* VI.1 (1025b1–18), he holds that a different kind of understanding (διάνοια) is responsible for making clear (δῆλον ποεῖν) the what-it-is and the if-it-is of those things which cannot be demonstrated. I think it is safe to assume that this is the sort of cognition which is relevant in the present passage when Aristotle talks about those things which are immediate first principles because they cannot be made clear through demonstration. With regard to these indemonstrable things, he says that their being (εἶναι) and their what-it-is must be either hypothesized (ὑποθέσθαι) or made clear in some other way (ἄλλον τρόπον φανερὰ ποιῆσαι).[87]

It is no coincidence that the language of *Metaphysics* VI.1 is almost identical with that used in the *Posterior Analytics* passage under scrutiny, and it is very significant that here Aristotle should turn to mathematics in order to illustrate the option of hypothesizing one's first principle. He cites the example of the arithmetician who assumes (ὑποτίθεται) both what the unit is (τί ἐστι τὴν μονάδα) and that it is (ὅτι ἔστιν). This begs comparison with the passage at *Posterior Analytics* I.2, where the arithmetician is said to posit (τίθεται) the definition of a unit as 'what is indivisible with respect to quantity.' Within the present context, such a definition would give an appropriate answer to the what-is-it question about the unit. In I.2 Aristotle also insists that a definition is not a hypothesis because 'what a unit is and that a unit is are not the same.'[88] The striking parallel with the passage in II.9 suggests that hypothesizing the being of a unit is a response to the if-it-is question.

V. *A science of being simpliciter*

I think that Aristotle is continuing the Platonic tradition when he accepts as a task for first philosophy the inquiry into the first principles of mathematics. While such a science is not discussed in the

thunder (e.g. a certain noise in the clouds) presupposes knowledge of its existence in the sense of an ability to discern genuine instances when confronted with them.

87 Leszl (1981) takes these to be exclusive options conforming to two different kinds of evidence or justification that are acceptable in mathematics and physics, respectively.

88 τί ἐστι μονὰς καὶ τὸ εἶναι μονάδα οὐ ταὐτόν, *APst.* 72a25.

Posterior Analytics, it emerges as a project in the *Metaphysics.* For instance, when he introduces the project of first philosophy in VI.1, he begins in the following way:

> We seek the principles and causes of beings, and obviously of them qua beings. For there is some cause of health and of fitness, and there are principles and elements and causes of the objects of mathematics. In general, every thinking and thought-partaking discipline concerns itself with causes and principles, whether more recondite or more commonplace. But all of these disciplines circumscribe a particular being or genus (of beings) and deal with this rather than with being simpliciter or with being qua being. Furthermore, they do not give any account of the what-it-is but start from that—either having made it obvious through perception or making a hypothesis of the what-it-is. In this way they go on to demonstrate, more or less rigorously, that some attributes belong per se to the genus with which they are dealing. Wherefore, from such an induction it is clear that there is no demonstration of substance or of the what-it-is but rather some other way of showing it forth. Similarly, these disciplines do not say anything about whether the genus with which they are dealing *is* or *is not,* since it belongs to the same kind of thinking to make clear both the what-it-is and the if-it-is.[89]

The passage outlines some similarities and differences between the special sciences, such as mathematics and medicine, and the general science which is being proposed. All sciences share an interest in causes (αἰτίαι) and principles (ἀρχαί) but the range of their interests is different.

In fact, as Aristotle points out, all theoretical (διανοητική) disciplines share this interest in causes and principles, whether these be more exact (ἀκριβεστέρας) or more simple (ἁπλουστέρας). Presumably, he is comparing the more abstruse principles of mathematics with the more ordinary causes sought by medicine.[90] In a passage from the *Topics* (141b3 ff.), we have already seen Aristotle contrasting ordinary intelligence with the precise and extraordinary understanding (ἀκριβοῦς καὶ περιττῆς) required by mathematics. There is

[89] *Met.* VI.1, 1025b3–18: author's translation based on Ross (1908) & Kirwan (1971).

[90] Leszl (1981) 321–2 points out that in the Hippocratic tradition hypotheses are frequently cited in the account of medicine as a *techne,* even though some medical writers (e.g. *On Ancient Medicine*) rejected the use of hypotheses in favor of empirical observation. Such a rejection shows that medicine was often treated in a dialectical way and that Plato was historically justified in seeing it as a science grounded in Forms like mathematics; cf. *On Forms.*

good reason to believe that such a contrast was part of the legacy of Plato (*Rep.* 526A–B) who made mathematics an integral part of an educational curriculum that leads the rulers toward truth. It is unclear, however, whether Aristotle would accept or reject the conclusion of the *Philebus* (55D ff.) that dialectic is a purer and more exact (ἀκριβεστέρη) science than mathematics. While he does not address the issue directly in the above passage, perhaps we can draw some inferences from the way he differentiates first philosophy from the special sciences.

The significant differences may be brought under two general headings: (i) method, in the Greek sense of 'way of inquiry' or subject-matter; (ii) foundational inquiry, i.e. answers to what-it-is and if-it-is questions. Under the first heading, Aristotle differentiates first philosophy from all of the special sciences in terms of the kind of subject-matter with which they deal. For instance, he says that these sciences mark off some kind of entity (ὄν τί) or some genus (γένος τι), thereby investigating a part of being rather than being simpliciter (ὄν ἁπλῶς) or being qua being (ὄν ᾗ ὄν). First philosophy, by contrast, searches for the principles and causes of beings (τῶν ὄντων) qua beings (ᾗ ὄντα). Here Aristotle seems to be drawing an implicit contrast between a science that deals with being in general and the special sciences that deal with particular genera of being.

But this is not the same as Plato's contrast at *Philebus* 58C, which appears to depend upon dialectic being superior to the other sciences in clarity (σαφές), accuracy (ἀκριβές) and truth (ἀλεθές). Thus, from such circumstantial evidence, one might guess that Aristotle has abandoned the Platonic project of making dialectic the 'mistress' science which unifies all other fields of inquiry. A similar anti-Platonic tendency is implicit in Aristotle's distinction between the theoretical and practical sciences with respect to their different goals of inquiry. In addition, the division of theoretical sciences by means of their different subject-genera reflects his decision to replace dialectic with first philosophy.

In spite of these differences, however, one can still see the Platonic influence on Aristotle's thinking about the relationship between this highest science and the particular sciences. For instance, in *Metaphysics* VI.1, he says that the special sciences do not give any account of the what-it-is (τοῦ τί ἐστιν) but simply start from this; some of them having made it obvious through perception, while others are taking the whatness as an hypothesis. The *Posterior Analytics* also lists these

two different ways of reaching the starting-points of particular sciences like mathematics and physics. Beginning with such principles, the sciences demonstrate that certain *per se* attributes belong to the genus of things with which they are concerned.

What is also clear from Aristotle's model of science is that there cannot be any demonstration of first principles, and he repeats this point here in VI.1 when he says that there is no demonstration of substance (οὐσίας) or of the what-it-is (τοῦ τί ἐστιν) but rather some other way of making it clear. Similarly, he says, the special disciplines do not say anything about the existence or non-existence (εἰ ἔστιν ἢ μὴ ἔστι) of the genus of things with which they are dealing. The reason for this, he explains, is that it belongs to the same kind of thinking (τῆς αὐτῆς διανοίας) to make clear both the what-it-is and the if-it-is. Initially, it seems odd for Aristotle to claim that the special sciences say nothing at all about their subject-genera, given that the first principles include definitions and hypotheses. But he must mean that the kind of thinking involved in demonstrative sciences does not concern itself with giving any reasoned account of the what-it-is or the if-it-is of their basic subject-genera. Thus the clear implication of this whole passage is that giving such an account is an appropriate task for the kind of thinking involved in first philosophy.

Finally I want to consider the character of the account to be expected from first philosophy. Given the Platonic background to the problem about 'firsts,' it is natural to expect that the account will be dialectical in character. In fact, there is some evidence that Aristotle himself may have assigned this task to dialectic before he discovered first philosophy. For instance, near the beginning of the *Topics* (I.2), he includes it among the many uses of dialectic:

> For the study of the philosophical sciences it is useful, because the ability to puzzle on both sides of a subject will make us detect more easily the truth and error about the several points that arise. It has a further use in relation to the ultimate bases of the principles used in the several sciences. For it is impossible to discuss them at all from the principles proper to the particular science in hand, seeing that the principles are primitive in relation to everything else: it is through reputable opinions about them that these have to be discussed, and this task belongs properly, or most appropriately, to dialectic: for dialectic is a process of criticism wherein lies the path to the principles of all inquiries.[91]

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[91] *Topics* I.2, 101a34–b3: tr. Pickard-Cambridge in Barnes ed. (1984).

Even though the passage seems to be separating two different uses of dialectic, I think that they are sufficiently connected to constitute a single order of inquiry. First, it is said to be useful with reference to the philosophical sciences (πρὸς τὰς κατὰ φιλοσοφίαν ἐπιστήμας) because, if we are able to raise difficulties on both sides (πρὸς ἀμφότερα διαπορῆσαι), we shall more easily discern the true and the false on every point. Here I think that the expression 'philosophical sciences' does not cover dialectic itself and must refer to theoretical sciences like mathematics; cf. *Phil.* 57B–C.

When taken this way, Aristotle's statement gives us a hint of the role which dialectical questions and objections play in a demonstrative science; cf. *APst.* I.12. It also confirms the connection between demonstrative syllogisms and the practice of dialectic, which Kapp (1942) established in his book on the development of Aristotelian logic. Even the formal theory of syllogistic demonstration was never completely divorced from the question-and-answer technique that was characteristic of the early development of dialectic within the Platonic Academy. We find traces of it, for instance, in Aristotle's distinction between an absolute hypothesis (ἁπλῶς ὑπόθεσις) and a hypothesis relative to the student (πρὸς ἐκεῖνον); cf. *APst.* I.10, 76b29–30. The latter is a starting-point which is more familiar to us, as distinct from one which is more familiar absolutely. Thus, as Thomas Upton (1984) has pointed out, the scientific demonstrator would begin in the middle of his science and propose scientific questions that are closer to the knowledge of his audience.

In other words, the scientist would begin with questions based on hypotheses that are provable by appeal to higher hypotheses or, ultimately, by appeal to absolute hypotheses. Such ultimate hypotheses are none other than the proper ἀρχαί of a science, which command trust (πίστις) in and through themselves and which are self-explanatory to those involved in the science; cf. *Topics* I.1, 100b18–22. In this regard I take issue with Cherniss (1944, 66n52) when he makes a sharp distinction between Plato and Aristotle as to their respective attitudes towards scientific hypotheses. When Plato says (*Rep.* 511B–C) that the dialectician should try to reach the ἀνυποθέτον, Cherniss takes him to be thereby suggesting that hypotheses are inadequate as explanations. Aristotle, on the other hand, is taken to be treating scientific hypotheses as genuine ἀρχαί which do not need the support of some 'higher' reality, since they assert the existence of their

subject-genera.[92] Thus it would appear that, by compartmentalizing the sciences, he has emancipated them from the slavery in which they are held by the 'mistress' science of dialectic. It is this claim for the 'liberation' of the special sciences from any need for foundational inquiry that I question as an interpretation of Aristotle.

In the *Topics* passage quoted above, the second use of dialectic is said to be in relation to the ultimate bases of the principles for the several sciences (πρὸς τὰ πρῶτα τῶν περὶ ἑκάστην ἐπιστήμην [ἀρχῶν]). Even if we delete ἀρχῶν,[93] the Greek text clearly states that dialectic is useful with reference to the 'firsts' of each particular science, and it is obvious from the subsequent explanation that these are ultimate first principles. Aristotle explains that it would be impossible to say anything about them by starting from the proper principles (ἐκ τῶν οἰκείων τῶν ἀρχῶν) of the given sciences because the principles are 'the firsts' of everything else.[94] Thus to resolve the problem of where to start in talking about the principles of the particular sciences, he appeals to the dialectical practice of beginning with reputable opinions (ἐνδόξα). The clear implication here is that it is necessary to give some account of these first principles and this means that, in some sense, they themselves need clarification.

J.D.G. Evans (1977, 31 ff.) draws attention to this passage in support of his argument that dialectic helps towards the discovery of their first principles through its ability to debate both sides of a question. Admittedly, there is strong evidence for his interpretation in Aristotle's description (101b3–4) of dialectic as a process of review (ἐξεταστική) which paves the way for the first principles of all special inquiries. When the metaphor of a military review (ἐξέτασις) is combined with ὁδός, it gives the impression of movement towards a goal that could well be the discovery of first principles for the special sciences. However, it could also involve the justification and discussion of first principles in a foundational inquiry such as that envisaged by Plato. In a significant passage from the *Nicomachean Ethics* (1095a28–b1), this method of reviewing the common opinions is explicitly connected with Plato's distinction between the way (ἡ ὁδός) to the principles (ἐπὶ τὰς ἀρχάς) and the way from the principles (ἀπὸ τῶν ἀρχῶν). It is obvious from the context that Aristotle sees the review

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92 Cf. *APst.* 71a20–21, 76b17–19, b35–36, 93b22 ff., 90b30 ff.; *Met.* 1025b4 ff., 1064a7–10.

93 Some manuscripts (Cf) & (B) do not include ἀρχῶν, most others do.

94 ἐπειδὴ πρῶται αἱ ἀρχαὶ ἁπάντων εἰσί, *Top.* 101a37.

of opinions as being on the way to the principles, but it is not clear whether this is a method of discovery or a process of justification or both. None of these possibilities can be ruled out except through a careful scrutiny of Aristotle's own procedure.

In *Physics* I, 2, for instance, Aristotle begins with a review of the opinions of his predecessors on the question of whether there is one principle (ἀρχή) or many. For both options he sets up a web of dichotomies that covers all the logical possibilities and also categorizes the collected *doxa*. If the principle is held to be unique, for example, then it must be either immobile (ἀκίνητον), as Parmenides and Melissus claim, or mobile (κινουμένην), as the physicists say. We should notice that, in the way he sets up this dichotomy, Aristotle already discriminates between the Eleatics and those predecessors who are more properly concerned with nature. Obviously he is preparing the way for his subsequent critique of those people (i.e. the Eleatics) who postulate a single and immovable principle:

> Now to inquire whether being is one and immovable is not to inquire about nature; for just as the geometer has no arguments at all against one who rejects the principles of geometry, seeing that their discussion belongs to another science or to a science common to all others, so too in the case of principles; for if being is only one and is one in this manner (immovable), no principle exists at all, seeing that a principle is a principle of some thing or things.[95]

From my point of view, the chief interest of the passage lies in its clear assertion that special sciences like physics and geometry do not engage in any discussion about their own first principles. Such a discussion is unambiguously said to belong to 'another science' (ἑτέρας ἐπιστήμης) or to a science 'common to all' (πασῶν κοινῆς). There is, however, no clear identification of this other science whose task it is to discuss or defend the principles of the special sciences. What is clear is that, *pace* Cherniss, Aristotle does envisage the possibility that such principles may be open to dialectical objections. But, as Upton (1984, 254n20) points out, the scientist qua scientist cannot and need not respond to these objections. The objector has effectively removed himself from the methodical sphere of the science of nature, for instance, by challenging its fundamental assumptions. This is what Aristotle appears to mean when he says that those who inquire about a single and immovable being are not inquiring about

[95] *Phys.* I.2, 184b25–185a5: tr. Apostle (1969).

nature. Later (185a12–13) he makes this more explicit when he asserts that 'we' (physicists) assume that either all or some of the things existing by nature are movable (τὰ φύσει . . . κινούμενα εἶναι). Similarly, if we take the infinite divisibility of the continuum to be a fundamental principle of geometry, then people like Democritus who deny this principle would not be accepted as geometers. Such a conclusion might also help to explain the Academic neglect of the mathematical work of Democritus.

To conclude this chapter, let me focus briefly on the question about the identity of the science which must discuss the first principles of the special sciences. In the *Physics* passage quoted above, there are some indications that dialectic may be the science in question. For instance, this science is said to be different from the special sciences but 'common to all.' The context seems to suggest that 'all' may refer to all the special sciences but it may also be referring to all beings, as distinct from the particular genus of beings which each science marks off. If we take the particular sciences as referents, then we have something like the Platonic conception of dialectic as a 'mistress' science that provides a foundational account for these sciences. Indeed, there may be some evidence to support such a conception in the latter part of the passage and in what follows. Aristotle declares that, in the discussion of principles, there will be further difficulties for the Eleatics who hold that being is one and immovable. Those who hold such a thesis have eliminated all principles because the very meaning of principle implies that there must be something else of which it is the principle. So Parmenides and Melissus seem to be either making an eristic argument (λόγον ἐριστικόν) or holding a thesis just for the sake of argument. In either case, it would appear that dialectic is given the task of discussing (διαλέγεσθαι) such positions.

Thus one might reasonably expect that such a dialectical task is being undertaken in subsequent passages when Aristotle constructs arguments against the claims of Parmenides and Melissus. Yet this assumption is undermined by the way he introduces his discussion: "However, since these thinkers discuss problems in physics even if their subject is not nature, perhaps it is well to go over their views somewhat; for such inquiry has philosophic value."[96] From the concessive particles (οὐ μὴν ἀλλ') at the beginning of the passage, it would

[96] *Phys.* 185a17–20: tr. Apostle (1969).

appear that Aristotle is reluctant to concede that a discussion of Eleatic views has any philosophical interest. Yet he admits that the Eleatics raise some physical puzzles (φυσικὰς ἀπορίας), even though their subject-matter is not about nature. At this point the reader can hardly avoid asking: But what then *is* their subject-matter? From the immediate context the obvious answer is that 'being' (τὸ ὄν) is the subject of their inquiry. If this is taken as established, however, the evidence begins to point towards first philosophy as the science whose task it is to discuss difficulties about the principles of the special sciences.[97] It also seems likely that a dialectical review of opinions will be part of the method of that science.

In my next chapter I will argue that such a dialectical procedure is part and parcel of Aristotle's projected science of first philosophy, and this will involve a detailed scrutiny of his treatment of selected *aporiai* from *Metaphysics* III. Although the 'puzzles' collected in that book give the impression of being a miscellaneous lot, quite a few of them turn out to be difficulties about the foundations of the special sciences. Significantly enough, this is consistent with what is said in the *Topics* and the *Physics* about the inability of the particular sciences to give any account of their own first principles. When we connect this with the project for a first philosophy, which is set out in *Metaphysics* VI.1, we can see that such an account is one of the tasks of this science of being qua being. There Aristotle asserts that it is the same kind of thinking (διάνοια) which makes clear both the what-it-is and the if-it-is of the subject-genera with which the special sciences deal. Such questions are not adequately answered by these sciences through assumption or hypothesis, since absolutely prior questions cannot be answered through demonstration. This implies that the characteristic method of first philosophy will *not* be demonstrative, unless it can find more ultimate first principles from which the principles of the special sciences can be deduced. But Aristotle did not accept the Platonic vision of dialectic as an overarching science from which the particular sciences obtain their deductive grounding.

Furthermore, the general consensus among Aristotelian commentators is that the compartmentalization of these sciences frees them

[97] Gadamer has suggested (in personal conversation) that physics may be conceived of as first philosophy in Book I of the *Physics*, so that its method would naturally be dialectical. But this would be consistent with Aristotle says in *Metaphysics* VI.1 only if his conception of the divine as supersensible substance does not enter the picture in *Physics* I.

from the demand for ultimate foundations. Yet this does not mean that Aristotle was unaware of the need for some kind of account of their first principles, particularly where dialectical objections and other difficulties were associated with them. As I have shown from a number of different passages, there is cumulative evidence that he was aware of the need for such a dialectical account. In my subsequent chapters, I hope to show that this is conclusively confirmed by the way in which Aristotle proceeds with his many treatises on special problems in first philosophy, especially the problem about the ontological status of mathematical objects.

Conclusion

In this chapter I have shown how a problem about the foundations of mathematics is already implicit in the Platonic tradition which serves as the starting-point for Aristotle's own philosophical development. Insofar as Aristotle accepts some of Plato's fundamental assumptions about scientific knowledge, he also inherits the attendant problems about its objects. Thus, contrary to what Jaeger and other commentators would have us believe, Aristotle does not inherit a dogma but rather a set of problems. Given what we know from Plato's dialogues about his manner of doing philosophy, this would seem to be a characteristic legacy. A review of some key passages in the *Topics* and the *Posterior Analytics* also shows that Aristotle is influenced by the Platonic assumption that the more exact sciences demand intelligible entities that are distinct from and independent of sensible things. Even in those early works where he is clearly rejecting Platonic Forms as objects of science, we find a similar vagueness about the ontological status of universals that are posited to replace them. Thus we might suspect that Aristotle has not yet brought the issue into clear focus, and this is partially confirmed by his ambiguous and unsatisfactory treatment of quantity in the *Categories* and in *Metaphysics* Delta.

CHAPTER FOUR

ARISTOTLE'S DIALECTICAL METHOD

In this chapter I focus primarily on *Metaphysics* III, the so-called 'aporetic' book which has only recently been given some of the attention it deserves.[1] In terms of my whole project, this book is important because it contains at least two, if not three, aporiae that have a direct bearing on problems about the ontological status of mathematical objects. Since these problems belong to first philosophy, I will consider the role of dialectical method in metaphysics or first philosophy.[2] I hold this methodological approach to Aristotle to be justified by the fact that he devotes a lot of attention to questions about appropriate methods for different inquiries.[3]

Therefore I begin with his claim that dialectic can provide a way to the first principles of the philosophical sciences. Secondly, from the list of aporiae given in *Metaphysics* III, I select three aporiae which are relevant to the problem about mathematical objects. Subsequently, I examine Aristotle's review of the difficulties connected with one of these aporia which he describes as the most difficult yet most necessary to resolve.[4] Finally, I consider the difficulties associated with two aporiae which bear directly on the whole problem about mathematical objects. The fact of there being equal difficulties on both sides of a question is his main reason for describing something as an impasse (ἀπορία).[5] In general, his preliminary treatment of every problem in first philosophy is aporetic in the sense that he gives an even-handed

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[1] Cf. Halper (1978), Code (1982 & 1984), Cleary (1982 & 1987b), Madigan (1986). Some scholars (e.g. Code) prefer the term 'aporematic' but nothing much depends on terminology in this case.

[2] Terence Irwin (1988) has recently suggested that a 'strong' dialectic is developed by Aristotle for the science of first philosophy, as distinct from the 'pure' dialectic that is described in the *Topics* and elsewhere. But I can find little evidence to support such a sharp distinction between types of dialectic.

[3] In addition to the Organon, which is largely devoted to dialectical and demonstrative methods of inquiry, methodological reflections are to be found in almost every treatise. Furthermore, he sometimes refers to another Aristotelian work called *Methodica* which is not extant.

[4] Πάντων δὲ καὶ θεωρῆσαι χαλεπώτατον καὶ προς τὸ γνωναι τἀληθὲς ἀναγκαιότατον, 1001a4–5.

[5] For a definition of ἀπορία as the 'equality of opposing arguments' (ἰσότης ἐναντίων

review of conflicting opinions without attempting to break the resulting impasse.[6]

I. *Methodology*

At the beginning of *Metaphysics* III, Aristotle makes some methodological remarks which diverge considerably from what he says in the *Analytics* by way of logical analysis of the demonstrative sciences. Since that analysis often does not fit with his own procedure, we cannot simply assume that the *Analytics* serves as a complete guide to Aristotle's way of doing physics and metaphysics. For instance, it is rare to find syllogistic argumentation in any of his scientific works, even though he appears to hold that a finished science should take such a form. At least two solutions to this conundrum have been proposed.

Barnes (1975 & 1981) concedes that syllogistic reasoning plays no role in Aristotle's scientific research, and suggests instead that the *Analytics* is a formal analysis of the best pedagogic means of presenting an existing science like mathematics. This approach is rather unsatisfactory however because it does not reduce the gap between Aristotle's theory and practice. Therefore other scholars[7] have suggested that the *Posterior Analytics* does not set rules which scientists must follow, but rather that it describes the underlying logical character of scientific understanding. Another possible compromise (which I favor) is to distinguish in a Platonic fashion between the way *to* the principles and the way *from* the principles; i.e. between induction and deduction. Thus syllogistic reasoning may be the way *from* the princi-

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λογισμῶν) see *Topics* VI.6, 145b2, 16–20. Cf. also *De Caelo* I.10, 279b7. In addition, Aristotle describes dialectical problems as arising where syllogisms come into conflict and generate an aporia because there are strong arguments on both sides; cf. *Topics* I.11, 104b13 ff.

[6] Syrianus (*in Metaph.* 1.19–20) also recognizes that one should not look for Aristotle's own solutions in *Metaphysics* III, since the book is entirely aporetic in character. Julia Annas has suggested to me (in correspondence) that Aristotle's aporetic procedure here is quite similar to that of later Sceptics like Sextus Empiricus, since both of them think that a thorough investigation of several important philosophical issues yields convincing reasons for believing both p and not-p. Of course, as Annas acknowledges, the major difference is that for the Sceptics this is the end of the story; whereas for Aristotle it is only the beginning because he thinks that one can break the impasse by spotting a bad argument or by making a distinction or perhaps by appealing to some reputable opinion.

[7] Cf. Gotthelf (1987), Lennox (1987), & Wians (1989).

ples, while Aristotle's dialectical method can be an inductive way *to* the principles.[8]

I.1. *The need to review the difficulties*

At the beginning of *Metaphysics* III, Aristotle emphasizes the necessity of first discussing the problems associated with first philosophy, which is described as 'the science that is being sought' (τὴν ἐπιζητουμένην ἐπιστήμην).[9] There are at least two implications of this deliberately vague description; i.e. that the science in question has yet to be discovered, and that the discussion of aporiae will contribute to its discovery.[10] Thus one gains access to the subject-matter of this unnamed science primarily through the conflicting opinions of predecessors, and this is also consistent with the role which the *Topics* gives to dialectic as the way to the principles of every inquiry, if metaphysics is included among the so-called 'philosophical sciences.' In support of this interpretation, consider the beginning of *Metaphysics* III:

> With regard to the science which is the subject of our inquiry, we must first state the problems which should be discussed first. They are concerned with matters about which some thinkers expressed different beliefs, and besides them, with some other matters which may happen to have been overlooked.[11]

The repetition of πρῶτον here emphasizes that every inquiry should begin by first setting out the puzzles, which are either derived from conflicts of views or from puzzling matters on which no one has expressed any views.[12] This first stage of inquiry is completed in *Metaphysics* III when Aristotle first lists the puzzles that arise for his

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[8] My conjecture is helped by von Fritz (1964) who treats the collection of opinions as a type of induction which is appropriate for dialectical or metaphysical inquiry.

[9] This introduction seems to support the argument of Evans (1977) 51 that the art of dialectic, with its capacity for making such a general review of difficulties, precedes (and grounds) the universal science of ontology. In this respect first philosophy is different from special sciences like mathematics, which are established disciplines with their own methodical procedures independent of dialectic; cf. Gigon (1961).

[10] Alexander of Aphrodisias (*in Metaph.* 171.5–12 (Hayduck)) thinks that the aporetic inquiry of *Metaphysics* III can help us to discover the nature of first philosophy and its characteristic objects. According to him, the projected science can be called wisdom or theology, though it goes under the general name of metaphysics.

[11] *Met.* 995a24–27: tr. Apostle (1966).

[12] If an aporia is defined as the 'equality of contrary arguments' (*Top.* VI.6, 145b2)

projected science, and then reviews the difficulties associated with each aporia.

Before examining the list of aporiae, let us briefly consider Aristotle's stated purpose in collecting the aporiae and in reviewing difficulties associated with them:

> Now those who are seeking answers will find it useful to go over the difficulties well; for answers successfully arrived at are solutions to difficulties previously discussed, and one cannot untie a knot if he is ignorant of it. The difficulty raised by thought makes clear the knot in the subject-matter: insofar as thought is in difficulties, it is like those who are bound; and in both cases one cannot go forward.[13]

In the Greek there is an explicit contrast between εὐπορία, which is an intellectual facility for resolving difficulties, and ἀπορία which indicates the lack of such means. Here ἀπορία seems to have a specific methodological meaning but its derivation from the verb ἀπορέω also suggests the intellectual experience of 'being at a loss' or 'being stuck.'[14] Thus the metaphor of being bound is quite appropriate and Aristotle can describe a solution as 'loosing the bond' (λύειν τὸν δεσμόν).[15] In addition, the derivation of εὐπορία from the verb εὐπορέω suggests that the 'resourceful' dialectician may be described as being well-endowed with the intellectual means for solving puzzles.[16] Such linguistic resonances abound in the suggestive contrast between ἀπορία and εὐπορία.

Furthermore, this contrast supports the claim that a thorough review of the difficulties (τὸ διαπορῆσαι καλῶς) is useful for those who

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then one can see how it derives from a conflict of views, but it is less easy to see how an aporia can arise about a matter on which no one has expressed any views. However, Aristotle does claim to have discovered entirely new aporiae, especially in the more empirical disciplines that were neglected by predecessors.

[13] *Met.* 995a27–33: tr. author.

[14] Cf. Bonitz (1870) 85a16–b24. Aubenque (1961) finds two distinct meanings in this passage. The first refers to the methodological procedure of going through the difficulties (ἀπορῆσαι) in order to reach clarity, whereas the second refers to the experience of being struck dumb like Socrates' interlocutors when they are made painfully aware of their ignorance. Madigan (1992) 87 notes that Alexander uses ἀπορία in at least 4 different senses: 1) a physical impediment to movement; 2) a state of perplexity; 3) a problematic object or issue that causes perplexity; 4) a philosophical discussion that argues on both sides of the issue.

[15] Cf. *Meteor.* 354b22, *Met.* 1061b15, 1062b31, *EN* 1146b6–7, *MM* 1201b1, *Pol.* 1281b22.

[16] Along with the verb and noun forms, the contrasting adjectival forms (ἄπορος / εὔπορος) may also have connotations of being wealthy or of being poor; cf. LSJ. & *Pol.* 1279b9, b19, 38, 1289b30, 1290a10, 1291b8, b33, 1292a40, 1293a5, 1295b2, 1296a17, 1296b25, 1297a14, 1302a2, 1303a2.

wish to resolve them, since the consequent free passage of thought is simply the resolution of previous difficulties.[17] Mixing his metaphors somewhat, Aristotle insists (995a28–29) that it is not possible to loose a knot with which one is not acquainted.[18] This striking metaphor suggests that a survey of difficulties is not only useful but essential for the solution to a problem and I think that this is borne out by Aristotle's own practice. For instance, almost every treatise begins with a doxographical review which identifies the leading difficulties about the topic in hand.[19] Furthermore, the opinions of 'the many and the wise' belong to the phenomena to be 'saved' by any definitive solution.[20] Thus I think that Aristotle's methodological influence on Theophrastus is reflected in his collection of 'Physical Opinions' (φυσικῶν δόξαι) from which some fragments survive.[21]

I.2. *The primacy of the dialectical review*

From the introduction to *Metaphysics* III, therefore, it appears that Aristotle might even insist upon the generation of an aporia as a necessary first stage in any theoretical inquiry, especially if it generates the kind of wonder that stimulates philosophical thinking; cf. *Met.* I.2, 982b11–21. In any case, as he points out (995a30–31), intellectual puzzlement reflects some problem in the matter at hand. But the best way of becoming familiar with a problem is to review what others have said about it:

> Accordingly, one should first study all the difficulties both for the purposes stated and because those who inquire without first going over the difficulties are like those who are ignorant of where they must go;

[17] ἡ γὰρ ὕστερον εὐπορία λύσις τῶν πρότερον ἀπορουμένων ἐστί, *Met.* 995a28–29.

[18] Aubenque (1961) 5n5 rejects 'knot' as a translation of δεσμός on the grounds that it does not carry the right connotation of being bound with chains, so that one cannot advance towards a solution. But I think this is a matter of linguistic taste, because it makes good sense in English to say that one is 'tied up in knots' when one cannot make any headway with a difficult problem.

[19] Aristotle thinks that there are difficulties peculiar to each kind of inquiry, so that such a review is also an initial exploration of the subject-matter; cf. *EE* 1215a3.

[20] Cf. Owen (1961) & Nussbaum (1982) 267–94. For a different approach see Irwin (1987 & 1988).

[21] Cf. Wimmer ed. (1866) 321–40. Alexander (*in Metaph.* 196.19) and Syrianus (*in Metaph.* 23.9) both suggest that Theophrastus may have added Book I to the *Metaphysics,* and this would be consistent with Alexander's report (172.22) of a tradition that held Book III to be the first book. In addition, the character of the *Metaphysics* of Theophrastus shows that setting out aporiae was standard practice in the Lyceum as an introduction to physical and metaphysical inquiry; cf. Laks & Most eds. (1993).

> besides, such persons do not even know whether or not they have found what they are seeking, for the end is not clear to them, but it is clear to those who have first gone over the difficulties. Further, one who has heard all the arguments, like one who has heard both parties in a lawsuit or both sides in a dispute, is necessarily in a better position to judge truly.[22]

In each discipline, therefore, the inquirer must become acquainted first with the characteristic problems and difficulties of its subject-matter. This seems to be even more essential for the projected science of first philosophy, since Aristotle is dealing with a subject-matter (i.e. being as such) about which previous thinkers have said many different and conflicting things. Hence a review of previous opinions serves two related functions: (a) it helps one to become familiar with the object of inquiry; (b) it serves to identify the problems to be resolved. In other words, an historical survey of conflicting opinions guides our approach to a problem and also gives us a yardstick for measuring our own success; namely, whether our solution can save the most reputable opinions and resolve all the outstanding difficulties. Within a dialectical context, this is what Aristotle means when he talks about 'saving the appearances' as a general criterion for the adequacy of a solution.[23] Such a criterion naturally presupposes the methodical survey of the phenomena to be preserved, chief among these being the reputable opinions about a question.[24]

The final argument in the above passage appeals to forensic prac-

[22] *Met.* 995a33–b4: tr. Apostle (1966). Grammatically, the perfect infinitive (τεθεωρηκέναι) combined with the temporal adverb πρότερον serve to emphasize that the review must already have been completed before the inquiry proper begins. In addition, the adjective πάσας drives home the point that the review of difficulties must be comprehensive—a point reiterated by πάντων at the end of the whole passage. We should also note that, in referring to these difficulties, Aristotle uses the term δυσχερείαι rather than ἀπορίαι. This fits his practice of gathering difficulties in order to generate a specific puzzle (ἀπορία), which is a feature of *Metaphysics* III.

[23] At *EN* VII.1, 1145b2–7, Aristotle describes a method which begins by 'setting out the phenomena' and then reviews the difficulties, with a view to reaching a conclusion by resolving the difficulties, while leaving the most important *endoxa* standing. This description fits quite well with Aristotle's dialectical procedure throughout the *Metaphysics* and the *Physics;* cf. IV.4, 211a7–11.

[24] Irwin (1988) claims that Aristotle's dialectical method fails to satisfy his realist assumptions about inquiry because its criterion of truth must be internal coherence rather than correspondence with any independent facts such as are provided by sense perception in empirical inquiry. As against this, Berti has suggested (in unpublished mss.) that *endoxa* may constitute a class of opinions which are universally accepted by the experts and which can serve as touchstones of truth and objectivity in a dialectical inquiry.

tices as showing the necessity for a complete review of the difficulties. One must be in a better position to judge, Aristotle insists (995b2–4), when one has heard (ἀκηκοότα) all of the contending arguments (τῶν ἀμφισβητούντων λόγων) as if they were parties to a legal dispute. This ideal of impartiality, which is borrowed from forensic oratory, exerts a notable influence upon Aristotle's concept of dialectical inquiry. The ideal stipulates that the seeker after truth and justice must give a fair hearing to both plaintiffs (ἀντιδίκοι) before he exercises his function as an arbitrator (διαιτητής).[25] Although Cherniss has cast doubt on whether Aristotle himself lives up to this ideal, yet for him it is undeniably a model of disinterested inquiry.[26]

II. *Some metaphysical aporiae*

Scholars disagree about the exact number of aporiae which are treated in *Metaphysics* III but this is not a matter that demands precision.[27] Following his own methodological advice, Aristotle first lists the aporiae of his projected science and then discusses each one in detail. He does not appear to worry about the order in which these aporiae are treated nor is he careful about numbering them. In fact, there are internal connections between some of the aporiae such that they could be numbered either as two connected aporiae or as one continuous aporia. But, even though Aristotle attaches no importance to their number, it is quite possible that he intended these aporiae in *Metaphysics* III to constitute a comprehensive program for the projected science of first philosophy.[28] While most of the aporiae are taken up again in later books, a few are not given any detailed treatment in

[25] In *Rhetoric* I Aristotle distinguishes between a judge, who simply follows the written law, and an arbitrator who pays attention to the unwritten law in an attempt to find an equitable resolution of a conflict between two litigants who appear before him prior to bringing their case to court. Perhaps this distinction can provide an important insight into Aristotle's model of dialectical inquiry.

[26] Cf. *Phy.* 206a12–14, *Cael.* 279b7–12, *Respir.* 470b10–12, *Met.* 987a2–3, *EN* 1095b21.

[27] I tend to follow the Aristotelian maxim that one should only demand precision where that is appropriate to the subject-matter. Therefore, while it may not be necessary to count the exact number of aporiae, one is required to be precise about the content of metaphysical aporiae since that will dictate the subject-matter of the science of metaphysics.

[28] Perhaps the greatest obstacle to such a reading of the *Metaphysics* is that the central books on substance (i.e. VII to IX) do not appear to be guided by any of the aporiae found in Book III or XI. Irwin (1988) has cleverly avoided this obstacle

the *Metaphysics*.[29] I am not here concerned with this question beyond showing that certain aporiae bear directly on the problem about the foundations of mathematics, and that they are part of the subject-matter of metaphysics.

Therefore, I will take each of these relevant aporiae separately and examine Aristotle's initial formulation of it in *Metaphysics* III.1. In analysing these formulations, my goal is to establish a direct link with the problems that I have already discussed as facing Platonism with regard to the mathematical sciences. The first aporia which deserves scrutiny goes as follows:

> And we must also inquire into this, (4) whether sensible substances alone should be said to exist or besides these also others, and if others also, whether such substances are of one genus or of more than one; for example, some thinkers posit the Forms and also the Mathematical Objects between the Forms and the sensible things.[30]

One can see immediately from the introduction to this aporia that it is implicitly connected with the previous problem about whether there is a single science dealing with all substances.[31] In fact, the introductory phrase (καὶ τοῦτο δ' αὐτό) points back to the second option in the previous aporia (995b10–13): If science is concerned with substance, whether there is one or more science dealing with all substances; and, if more than one, whether they are all akin or whether some of them should be called wisdom and others something else.

Among the different options constituting this complex aporia, the one which led Aristotle to formulate the subsequent aporia may be the question about the appropriate names for the sciences dealing with substance; i.e. whether they should all come under the name of wisdom (σοφία) or whether they should be called some other names.

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by treating the central books as a preliminary inquiry into the nature of substance itself, which was not envisaged in Book III but which became necessary by way of preparation for answering the aporiae about substance listed there.

[29] In his notes on the *Metaphysics,* Apostle (1966) 272–3 gives a list of passages where each of the fourteen aporiae which he has counted are both discussed and answered. In some cases, the textual basis for his claims is rather weak.

[30] *Met.* 995b13–18: tr. Apostle (1966). The parallel aporia in XI.1 (1059a37 ff.) goes as follows: "In general, there is this problem, whether the science we now seek is concerned at all with sensible substances or not, but rather with some other substances. If with others, it would be either with the Forms or with Mathematical Objects"—tr. Apostle (1966)

[31] Alexander, *in Metaph.* 175.14–176.16. Syrianus (*in Metaph.* 2.15 ff.) goes one better by combining three aporiae together, though he quotes and discusses each one separately.

This raises the further question of how many different kinds of substance there are because, if only natural substances exist, the science should be called physics.[32] On this reading it is easy to see how the question about mathematics follows: Does it also involve knowledge of substances?

From such a perspective, therefore, we should analyse the fourth aporia with its characteristic structure of two distinct questions, which reflect its origins in dialectical practice. In this particular case, the first appears to be a straightforward question about whether or not there exist any other substances besides sensible substance.[33] Indeed, it is formulated in such a way as to underline Aristotle's lack of doubt about the existence of sensible substances. So the question is not whether *any* substances exist but whether sensible substances alone should be said to exist.[34] One should also note that this assumption about sensible substances establishes the philosophical framework for the subsequent question; namely, if there are other substances, whether they are of one kind or of more than one kind. As examples of different kinds of substance, Aristotle cites the Forms and also the Mathematical Objects posited by some people as intermediate (μεταξύ) between sensible things and Forms.

The people in question could be either Platonists or just Plato himself, as Alexander[35] seems to think. The commentator may have in mind the passage in *Metaphysics* I.6 where an almost identical doctrine is explicitly attributed to Plato:

> Further, he says that besides the sensible things and the Forms, and between these, there exist the Mathematical Objects, differing from the sensible things in being eternal and immovable, and from the Forms in that there are many alike; whereas the Form itself corresponding to these is only one.[36]

Unlike Cherniss I think that Plato is committed to the existence of

[32] Cf. *Met.* VI.1, 1026a27–29 & XI.7, 1064b9–13.

[33] Syrianus (*in Metaph.* 2.15 ff.) takes the question to be whether there are other intelligible substances (νοηταί οὐσίαι) along with sensible substances like the heavens, the earth, animals, plants.

[34] πότερον τὰς αἰσθητὰς οὐσίας εἶναι μόνον φατέον, *Met.* 995b14–15. The parallel passage in XI.1 (1059a37 ff.) leaves open the possibility that first philosophy might not be about sensible substances.

[35] Cf. *in Metaph.* 176.9. Syrianus (*in Metaph.* 4.3 ff.) identifies the relevant Platonic doctrines in terms of the divided line in the *Republic* and the psychogony in the *Timaeus*.

[36] *Met.* 987b14–18: tr. Apostle (1966).

two kinds of entities called Forms and Mathematicals, which are held to be substances independent of sensible things. According to Aristotle's report, Mathematicals are 'many alike' (πόλλ' ἄττα ὅμοια), whereas the Form itself is in each case unique (ἕν ἕκαστον μόνον); e.g. there is only one Form of Triangularity but there are many mathematical triangles. So the Intermediates were posited to solve what Annas (1975) calls 'the uniqueness problem'; namely, that ordinary mathematics cannot function with single and unique Forms like Triangularity or Squareness. For instance, certain geometrical propositions seem to presuppose at least two perfect triangles which can be proved to be exactly congruent and, therefore, equal.[37]

Thus the distinction between Forms and Mathematicals reflects conflicting truth conditions within mathematics and dialectic. Dialectical consistency, on the one hand, demands a single Form of Triangle which is not an instance of itself, if such absurdities as the Third Man regress are to be avoided. But, on the other hand, mathematical truth requires a plurality of perfect triangles. Since images are unavoidable in mathematics, but especially in geometry, it is different from dialectic, which makes no use of images according to Plato in the *Republic*.

What is most important to notice, however, is that these entities are posited not on the basis of sense perception but as solutions to dialectical problems. In this way dialectical arguments give us access to hidden realities that would otherwise be inaccessible. Such was the very point of Socrates' questions about universals that so puzzled his empirically-minded contemporaries, and forced them to look towards the intelligible realm. The point is nicely brought out by the leading question of an aporia that occurs for the first time in *Metaphysics* III.6 (1002b12–32): Why should one search for other realities, such as the Forms we posit, besides sensible things and the intermediate mathematical objects? The answer is that knowledge requires a single basic object that is not identical either with sensible things, which are indefinite in number, or with mathematicals which are like ideal particulars. On the basis of the argument from the sciences, therefore, Plato posited Forms as objects that satisfy the demands of science for something that is simultaneously one in num-

[37] Wedberg (1955) 54 reconstructs a plausible argument for the necessity of perfect instances of Euclidean Forms to satisfy the assumption that geometry is true and has real objects.

ber and universal in scope, and which is completely intelligible. The crucial point is that the need for such entities does not become apparent except as a result of dialectical puzzles that demand solution in terms of Forms or universals. The same point applies in a different way to the mathematical sciences.

As I am primarily interested in the shape of Aristotle's problematic, however, I will concentrate on the information that he gives us about the separate status of Forms and Mathematicals because this is most important for understanding the aporia in *Metaphysics* III. Intermediate mathematical objects are said to differ from sensibles inasmuch as they are eternal (ἀΐδια) and unchanging (ἀκίνητα). The first distinguishing mark is already familiar from the arguments 'from the sciences,' where scientific objects are described as eternal in contrast with sensible things which are perishable. But this argument only proved the existence of Forms as objects of science separated from sensible things (παρὰ τὰ αἰσθητά). It would seem, however, that a similar argument must lie behind the claim that mathematical objects are eternal and exist apart from sensible things.[38] Yet the important point for Aristotle's problematic is that, if mathematical objects are thus separated, they must be independent substances just like the Forms. It is in this light, I think, that we should understand his formulation of the aporia in *Metaphysics* III, which assumes the existence of sensible substances when it asks whether there are other things apart from these (παρὰ ταύτας) such as Forms and Mathematicals, which can also be called substances.

The second aporia I want to consider is listed last in III.1, though it is closely connected with the aporia already outlined. That aporia covered mathematical objects in a general way under the question about different kinds of substance, whereas this deals more specifically with the problem about the ontological status of mathematical objects:

> Moreover, (14) are numbers and lines and figures and points substances in any sense or not, and if substances, are they separate from sensible things or are they constituents of them.[39]

Despite the different numbering schemes,[40] I see no good reason why two aporiae which are listed separately cannot be integrally connected

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[38] Cf. *Met.* XIV.3, 1090a35–b1.
[39] *Met.* 996a12–15: tr. Apostle (1966).
[40] Cf. Ross (1924 i) xvi–xvii, Brumbaugh (1954b), & Apostle (1966).

as general and particular forms of the same problem; i.e. whether or not the objects of all the sciences are independent substances.

Just as with the previous aporia, we can distinguish two separate yet connected questions within the passage; the second being contingent upon a positive response to the first. Taken literally, the first question (996a12–14) asks whether or not numbers (οἱ ἀριθμοί) and lines (τὰ μήκη) and figures (τὰ σχήματα) and points (αἱ στιγμαί) are kinds of substances (οὐσίαι τινές). Since this list contains many kinds of mathematical entities, we might take the question to be closely connected with the general question in the previous aporia; namely, whether mathematical objects are substances distinct from sensible substances. This connection appears to be firmly established by the subsequent question in the present aporia, which begins by explicitly assuming a positive response to the first question. If these aforementioned things are substances, the question is whether they are separated (κεχωρισμέναι) from sensible things or are present in them (ἐνυπάρχουσαι ἐν τούτοις).

The presupposition of this question is exactly the same as that which guided the previous aporia; namely, the self-evident existence of sensible substances. In order to establish the existence of other substances, therefore, it will be necessary to clarify their relationship to sensible things. That is the reason why the present aporia goes on to ask about mathematical objects (as putative substances) whether they are entirely separate from sensible things or are contained in them as independent substances. When Aristotle tries to resolve this aporia in *Metaphysics* XIII, he considers precisely the same two options for mathematical objects as substances. There he also attributes each option to some contemporary thinkers, including the Platonists, though Aristotle has changed the framework with his assumption about the primacy of sensible substances.[41]

In concluding my chosen list of aporiae, let me briefly consider one which seems to have no direct bearing on the problem about mathematical objects but which turns out to be important, in a general way, for Aristotle's philosophical reflections upon mathematics. Significantly, he identifies it as the most difficult aporia stimulated by the conflicting views of his predecessors:

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[41] Perhaps it is by way of reaction against this assumption that the Neoplatonic commentator, Syrianus (*in Metaph.* 12.25 ff.) adopts the strategy of simply asserting the Platonic order of priorities, beginning with the Forms of mathematical objects and concluding with their appearance in sensible things.

> Again, (11) there is the most difficult and perplexing problem, whether the One or Being, as the Pythagoreans and Plato used to say, is not some other thing but is itself the substance of things, or this is not so but the underlying subject is something else, for example, Friendship, as Empedocles says, or Fire, or Water, or Air, as others say.[42]

It is not immediately obvious why this aporia should be said (996a4–5) to contain the greatest difficulty (χαλεπώτατον) and to create the greatest impasse (πλείστην ἀπορίαν), though we must take this description seriously and seek to understand why it is said to be the most difficult to resolve.

Structurally, the aporia itself is simpler than the ones already listed in that it does not contain a subsequent question but merely lays out two options as follows: (i) either One or Being (τὸ ἓν καὶ τὸ ὄν) is the substance of things (οὐσία τῶν ὄντων); (ii) or not this but something else is the subject (τὸ ὑποκείμενον). While this appears to capture the basic structure of the aporia, a number of complications must be taken into account before the whole picture emerges. For instance, the first option may be translated more literally as: "whether One and Being, as the Pythagoreans and Plato used to say, is not something else (οὐχ ἕτερόν τί ἐστιν) but (is) the substance of things." In the second option 'something else' (ἕτερόν τι) seems to be connected with some underlying subject (τὸ ὑποκείμενον), such as Friendship according to Empedocles, or Fire or Water or Air, according to others. In this light, let me canvass some possible ways of understanding the whole aporia.

Given the historical figures named, the first and most obvious interpretation is to take it as a puzzle about whether the substance of things is as the Pythagoreans and Platonists say it is or as the natural philosophers hold it to be. The former group posit some completely general principle, like One or Being, as the substance of things; whereas the latter put forward different physical principles like air, water, fire, and Friendship (φιλία). Friendship, however, does not fit easily into a list of material principles and this forces us to seek a more subtle interpretation of the aporia. According to Empedocles,[43] the four different 'roots' or elements can be either separated in Strife (Νεῖκος) or united in Love (Φιλότης). Thus Friendship

[42] *Met.* 996a4–9: tr. Apostle (1966). There is no exact parallel in *Metaphysics* XI but corresponding problems are found in two separate passages; cf. 1059b24 ff. & 1060a36 ff.

[43] Cf. Fr. 17 & 21 (DK), Kirk, Raven & Schofield eds. (1983) 287–93.

turns out to be a general principle of unity, analogous to One or Being for the Pythagoreans and the Platonists.[44] The crucial difference, in Aristotle's terms, is that Empedocles sees Friendship as the subject of unity; whereas the One and Being are themselves held to be unified substances.

From this historical perspective, we can give a fresh picture of the aporia under scrutiny. The question is (i) whether One and Being are nothing other than the substance of things or (ii) are not (the substance of things) but instead the attributes of something else as subject. Here I follow Alexander, who formulates the aporia as follows: (a) whether One and Being themselves are substances (οὐσίαι) and principles of things (τῶν ὄντων ἀρχαί).[45] While he offers no clarification of the phrase οὐχ ἕτερόν τί ἐστιν, Alexander does report the view of Plato that there is a Being Itself (αὐτό τι ὄν) and a One Itself (αὐτό τι ἕν). This suggests that the question is whether One and Being exist in their own right as substances, just as Plato held the Forms to exist, or whether they exist as attributes of other things.

But how are we to interpret this aporia which Aristotle characterizes as 'the greatest impasse' (πλείστη ἀπορία)? I suggest that it is connected with the leading problem that faces his own projected science of first philosophy; i.e. what are the substances of things. In fact, it is clear that variations on this problem occupy him throughout the *Metaphysics*. As the previous aporiae show, the clarification of the nature of sensible substance is an essential prerequisite for the treatment of mathematical objects. Therefore it will be necessary to deal with the aporia about the substance of things in a preliminary fashion before going on to treat the aporiae which are specifically relevant to the problem about mathematicals. According to the methodological advice given by Aristotle himself, the first step in any such treatment is to go through the difficulties that are constitutive of each aporia. This will be the task of the next section, beginning with the difficulties clustered around the last aporia I have outlined. As it turns out, such a chiastic order of inquiry is typical of classical Greek thinking.

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[44] Syrianus (*in Metaph.* 11.28 ff.) maintains that Love is nothing else than the One, whereas Strife corresponds to the Indefinite Dyad. But this seems to undermine the contrast that Aristotle wishes to make for the sake of his aporia, and it also appears to misrepresent the view of Empedocles. In general, I find Syrianus to be reading the aporia in an excessively Neoplatonic fashion; e.g. he describes the One as a transcendent One (τὸ ὑπερούσιον ἕν); cf. *in Metaph.* 11.20.

[45] Alex. Aphr. *in Metaph.* 179.28 ff.

III. *Working through the difficulties*

After listing the aporiae proper to first philosophy, Aristotle goes through the difficulties on both sides which constitute each aporia. For understanding the sticking points, therefore, it is important to follow his rehearsal of the difficulties that bring the mind to an impasse. First, we may avoid the temptation of projecting onto Aristotle what we consider to be philosophically interesting problems. Secondly, the review of difficulties (διαπορῆσαι) increases our familiarity with the intricacies of each problem, so that we can recognize what will count as a solution. As Aristotle himself put it rather laconically, we must become acquainted with the knot before we are in a position to untie it.[46]

In *Metaphysics* III.4 (1001a4–8) Aristotle again emphasizes that the aporia about One and Being is the most difficult of all and the most necessary for knowing the truth.[47] It may be paraphrased as whether Being and One are the substances of things (οὐσίαι τῶν ὄντων)[48] and whether each of them is nothing other than simply one and being, respectively; or whether they belong to some other nature as subject (ὡς ὑποκειμένης ἄλλης φύσεως), which is therefore the substance of beings. From the Greek text it seems clear that the two major options in this aporia are: (1) whether One and Being are self-identical unities like Platonic Forms; (2) or whether One and Being are attributes of some other underlying nature. This is verified by Aristotle's report on the conflicting opinions of his predecessors.

Plato and the Pythagoreans, on the one hand, are said to hold that Being and One are not something else (οὐχ ἕτερόν τι) but that this is their nature; i.e. their substance is simply to be one and being.[49] In support of Aristotle's interpretation, Alexander[50] says that

[46] In personal correspondence Julia Annas has made the interesting suggestion that, contrary to my assumption, Book III may have been composed *after* Aristotle had solved the problems listed there. So the whole book would have the pedagogical purpose of showing just what the problems are and how one can easily get into an impasse. As a result, the audience will better appreciate Aristotle's own solutions. While still being intrigued by this suggestion, I regard it as incompatible with most of Aristotle's references to the book of problems in *Metaphysics* XIII, for instance.

[47] With reference to the necessity of considering this aporia for the sake of truth, Alexander (*in Metaph.* 223.15 ff.) explains that it is of the greatest importance because it bears on the question of whether numbers can be posited as the principles and elements of beings.

[48] His use of the plural οὐσίαι is the only divergence from the previous formulation of the aporia.

[49] ἀλλὰ τοῦτο αὐτῶν τὴν φύσιν εἶναι ὡς οὔσης τῆς οὐσίας αὐτοῦ τοῦ ἑνὶ εἶναι καὶ ὄντι, *Met.* 1001a11–12.

[50] Cf. *in Metaph.* 223.36–224.6. Despite Aristotle's use of the plural, Alexander

Plato called Being Itself (αὐτοόν) and One Itself (αὐτοέν) Forms, thereby implying that One and Being are substances.[51] The alternative of treating One and Being as predicates of another subject is linked with natural philosophers like Empedocles who tried to say what the One is (ὅ τι τὸ ἕν ἐστιν) by identifying it with Friendship, as if he were reducing it to something more familiar (γνωριμώτερον).[52] Other natural philosophers however predicate unity and being of material elements like fire and air of which things consist and from which things come to be.[53]

From this brief initial survey of the *doxa,* we can reformulate the aporia more clearly as follows: (i) whether One and Being are independent and self-identical substances or (ii) whether they are attributes of some other subject (like Friendship or Fire or Air) which is itself the principle of unity and being for all things. Of course, the acid test of such a reformulation is whether it can clarify the difficulties which Aristotle outlines on both sides of the aporia.

First, assuming that one denies the independence and substantiality of unity and being, there arise the following difficulties:

> If we do not suppose unity and being to be substances, it also follows that none of the other universals is a substance; for these are most universal of all. If there is no unity-itself or being-itself, there will scarcely be in any other case anything apart from what are called the individuals. Further, if unity is not a substance, evidently number also will not exist as an entity separate from the individual things; for a number is units, and the unit is something whose essence is to be one.[54]

Whether we count two or three separate difficulties here, each of them arises from the denial of the first option (i.e. (i) above) as found in the protasis of the conditional: 'If someone does not posit 'unity'

(224.1 ff.) seems to think that One Itself and Being Itself are two different ways of referring to a single nature or essence, even though he acknowledges that they name two distinct Platonic Forms.

[51] This recalls the Academic process of ἔκθεσις, which involved the positing of Forms as separate substances; cf. *Met.* I.9, 992b10–12 & Alex. Aphr. *in Metaph.* 124.10–125.4. Berti (1979) 93n12 notes that this is very different from the ἔκθεσις mentioned by Aristotle in the Organon; cf. *APr.* 25a14–17, 28a23, *Soph. El.* 178b36–179a10.

[52] Aristotle's use of the word δόξειε may betray some hesitation about attributing to Empedocles the thesis that unity is identical with Love. This impression is confirmed by the fact that he feels it necessary to offer a subsidiary argument (1001a14–15) to the effect that, for Empedocles, Friendship (or Love) is the cause (αἰτία) of unity in all things.

[53] ἐξ οὗ τὰ ὄντα εἶναί τε καὶ γεγονέναι, *Met.* 1001a16–17.

[54] *Met.* 1001a19–27: tr. Ross in Barnes ed. (1984).

to be a substance.'[55] What is denied here is that One and Being are self-identical and independent substances, and the whole point of the objection is that if neither is a substance then the claim of the other universals to substantiality is undermined.

In order to understand the difficulties on one side of the aporia, it may help to schematize the objections as follows:

> I. (a) If One and Being are not posited to be some substances, then none of the other universals is a substance because unity and being are the most universal of all (καθόλου μάλιστα).
> (b) If there is not some One Itself and Being Itself then there will hardly be any of the others apart from the so-called particulars (παρὰ τὰ λεγόμενα καθ' ἕκαστα).
> II. If the One is not a substance, then neither does number exist as some separated nature of things (ὡς κεχωρισμένη τις φύσις τῶν ὄντων) because a number consists of units, while a unit is just what it is to be One (ἡ δὲ μονὰς ὅπερ ἕν τί ἐστιν).

The point of the first objection (I) seems to be that, since the most universal things (i.e. being and unity) are not substances, no universals can be substances. Ross (1924 i, 244–45) judges this to be a sound objection against Plato and the Pythagoreans, assuming that they posited either One or Being as substances because of their universality. Yet they might not have accepted the Aristotelian view of One and Being as universal predicates if this undermined their status as substantial elements or principles.

Of the two major objections, the second (II) is of greatest interest from my point of view. For the moment, let us assume that the argument is made in defence of Plato and the Pythagoreans against the denial of substantiality to Being and One. Thus Aristotle begins by accepting such a denial for the sake of argument and then he draws conclusions which would presumably be unacceptable to these people. When read in this light, the objection reveals certain fundamental assumptions about mathematical ontology which Aristotle took to be characteristic of Plato and the Pythagoreans. Such assumptions usually become visible only when they are implicitly denied, as happens here when the genitive absolute in the first clause concedes, for the sake of argument, that the One is not a substance.[56]

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[55] εἰ μέν τις μὴ θήσεται εἶναί τινα οὐσίαν τὸ ἓν καὶ τὸ ὄν, 1001a20. Alexander (224.19) records the variant reading τινὸς οὐσίαν, though he thinks it makes no difference to the meaning.

[56] This genitive absolute construction may be seen to parallel the conditional

Thus the objection may be formulated as follows (1001a24–27): If the One is not a substance then it is clear that number cannot exist as some separated nature of things, since number is (a collection of) units and a unit is just what it is to be One. Again, I think, the question is not about existence in any modern sense but rather it is about the mode of being of number. This is confirmed by the ὡς clause which specifies that the mode of being in question is that of some independent substance. Indeed, the argument will not work unless the modes of being denied to number and to the One are identical. It is also crucial that the numerical unit be identified with the One in some essential way. Such an identification is attempted in the last clause (1001a26–27) which declares that the unit is a certain kind of One (ὅπερ ἕν τι), or perhaps that the unit is identical with the One itself.[57] Since the unit is just what One is (goes the objection) and since number is a collection of units, it is obvious that number cannot have the mode of being of substance if One does not. But, as Alexander (225.1–3) points out, this conclusion would appear absurd to those who hold that number is some separate substance and also to whoever includes number among the mathematicals as some intermediate substances.

Since the denial of this view has led to absurdity, Aristotle concludes this section with a positive statement of the Platonic position together with some of its implications:

> But if there is a unity-itself and a being-itself, their substance must be unity and being; for it is not something else that is predicated universally of them, but just unity and being.[58]

The key to the interpretation here, it seems to me, is the reference to Platonic Forms which is contained in the words 'unity-itself and being-itself' (αὐτὸ ἓν καὶ ὄν), since such a reference carries with it the whole ontology of separated substances which Aristotle has already attributed to Plato. Thus the argument goes as follows: if One itself

construction of the previous objections, so that δῆλον ὅτι corresponds to συμβαίνει within the structure of similar arguments.

[57] Alexander (224.36–37) lists both as possible interpretations, even though he favors taking the unit to be One in the strictest sense (κυρίως). Still he concedes that it is possible to take the unit as a species of the One as its genus because One is predicated of it. Ross (1924 i) 244 adduces a parallel passage from the *Posterior Analytics* (83a24) to show that ὅπερ is systematically ambiguous in Aristotle between the particular and the species as referent.

[58] *Met.* 1001a27–29: tr. Ross in Barnes ed. (1984).

and Being itself exist as separated substances, then it must be the case that unity (τὸ ἓν) or being (τὸ ὂν) is the substance (οὐσία) of each of them because there are no other universal predicates (καθόλου κατηγορεῖται) of One and Being except these things (i.e. unity and being) themselves.[59]

Once again, for the sake of argument, Aristotle appears to accept the Platonic assumption that universality is the criterion of substantiality. When this is combined with the assumption that there are such entities as One Itself and Being Itself, it is easy to see how the argument concludes that unity and being must be the substances of these entities. Given that One and Being are themselves assumed to be substantial entities (as Platonic Forms) and that they are predicated of things in a completely universal fashion, the irresistible conclusion appears to be that unity and being are the substances of all things of which they are predicated.

If that was the sort of argument which influenced Plato to postulate either Being or One as the substance of all things, then it encounters the following objection:

> But if being itself and unity itself were something there is much *difficulty* as to how there can be something else besides these, that is, how things can be more than one. For what is distinct from being does not exist, so the statements of Parmenides must follow, namely, that all things are one and this is *Being*.[60]

The reference to Parmenides may provide a clue for understanding the point of this rather obscure objection. Ross (1924 i, 245) thinks that it may be a reference to a Parmenidean argument which could be paraphrased as follows: one can never forcibly hold 'what is not' to exist because there is nothing other than 'what is' (τὸ ἐόν).[61] When combined with some other assumptions about 'what is,' the logic of this argument leads inexorably to the conclusion that Being is One; so it would appear to be relevant to this particular *aporia* about unity and being as the substances of things.

As both the *Parmenides* and *Sophist* dialogues show, however, Plato was aware of the inadequacies of such a monolithic concept of Being

[59] Accepting Ross's reading of καθόλου at 1001a28 along with the manuscripts and commentators rather than Bonitz's conjectural καθ' οὗ which is also read by Jaeger but which only gives a good sense if One and Being are held to be identical with the subject of which they are predicated.

[60] *Met.* 1001a29–b1: tr. Apostle (1966).

[61] οὐ γὰρ μήποτε τοῦτο δαμῇ εἶναι μὴ ἐόντα, Fr. 7 (DK).

which excludes all plurality from the universe. Therefore, it is hardly fair of Aristotle to foist a similar notion on Plato himself in order to generate a difficulty for the view that being and unity are somehow identical. It would appear, then, that this objection is not merely dialectical but also eristic in the sense that it does not begin from shared assumptions.[62] Ross does his best to exonerate Aristotle by distinguishing between 'being' as a predicate and Being as a substance. If one treats 'being' as a predicate in the Aristotelian manner, Ross argues, then there is room for plurality in the universe. But, on the other hand, if Being is taken to be a substance and there is nothing other than Being, then the universe is a single substance without plurality.

Yet even Ross acknowledges that this latter position is not recognizable as one that Plato would espouse, since the *Sophist* explores a sense in which non-being exists; i.e. as otherness. Thus, insofar as Plato's concept of being is confused with that of Parmenides, I do not think that Aristotle's objection is legitimate. But perhaps Aristotle is arguing that Plato is forced by his assumptions into Parmenidean conclusions; e.g. that all things are one and that this is being.[63] Such a conclusion, of course, would constitute a difficulty for Plato who wants to say that there are many other Forms besides Being and One, and that seems to be the point of Aristotle's objection.

So the review of difficulties on both sides leads to an impasse:

> There are objections to both positions. For whether unity is not a substance or unity itself is something, a number cannot be a substance. If unity is not a substance, we have stated earlier why a number cannot be a substance; but if unity itself is something, the same difficulty arises as that concerning being. For from what will another unity besides unity-itself, come? It must be from not-one; but all things are either one or many, and of the many each is one.[64]

What Aristotle says here, quite literally, is that both ways are 'unsatisfactory' (δύσκολον) since, whatever assumption one makes about the One, it turns out that number cannot be a substance and especially

[62] In personal correspondence Julia Annas defends Aristotle's argumentative procedure here on the grounds that he thinks that Plato's arguments in the *Sophist* do not succeed against Parmenides. So she finds it reasonable for Aristotle to face Plato with Parmenides' problems, since he has no adequate argument to meet them. In her published commentary (1976) 205–7 on *Metaphysics* XIII & XIV, Annas tries to explain why Aristotle may be so unimpressed with Plato's *Sophist*, though she concedes that a failure in understanding gives rise to some distortion.

[63] ἓν ἅπαντα εἶναι τὰ ὄντα καὶ τοῦτο εἶναι τὸ ὄν, *Met.* 1001a33–b1.

[64] *Met.* 1001b1–6: tr. Apostle (1966).

not the substance of things. On the one hand, if the One (τὸ ἓν) is not a substance then number cannot be a substance because, as he has already (1001a25–27) explained, a number is a collection of units. If, on the other hand, the One is a substance then it would seem that there cannot be number at all because there is no source for a multiplicity of units apart from the One.

The final part of the objection is rather mysterious in many ways, not the least being the hidden assumptions on which it depends. The major assumption, I think, is that if the One is a substance then it is the only substance in the universe. This assumption finds its expression in the explicit analogy with Parmenidean Being which Aristotle draws when he says that one is facing a similar impasse in both cases. For the purposes of clarification here, we should refer back to the previous passage where he says that if Being-Itself and One-Itself are assumed to be something (τι) then there will be much puzzlement (πολλὴ ἀπορία) about how there can be something else besides these. When he explains there that the difficulty is how things can be more than one, he would appear to be assuming the identity of One and Being.

I think that he has returned to that difficulty in the present passage, in order to present a further explanation as to why it arises. The source of the difficulty appears to be that there is nothing else besides the One Itself (παρὰ τὸ ἓν αὐτὸ) from which another One (ἄλλο ἕν) can come to be except the not-One (μὴ ἓν). But all beings are either a unity (ἓν) or a multiplicity (πολλά), each of which is a unity. Such a division of beings (τὰ ὄντα) appears to exclude the not-One, which may also be described as not-Being. This is the obverse of a hidden assumption about the identity of Being and the One, which also underlies this objection.

Adopting an axiom from Zeno, the subsequent objection fastens upon the indivisibility of the One:

> Further, if unity itself is indivisible, according to Zeno's axiom it will be nothing. For he says that that which neither makes the sum greater when added nor the remainder less when subtracted is not one of the beings, clearly believing that being is a magnitude, and if a magnitude, then corporeal; for this is a being in all dimensions. The others, the plane and the line for example, will increase the result when added in one way but not when added in another way, but the point or the unit will not increase it at all.[65]

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[65] *Met.* 1001b7–13: tr. Apostle (1966).

It is hard to see how Zeno's axiom can be relevant here unless we take Aristotle to be assuming that if the One is indivisible then it cannot have spatial magnitude. This is an assumption based on his own definition of continuous magnitude but it is possible that Zeno shared the same assumption, though he may have used the axiom mainly in his arguments against plurality.[66] Of course, since it is an axiom governing 'things that are' (τὰ ὄντα), it can be used with reference to the ontological status of any entity, including the One. Yet, according to Simplicius, it was not Zeno's intention to destroy the One but rather to show the impossibility of plurality.

Aristotle is therefore using Zeno's axiom here for a different purpose, although he does not appear to misrepresent its original formulation. If we are to believe the report of Simplicius, Zeno proved that what has neither magnitude (μέγεθος) nor solidity (πάχος) would not even exist (οὐδ' ἂν εἴη). His argument is reported as follows: if this thing (without magnitude or solidity) were to be added (προσγένοιτο) to some other entity, it would not make it larger (μεῖζον). The reason given is that what would be added, being of no magnitude itself, cannot increase the other thing in magnitude and so what appears to be added really would be nothing.

In general, Zeno's axiom says that if something when taken away (ἀπογινομένου) does not decrease another thing nor increase it when added, then it is clear that what was added or subtracted is nothing. Aristotle formulates the axiom in similar terms when he says (1001b8–9) that whatever does not make something greater (μεῖζον) when added (προστιθέμενον), nor less when subtracted (ἀφαιρούμενον), is not 'among the beings' (τῶν ὄντων). Given the slight change in terminology, it seems that Zeno's axiom has been incorporated into the dialectical *topos* of 'the greater and the less.'[67] What is obvious from the above passage, however, is that Aristotle takes Zeno to be using the axiom as a criterion for being. For instance, he attributes to him the assumption that every being has a spatial magnitude.

There is some basis for such an attribution in the extant fragments of Zeno, especially where he says (Fr. 1) that what has no magnitude or solidity could not even exist. It must be such statements that Aristotle has in mind when he draws the further conclusion that, if some being is a magnitude, then it must be corporeal

[66] Cf. Simplicius, *in Phys.* 139.9 & 140.34.
[67] Cf. *Topics* III.5, 119a20 ff.

(σωματικόν). But then he seems to take a further step when he *argues* that the Zenonian axiom makes body a complete being, whereas planes and lines are only partial beings, at best. The argument begins with an explanatory assertion that this (i.e. body) is a being in every way (τοῦτο γὰρ πάντῃ ὄν). The meaning of this only becomes clear when body is contrasted with other things, like a plane or a line, which yield an increase when added in a certain way (πὼς μέν) but not when added in another way (πὼς δέ). Thus, as Alexander (227.16 ff.) explains, body is 'being' in an absolute sense because it is the only thing that causes an increase whichever way it is added to another thing. Planes and lines, by contrast, only yield an increase when they are put together at their limits and hence they are beings only in a certain way, according to the ontological axiom of Zeno. But a consistent application of this axiom yields the curious result that points and units are not beings, since their addition or subtraction does not lead to an increase or decrease in magnitude.

It is unclear from the above passage whether this was a conclusion which Zeno himself drew or whether it is an implication drawn by Aristotle. As the conclusion of a dialectical objection, however, it may have been generated to contradict the initial assumption (1001b7); i.e. that the One itself is indivisible. Yet the contradiction is not evident at first glance nor is it even clear that the objection succeeds. The key to the objection, I think, is that the point (στιγμή) and the unit (μονάς) are leading examples of the One which is indivisible. Thus, if their claims to existence are undermined by a consistent application of the Zenonian axiom, so is the claim that the One itself exists in a complete sense. By this axiom, it is body which exists in every way and, since body is divisible in every way, it would seem that divisibility is another criterion of physical being.[68] This would further undermine the claim to existence of the One, as exemplified by the point and the unit. The merit of such an interpretation is that it gives Aristotle a plausible dialectical objection.

It is clear from the subsequent passage, however, that Aristotle is not very happy with Zeno's argument, even though he thinks there is something legitimate about the objection itself when it is turned against the theory that magnitudes are generated in some way from the One:

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[68] Aristotle seems to espouse this view as his own in *De Caelo* I.1 & *Physics* V.3.

> But since he argues crudely, an indivisible thing can exist, so that the position can be defended even against him; for the indivisible when added will make the number though not the size greater. But how can a magnitude proceed from one such indivisible or from many? It is like saying that the line is made out of points.[69]

Aristotle's criticism of Zeno seems to be that he views the question in a crude manner (φορτικῶς), since he overlooks the possibility that some indivisible things can exist (εἶναι). Thus Aristotle offers a defense against the previous objection, yet continues to use Zeno's own axiom as a criterion of existence. Although he concedes that adding such an indivisible thing (e.g. the unit) will not make something greater in magnitude (μεῖζον), he insists that it will make it greater in number (πλεῖον). Hence, according to the Zenonian axiom itself, the unit must exist in a certain way, just like lines and planes. Thus Aristotle defends the existence of at least one kind of indivisible One; i.e. the unit of number.

But it would appear that no such defense can be offered for the point because, when it is added or subtracted from something, it does not yield an increase either in magnitude or in number. This may be what motivates Aristotle's final objection which is formulated as a rhetorical question: i.e. how can a magnitude (μέγεθος) come to be out of such a unit or even out of many such units? Obviously, he thinks (1001b17–18) that whoever holds this theory about the generation of magnitudes is also committed to the claim that a line consists of points. In his commentary on this passage, Alexander (228.5–10) identifies these theorists as Pythagoreans and attributes to them the claim that numbers are the principles (ἀρχάς) and elements (στοιχεῖα) of all things. He takes this to imply that numbers are also the principles of magnitudes. Alexander's interpretation is supported by Aristotle's explicit reference to the Pythagoreans at the beginning of this aporia about whether or not One and Being are the substances of things. Thus one might conjecture that we have here the number-atomism of the early Pythagoreans being subjected once more to the objections of Zeno.[70]

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[69] *Met.* 1001b13–19: tr. Ross in Barnes ed. (1984).

[70] Against any such conjecture, Owen (1958) argues that Zeno cannot have made his objections to Pythagorean theories of number-atomism since there is little evidence that such theories ever existed. By contrast, Burkert (1962) 258–9 thinks that a close relationship between the Pythagoreans and the Atomists is corroborated both by internal and external evidence.

Finally, even if a second principle of generation be added to the One, serious objections can be raised against the 'generation' of number and continuous magnitude from the same principles:

> But even if one believes, as some say, that a number is generated from unity itself and something else which is not-one, we must inquire none the less why and how the thing generated will be sometimes a number and sometimes a magnitude, if indeed the not-one is Inequality and is of the same nature. It is not clear how magnitudes could be generated either from the One and Inequality or from some Number and Inequality.[71]

Even though Aristotle does not explicitly identify the target of his criticism here, we might guess that it is some members of the Academy who posited the so-called 'Indefinite Dyad' (ἡ ἀόριστος δυάς) as a second principle of generation along with the One. In the *Metaphysics* passages where this view is outlined and criticised, there is some evidence for identifying the second principle with what is here called 'Inequality' (ἀνισότης).[72] Thus one may formulate Aristotle's first objection as follows: If the principles of generation are the same, how can it happen that the results are sometimes different? In other words, he is demanding an explanation for the clear difference between numbers and continuous magnitudes. The subsequent objection is slightly different in that it seems to demand an account of how (ὅπως) magnitudes can be generated either from the One and Inequality or from some number (ἐξ ἀριθμοῦ τινός) and Inequality.

In his commentary Alexander (228.10 ff.) takes the "either . . . or" here to indicate two different sets of first principles and, correspondingly, two different processes of generation. First, there is the One and the Indefinite Dyad, which are the principles of number and which, therefore, cannot also yield continuous magnitude. Second, there is some number and the Dyad which together are somehow thought to yield continuous magnitudes. Obviously, Alexander is going beyond the Aristotelian text in taking this second difficulty to be about the successive generation of number and magnitudes.[73] All that Aristotle does in the above-quoted passage is to set out two alternative sets of principles for the generation of continuous magnitudes; i.e. the One and Inequality, as distinct from some number and Inequality.

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[71] *Met.* 1001b19–25: tr. Apostle (1966).
[72] Cf. *Met.* 1087b5, 1088b32, 1089b–15, 1091b35.
[73] Cf. Alex. Aphr. *in Metaph.* 228.20–22.

Of course, one might assume (as Alexander appears to do) that 'some number' must already have been generated from the set of principles appropriate to number; i.e. the One and the Dyad. In this way it is possible to read the objection as being about the successive generation of numbers and magnitudes from a single set of principles; namely, the One and Inequality. But this would make the success of the objection depend upon the assumption that the theorists in question would consider the principles to be exactly the same in each case. As soon as we state it thus, it is clear how dubious the assumption really is and, therefore, how weak the objection becomes if we follow Alexander's interpretation. Yet it is not obvious how we might strengthen the objection beyond construing it as a general complaint about the difficulty of seeing how numbers and magnitudes could be generated from such principles.

Even though Aristotle does not give a summary of results at the end of this review of the difficulties, it may be useful to reflect on the nature of the impasse which results from it. The question at issue is whether One and Being are the substances of things or whether they have some other dependent ontological status. Whichever position we take on the issue, as Aristotle says, there are difficulties on both sides. On the one hand, if we assume that unity and being are not substances then none of the other universals can be substances, which presumably would be unacceptable to a Platonist. Similarly unacceptable would be the consequences of assuming that unity is not a substance because this would mean that number cannot exist as a separate nature. Furthermore, if either One itself or Being itself (as Platonic Forms) is not something, neither will any of the other universals be something besides (παρά) the so-called particular things. Yet, on the other hand, if One itself or Being itself is something, then unity and being must be the substance of each; for there is nothing else which is universally predicated of what is (i.e. being) or of what is one. We may suppose that this conclusion would also be unacceptable to Platonists who want to posit other Forms besides Being and One as the substances of things. Even less acceptable, we must suppose, will be the Parmenidean conclusion which Aristotle draws in the following way: if Being Itself and One Itself were something, there cannot be anything else besides Being and One because what is distinct from being does not exist.

After recognizing that there are difficulties on both sides, as in any typical aporia, Aristotle adds some other difficulties which have

a direct bearing on mathematics. If we suppose that unity is not a substance, it follows that number cannot be a substance. But even if we assume that One Itself is some substance, there is no source for plurality other than the not-One and this is excluded by the same Parmenidean argument as in the case of being. Furthermore, if unity itself is indivisible, according to the relevant axiom of Zeno it will be nothing. In addition, it is difficult to see how any magnitude can be generated from such a unity or even from a plurality of such units because that would be like saying that a line is made up of points. This latter remark leads one to suspect that the objection is directed against those Pythagoreanizing Platonists like Speusippus who are reporting as generating everything from the One as a principle. Even if one adds the Indefinite Dyad as a second principle, there is still the difficulty of how both numbers and magnitudes can be generated from the same principles. Such objections tend to undermine the mathematical cosmology of the Academy.

IV. *The difficulties about mathematical objects*

Proceeding chiastically, let us explore those difficulties which are associated with the specific aporia about mathematical objects that was given last in Aristotle's initial outline of the aporiae at *Metaphysics* III.1. At III.5, however, when he reviews its cluster of difficulties, the aporia itself is advanced somewhat in the order of inquiry: "A difficulty which follows the preceding is whether numbers and bodies and planes and points are substances or not."[74] Is there some significance in this aporia being described as 'following' (ἐχομένη) on the aporia about whether or not Being and One are the substances of things? While this may be simply marking a transition, yet there are some internal connections between the two aporiae. For instance, one of the implications of denying that One is a substance is that neither can number be a substance.

So this provides one link with the present aporia about whether or not numbers (as mathematical objects) are substances. In addition, an application of Zeno's axiom induced some uncertainty about the ontological status of such objects as the unit, the point, the line and the plane. While most of these entities (with the exception of the

[74] *Met.* 1001b26–28: tr. Apostle (1966).

point) may have survived the test, it is a natural to inquire next about whether or not they are substances. Finally, in connection with the previous aporia, there is the concluding set of difficulties about how numbers and magnitudes can be 'generated' either from a single principle (i.e. the One) or from two principles combined (i.e. the One and the Indefinite Dyad). Following these, one might raise a doubt about whether these completely universal principles could be substances or could generate other substances. All of these connections throw some light on why Aristotle brings forward the aporia about mathematical objects and treats it as subsequent in the order of inquiry.

The reformulation of the aporia itself is shorter than the original formulation and also shows small changes in terminology. The first word, τούτων, refers back to the difficulties associated with the previous aporia, while ἐχομένη characterizes the present aporia as following naturally from these. I think that Alexander (228.31–32) is correct to assume that Aristotle is referring back to the smaller set of difficulties about whether or not numbers are substances. From the point of view of terminology, the word used here for plane figures (ἐπίπεδα) seems to be more technical than the term (σχήματα) used in the previous formulation.[75] But there is the usual ambiguity in the use of σώματα to designate geometrical solids, since it can also refer to sensible solids. The more technical term would be στερέα, though Aristotle sometimes uses it also to designate sensible solids.

But the central question of the aporia is clearly about whether numbers and solids and planes and points are kinds of substances (οὐσίαι τινες) or not. No significance should be attached to the absence from III.5 of the subsequent question of whether they are separated from sensible things or belong in them as distinct substances. Even though this question seems to have dropped from sight, it is implicit in the leading question about whether or not mathematical entities are substances. For if mathematical objects are substances distinct from sensible substances then Aristotle sees only two possible relationships which they can have two sensibles; i.e. either they are separated from them or they are somehow in them. This is confirmed by his treatment of the so-called 'Intermediates' in the next aporia, and by his more general treatment of mathematical objects at *Metaphysics* XIII.2. In both cases, he treats as exhaustive these two

[75] Cf. LSJ. ἐπίπεδον, σχῆμα; Bonitz (1870) 275b50–276a12 & 739b.13–740a42.

possible relationships between mathematical entities and sensible things.

Having identified this aporia with that given last in III.1, we can now review the difficulties which Aristotle explicitly associates with it. Given that he sets them out as objections on both sides of the question, we cannot take any of them as positive arguments for Aristotle's own position, since that has not yet emerged and may not even be in play here.[76] In retrospect, it may turn out that some of them will support his position because these are the difficulties which he must resolve in the process of 'saving the phenomena.'[77] But here we must treat them simply as dialectical difficulties which are raised on either side of the question of whether or not mathematical objects are substances.

For instance, if one assumes that they are *not* substances then one encounters the following difficulty:

> If they are not, it baffles us to say what being is and what the substances of things are. For affections and motions and relations and dispositions and ratios do not seem to indicate the substance of anything; for all of them are said of some subject and none of them is a this.[78]

Despite being formulated in Aristotelian terms, the difficulty should be understood from a Platonic perspective as follows: If mathematical objects are not substances, it escapes us (διαφεύγει) what being is (τί τὸ ὂν) and what are the substances of things (τίνες αἱ οὐσίαι τῶν ὄντων). The difficulty itself centers on the 'what-is-being' (τί τὸ ὂν) question which, for Aristotle, leads directly to the question about the substance of things. In *Metaphysics* VII.1 (1028b4), for instance, he identifies them as two variations on the same historical question about the being of things. One of the reported answers is that One (ἓν) is the substance of things, and this tends to confirm the link between this aporia and the previous one (1001a4 ff.) about whether or not Being and One are the substances of things. In that aporia the difficulty (1001a25–26) of denying that One is the substance of things

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[76] Even if Aristotle had already reached his own solution by the time he wrote *Metaphysics* III, as Annas suggests, he deliberately suppresses it here and conducts an aporetic inquiry into each problem as preparatory to such a solution. But why should he use such an elaborate pretence, unless he felt the aporetic phase to be absolutely essential to the whole inquiry?

[77] Elsewhere (Cleary 1994a), I have tried to show the parallel between Aristotle's methods of treating a problem in both the empirical and dialectical sciences.

[78] *Met.* 1001b28–32: tr. Apostle (1966).

is that neither will number be substance. Since this is a difficulty only for a Platonist, the same perspective applies in the present argument which assumes that mathematical objects have a better claim to being substances than the other candidates.

This is the implicit point of his claim (1001b30–31) that neither affections nor motions nor relations nor dispositions nor ratios appear to indicate the substance of anything, since they are all predicated of some subject and so none of them is a 'this.' Here Aristotle seems to be using the *Categories* criterion of substance; i.e. that it must be a 'this' (τόδε τι), just like a particular man or horse.[79] Thus, in the case of primary substances like a man or a horse, it is certainly true that each of them signifies a certain 'this' because the referent is something individual (ἄτομον) and numerically one (ἓν ἀριθμῷ). So mathematical objects, if they come under the category of quantity, should have a dependent status similar to that of quality. Yet the first objection (above) assumes that these objects have the strongest claim to being the substances of things, and this suggests that a Platonic meaning of substance is intended here.

For clarification of this meaning it is reasonable to look to *Metaphysics* V.8, where Aristotle collected the various senses of substance. For instance, 'substance' in one sense refers to the simple bodies such as earth, fire, water, and such things. In general, this kind of substance includes elemental bodies and the things constituted out of them; e.g. animals and their parts. Aristotle goes on (1017b13–14) to explain that all of these are called 'substance' because they are not predicated of a subject, whereas everything else is predicated of them. Although he adopts a criterion of substance from the *Categories*, it does not follow that he is merely putting forward his own view here, because it is clear that bodies were regarded as most substantial by the natural philosophers.

However, since the third meaning of substance is illustrated in terms of mathematical form, it appears to be derived from the Pythagorean tradition. According to this view, those things are substances which are constituent parts in such things as are not predicated of a subject and which are defining and signifying a 'this.'[80] Furthermore, these constituent parts are such that when they are

[79] Πᾶσα δὲ οὐσία δοκεῖ τόδε τι σημαίνειν, *Cat.* 3b10.
[80] ὁρίζοντά τε καὶ τόδε τι σημαίνοντα, *Met.* 1017b17–18.

destroyed the whole is also destroyed with them.[81] In view of his talk about constituent parts, one is surprised by the mathematical examples which Aristotle supplies with this third meaning of substance. Furthermore, he assigns this way of thinking about substance to other (unidentified) thinkers or, at the very least, he implies that they order things by such a criterion of substance. For instance, they say that the destruction of the plane will destroy the body and, similarly, that if the line is eliminated then so is the plane. Such a criterion leads some thinkers to the conclusion that number is the substance of all things because it determines everything and, if it is eliminated, there is nothing without it.[82]

This way of thinking about mathematical forms as substances may be the perspective from which Aristotle introduces the first difficulty in *Metaphysics* III.5, which appeals to two criteria of substance which quantities do not satisfy according to his own categories. But mathematical objects such as the line and the plane satisfy the first criterion for substance, since they do not seem to be predicated of some other subject. In addition, they also appear to satisfy the second criterion because, referring to mathematical objects as constituent parts, Aristotle says that they are things signifying a 'this' (τόδε τι σημαίνοντα); cf. *Met.* 1017b17–21. Thus, contrary to initial appearances, the first objection is internally consistent, since mathematical objects like planes and lines may be taken to satisfy the criteria for substantiality that qualities and relations fail to satisfy. Of course, we should keep in mind that Aristotle's method here is peirastic in the sense that he is testing certain assumptions about substance, while using common criteria of substance to guide the argument at this stage of his inquiry.

Some of these criteria continue to hold when Aristotle goes on to consider entities that appear to have a better claim to substance than mathematical objects:

> As for objects which most of all might seem to indicate substances, such as water and earth and fire and air, of which the composite bodies consist, their heat and coldness and such are affections and not substances, but the body alone which is affected by these persists as a sort of a thing and a substance.[83]

[81] ὧν ἀναιρουμένων ἀναιρεῖται τό ὅλον, *Met.* 1017b18–19.
[82] ἀναιρουμένου τε γὰρ οὐδὲν εἶναι καὶ ὁρίζειν πάντα, *Met.* 1017b20–21.
[83] *Met.* 1001b32–1002a4: tr. Apostle (1966).

If my reading so far is correct, then it is no coincidence that the objects listed here are identical with the so-called 'simple bodies' which are named in V.8 as primary examples of substance. Thus the verb δόξειε, as used in the above passage, may be taken in either of two ways. On the one hand, it could be a reference to the fact that the four elements simply appear to be substantial.[84] But, on the other hand, it might refer to the opinion (δόξα) that the elemental bodies indicate the substance of things because they are the basic constituents out of which the composite bodies are compounded.[85]

I find the latter interpretation more plausible, especially in the light of Aristotle's introductory remarks about the need to review the *doxa.* Here we find him pitting one historical opinion against another, in order to discern the truth from their harmony and discord. Thus there would seem to be some continuity with the previous difficulty which arose from denying that mathematical objects are substances. If they are not, as Aristotle has argued dialectically, then we are baffled as to what are the substances of things. Now (he continues) suppose that we take such things as water and earth which appear to be most substantial of all, we can still ask what are their substances. If someone answers in terms of hotness or coldness or such things, one can object that these are affections (πάθη) and not substances (οὐσίαι). Since attributes like 'hot' and 'cold' are predicated of some subject, it is reasonable to assume that they are not substances. But some criterion of permanence seems to be introduced when Aristotle says that the body affected by these opposite attributes is the only thing that remains as something real and as a substance.[86] Of course, there is no reason why this permanent substratum of changing qualities should not satisfy the other criteria for substance, since a form like water is predicated of some subject like 'body' which is a 'this' in appearance (*Met.* 1070a9–10). But, as we shall see from the subsequent difficulty, body (σῶμα) does not have the best claim to substance, despite the support of sense experience and of general opinion.

Clearly, the next difficulty is introduced from the point of view of those Pythagoreanizing Platonists who gave primacy in substance to

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[84] μάλιστ' ἂν δόξειε σημαίνειν οὐσίαν, *Met.* 1001b32–33; cf. also *Met.* 1028b7–13.
[85] ἐξ ὧν τὰ σύνθετα σώματα συνέστηκε, *Met.* 1002a1; cf. also *Met.* 1017b10–13.
[86] τὸ δὲ σῶμα τὸ ταῦτα πεπονθὸς μόνον ὑπομένει ὡς ὄν τι καὶ οὐσία τις οὖσα, *Met.* 1002a3–4.

mathematical form. Such a perspective is confirmed by the historical remarks which are appended in the following passage:

> What is more, a body is a substance to a lesser degree than a surface, and a surface than a line, and a line than a unit or a point; for a body is defined in terms of these, and these seem to be capable of existing without a body, but a body cannot exist without these. It is because of these arguments that (a) the common people and the earlier thinkers considered a body to be a substance and a being, and the rest as attributes of it, and hence the principles of bodies to be the principles of things, but that (b) the later and wiser thinkers considered numbers to be substances.[87]

The initial combination of particles (ἀλλὰ μὴν . . . γε) gives a strong adversative sense which is consistent with Aristotle's method of conjoining two opposing opinions about substance. Previously, by appealing to the criterion of permanence, he had given some grounds for the popular view that the elemental bodies are substances. But now, citing a different criterion, he argues that body is less substantial (ἧττον οὐσία) than surface and this, in turn, is less substantial than the line and the line less so than the point and the unit. The new criterion is implicit in the explanation that these determinants (ὥρισται) of the body appear to be capable of existing independently, whereas body cannot exist without them.[88] The Greek reveals an application of the criterion for priority with respect to nature and substance, which Aristotle explicitly attributes to Plato at *Metaphysics* V.11. Even though no examples of this kind of priority are supplied there, it is fairly easy to connect the criterion with a similar way of thinking about substance in V.8 which outlines a system of non-reciprocal dependency between unit, point, line, plane, and solid. Thus we can see how the criterion applied in the above passage can be made to yield this schema of logical and ontological dependency, according to which one might claim that number is absolutely prior to everything else, since everything depends on the unit for its definition.

In fact, this is almost exactly the claim that Aristotle attributes to those 'later and wiser' (ὕστεροι καὶ σοφώτεροι) thinkers who puzzled over the question of substance. At the end of the above passage, these people are contrasted with the many previous thinkers who

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[87] *Met.* 1002a4–12: tr. Apostle (1966).

[88] τὰ μὲν ἄνευ σώματος ἐνδέχεσθαι δοκεῖ εἶναι τὸ δὲ σῶμα ἄνευ τούτων ἀδύνατον, *Met.* 1002a6–8.

dealt with the same question about substance and being. According to Aristotle, these natural philosophers took the ordinary view of most people that body is substantial, while other things are merely attributes of this. Presumably this is the same view of substance which is justified by appeal to the criterion of permanence. The logical conclusion (1002a10–11) of such a view seems to be that the principles of bodies are identical with the principles of beings. It is clear from *Metaphysics* I.3 (983b6 ff.) that Aristotle considers this to be an accurate account of his predecessors' way of thinking about being. Similarly, at I.5 & 6, the account which he gives of the Pythagoreans and their influence on Plato has many points in common with the position here assigned to the 'later and wiser' thinkers. For instance, the Pythagoreans are credited with the claim that the principles of number are the principles of all things; cf. *Met.* 985b25 & 986a1–2. Plato is reported to have agreed with them that numbers are the causes of the substance of other things, just as he accepted that One is itself a substance and not a unit predicated of something else.[89]

Incidentally, this passage also provides a striking confirmation of the close connection between the aporia about whether or not One is a substance and the aporia about whether or not Number is the substance of things. In view of Aristotle's own negative response to both questions, it is possible that there is an ironic edge to his description of these later thinkers as 'wiser' (σοφώτεροι). The fact that the Pythagoreanizing Platonists were skilled in the science of mathematics led them to seek abstruse and hidden causes, whereas the more naive natural philosophers stayed on the level of ordinary sense experience. But the simple-minded natural philosophers grasped an 'elementary' truth that escaped the lofty-minded Platonists. Whether or not any such irony is intended, it is clear that he wishes to contrast these two different historical ways of thinking about substance and being.

Thus we can see that his review of conflicting opinions about the same question is part and parcel of Aristotle's dialectical method, since its purpose is to generate an aporia. This is the state of puzzlement reflected in the following summary: "As we said, then, if these are not substances, there is no substance and no being at all; for

[89] τὸ . . . ἓν οὐσίαν εἶναι, καὶ μὴ ἕτερον τι ὂν λέγεσθαι ἕν . . . τοὺς ἀριθμοὺς αἰτίους εἶναι τοῖς ἄλλοις τῆς οὐσίας, *Met.* 987b22–25.

indeed the attributes of these do not deserve to be called beings."[90] If I am right about his aporetic method, this summary presents an impasse that arises from denying that either numbers or points or planes or bodies are substances. By means of a familiar ambiguity in the term σῶμα, both physical and mathematical bodies are implicitly included in this list of entities. Thus if all of the listed entities are rejected as substances, it would seem that there is no substance or being at all (ὅλως) because only accidents are left and they do not deserve to be called substances. The rejection of accidents as 'beings' is justified only if we take ὄντα to mean substances and if we refer back to the criterion of permanence used by the natural philosophers. According to that criterion, attributes like hot and cold cannot be 'beings' because they do not remain, whereas the body which is affected does remain as some being and some substance.[91] Thus attributes fail to be substances, whether we apply the criterion of permanence or the criterion of predication or even the requirement that substance be a 'this.' Within this dialectical context, therefore, it is plausible for Aristotle to conclude that if mathematical entities (including bodies) are rejected as substances then we are at a loss to know what substance or being is.

So far in this review of difficulties, everything seems to support the view that mathematical entities are substances because they appear to satisfy the standard criteria. But at this point in the review Aristotle brings forward difficulties which arise from accepting such a position:

> Yet if it is agreed that lines and points are substances to a higher degree than bodies, but we do not see in what kind of bodies these can exist (for they cannot exist in sensible things), a substance could not exist at all.[92]

Once again, the prominent initial position in the Greek text of the combined particles ἀλλὰ μὴν emphasizes their adversative sense and thereby marks a clear transition to an opposing argument. This opposition is further underlined by the structure of the conditional which introduces one of the difficulties arising from the adoption of the position indicated. Thus the *protasis* of a conditional explicitly states the position which is being accepted for the sake of argument,

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[90] *Met.* 1002a12–14: tr. Apostle (1966).
[91] . . . . ὡς ὄν τι καὶ οὐσία τις οὖσα, *Met.* 1002a3.
[92] *Met.* 1002a15–18: tr. Apostle 1966.

while also introducing considerations which tell against such a position, and the *apodosis* concludes that there cannot be any substance. One might paraphrase the whole argument as follows: even if one accepts that lines are more substantial than bodies and that points are substances more so than lines, yet it is hard to see to what kinds of bodies they could belong. They cannot be in sensible bodies and hence (the argument concludes) there cannot exist any substance.

As it stands, however, the statement of the difficulty has a number of puzzling features which are perhaps due to some hidden presuppositions. The first seems to be the assumption that lines and points must belong to some kind of body if they are to be substances at all.[93] Such an assumption seems to be necessary for making the logical step from the difficulty of seeing how these entities can belong to any sort of body, to the conclusion that no substance exists. Yet this assumption is tantamount to denying the Platonist position that was ostensibly accepted for the sake of argument in the antecedent; i.e. that planes, lines, and points can be without bodies since they determine them. If this is the case, however, the difficulty arises from the conflict between Platonic views and some of our basic intuitions about substance. For instance, while accepting for the sake of argument that points, lines and planes are more substantial than bodies, we are still tempted to ask about the kind of body to which they belong.

Obviously, the implicit assumption here is that they must belong to some kind of body by their very natures, and the argument itself hinges on the difficulty of seeing to what sort of body these substantial points and lines can belong. By way of parenthetical explanation, we are told that they cannot be in sensible bodies (ἐν . . . τοῖς αἰσθητοῖς). Even though the Greek word for bodies is not included in this explanation, I think it is clear from the context that αἰσθητοῖς is meant to designate sensible bodies.[94] It would also be consistent with Platonism to deny that mathematical entities are in the sensible

[93] Alexander (*in Metaph.* 230.20 ff.) seems to identify this assumption when he explains that, if point and line and plane are beings and substances, it is clear that their reality (ὑποστάσις) is also in bodies; and if in bodies then in sensible bodies.

[94] It is unclear to me whether Aristotle ever accepted a kind of body which is not sensible, in the sense that it is not possible in principle for the senses to perceive it. The most likely candidate would seem to be *aither,* which plays a prominent role in the *De Caelo,* but its perceptibility is not discussed there in detail. In a discussion of the sphericity of the universe, however, Aristotle claims that the uniformity and fineness of *aither* makes it capable of embodying the spherical shape more accurately than any of the sublunary elements that are visible to us; cf. *Cael.* 287b15–22.

world, in response to the Protagorean objection against the truth of mathematics. For instance, we cannot find anything in the sensible world which corresponds to the definition of a point nor can we find such a thing as length without breadth, which might satisfy the definition of a line. But, as Alexander explains, if these entities were 'beings' (ὄντα) and 'substances' (οὐσίαι), they would be in sensible bodies because the latter are "the only things in subsistence."[95] Despite its non-Aristotelian character, we might agree that Alexander has identified the basic assumption behind this objection; namely, that sensible bodies are the only things that have the mode of being of substance. If that is so then the objection seems to be made from the perspective of the natural philosophers who considered only physical bodies to be real.

In fact that perspective is obvious from the subsequent argument, which looks like an addendum: "Further, these appear to be divisions of bodies, one in width, another in depth, and another in length."[96] It is clear from the Greek text that the word ταῦτα must refer only to planes, lines, and points, since these are apparently divisions of body (διαιρέσεις . . . τοῦ σώματος). What is less obvious but also implicit in the passage is an alternative view of their ontological status. Whereas previous thinkers had seen them as independent entities which are more substantial than body, Aristotle suggests that they are merely entities (ὄντα) that divide a body. For example, the plane divides a body with respect to depth, while the line divides with respect to width and the point with respect to length. Perhaps it is this perspective which motivates the previous difficulty because, if one sees these mathematical entities as divisions of body, one will naturally be tempted to ask about the kind of body to which they belong. Since this view is not ascribed to any historical thinker but is simply described as what appears (φαίνεται) to be the case, it must be treated as one of the φαινομένα to be 'saved' by any alternative account of the ontological status of planes, lines, and points. But no such account is given here, since to call these entities 'divisions of body' is merely to insist on a relationship to some kind of body. Yet it does pose a difficulty for the Platonic view which held them to be more substantial than and hence independent of all body.

Another difficulty for this view arises from the fact that there are

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95 ταῦτα γὰρ ἐν ὑποστάσει μόνα, *in Metaph.* 230.25–26.
96 *Met.* 1002a18–20: tr. Apostle (1966).

many possible divisions of a body that can yield many different figures, none of which has a greater claim to being in the body:

> Besides this, any figure is as much in a solid as any other figure, or else no figure is in it at all; accordingly, if Hermes is not in the stone, neither will half of the cube be in the cube as something definite, and so, neither will the surface be in the cube; for if any sort of surface were in it, the surface which marks off the half cube would be in it too. The same argument applies to any line or point or unit.[97]

This is an important objection to which Aristotle returns at *Metaphysics* XIII.2 in refuting those thinkers who claimed that mathematical objects are in sensible bodies as substantial and determinate (ἀφωρισμένα) entities. Here, by means of a comparison with the potential statue of Hermes not yet hewn from the stone, he makes the commonsensical claim that any sort of figure (ὁποιονοῦν σχῆμα) has an equal claim to being in the solid. The supporting argument is that if the statue is not in the stone as something determinate (ὡς ἀφωρισμένον), then neither is the half-cube in the cube as a definite entity.

Similarly, therefore, neither will a surface be marked off as something definite in the cube because, if any sort of surface were in it, then the surface which is marking off (ἀφορίζουσα) the half-cube would be in it too. If they are the determinate boundaries of bodies (the argument goes), then the surface of the half-cube has just as much (or just as little!) claim to being a determinate entity as the surface of the cube. Aristotle says that similar difficulties can be brought against the treatment of lines, points, and units as determinate entities. The strength of such objections lies in the fact that there does not seem to be any specific mathematical figure which can lay exclusive claim to being the defining boundary of any (physical) body.

In his commentary on this passage, Alexander (230.32 ff.) goes well beyond Aristotle's text when he declares that it is 'by reflection' (ἐπινοίᾳ) that these things (i.e. mathematical entities) are said to be present in bodies, as it is not by virtue of self-subsistence or any capacity to exist in separation.[98] It remains to be seen whether or not Alexander proves to be correct about Aristotle's own view in *Metaphysics* XIII.3 but such remarks are certainly premature at this point in the dialectical process. Since Aristotle expresses his intention of first reviewing the difficulties in an even-handed manner before

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[97] *Met.* 1002a20–25: tr. Apostle (1966).
[98] οὐ γὰρ δὴ τῇ ὑποστάσει καὶ τῷ χωρίζεσθαι δύνασθαι, *in Metaph.* 230.34–231.1.

trying to resolve the problem in hand, it would be somewhat underhanded for him to insert his own definitive views into such a review.[99] While commentators like Cherniss are quite willing to accuse Aristotle of such sleight-of-hand, there must be convincing evidence in the text before we convict him. Even though Alexander is obviously adopting an Aristotelian stance, I think that his tendency to introduce 'solutions' from elsewhere into his commentary on *Metaphysics* III misrepresents the peirastic character of Aristotle's review of difficulties.

As evidence for the purely aporetic character of Book III, consider Aristotle's general summary of his review of difficulties on both sides of the question about the substantiality of mathematical objects:

> Accordingly, if bodies are in the highest degree substances, but planes and lines and points are substances to a higher degree than bodies, and if the latter do not exist or are not substances, we are baffled as to what being and what the substance of things is.[100]

This passage encapsulates the conflicting conclusions of the previous review and shows how it leads to a complete impasse about being and substance. On the one side, the argument that substance is the permanent substratum of change yielded the conclusion that body remains as some being and some substance (1002a3–4). This conclusion is now condensed into the first part of the *protasis* as: "If, on the one hand, the body is in the highest degree substance" (1002a26). By contrast, the other part of the *protasis* summarizes the argument for the greater substantiality of points, lines, and planes, together with the objection which undermines this view of substance and being. As a result of the conflict of views contained in the *protasis,* complete puzzlement about being and substance is expressed in the *apodosis* as follows: "it baffles us to say what being is and what the substance of things is."[101] This repeats almost *verbatim* the statement made at the very beginning of III.5, where the central question of the aporia is set out: If mathematical objects are not some substances, then it baffles us to say what being is and what are the substances of things.[102]

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[99] On this point Annas (in personal correspondence) suggests that Aristotle may be justified in inserting his own position as that of any reasonable person who might be party to the debate. But, despite Alexander's best efforts, the most that can be squeezed from the text of *Metaphysics* III is an objection put forward from a commonsense point of view that may have been shared by Aristotle.

[100] *Met.* 1002a24–28: tr. Apostle (1966).

[101] διαφεύγει τί τὸ ὂν καὶ τίς ἡ οὐσία τῶν ὄντων, *Met.* 1002a27–28.

[102] διαφεύγει τί τὸ ὂν καὶ τίνες αἱ οὐσίαι τῶν ὄντων, *Met.* 1001b28–29.

Thus, *pace* Alexander, I think that Aristotle is not explicitly taking up any position on this question but is deliberately confining himself to the elucidation of the aporia itself, as he indicated in his remarks on method at the beginning of Book III.

After reviewing the difficulties associated with the view that mathematical entities are substances, Aristotle lists some additional difficulties concerning their generation and destruction:

> In addition to what has been said, unreasonable consequences follow even in matters concerning generation and destruction. For it seems that a substance, if it did not exist before but it now does, or if it existed before but now it does not, undergoes these changes by being in the process of becoming or being destroyed, respectively; however, points and lines and surfaces cannot be in the process of becoming or of being destroyed, but at one moment they exist and at another they do not.[103]

This objection appears to be an afterthought. Unlike previous difficulties, it does not emerge from the conflict between historical opinions, even though it may be assumed to be directed against the view that points, lines, and planes are substantial. For this reason it comes under the so-called 'antithesis' in Aristotle's dialectical practice of considering both sides of any question.[104]

The general strategy of this objection to the view that mathematical entities are substances is to develop unreasonable (ἄλογα) consequences concerning generation and corruption. The structure of the whole passage underlines the fact that the difficulty depends on a contrast between the appearances (δοκεῖ μέν . . .) related to the generation of substances and the logical situation (τὰς δέ . . .) with respect to the generation of mathematical objects. The appearances related to the generation and destruction of substances involve a process over time, which is indicated here by πρότερον and ὕστερον.

By contrast, mathematical entities such as points, lines, and planes, cannot be either generated or destroyed in this sense of undergoing a temporal process because at one moment they are (ὁτὲ μὲν οὔσας) and at another moment they are not (ὁτὲ δὲ οὐκ οὔσας). Here the use of the present participles on both sides of the contrast underlines the point that no process over time is involved. This complete lack of

[103] *Met.* 1002a28–34: tr. Apostle (1966).

[104] With some caveats, I consider the best guide to such dialectical practice to be still the massive commentary on Aristotle's *Metaphysics* provided by Thomas Aquinas.

temporal process in the so-called 'generation' of mathematical objects may be another one of the phenomena to be saved by Aristotle's own positive account of their ontological status. But here he uses it simply to make an objection against the view that they are substances, since the generation and destruction of substance seems (δοκεῖ) to involve a process over time. The implicit assumption which appears to ground the objection is that corruptible substance is the only kind of substance. Since this is hardly an assumption that the Platonists would accept and, since Aristotle himself acknowledges the existence of other kinds of substance, the whole objection seems to be eristic in character, although it points towards his own proposed solution.

Aristotle subsequently explains why mathematical entities do not appear to undergo a process of generation and corruption over time:

> For whenever bodies come into contact or are divided, the limits become instantaneously one when the bodies come into contact, or instantaneously two when they are divided; so, when the limits are together they do not exist but have been destroyed, and when the parts of a body are divided, limits now exist which did not exist before. For, certainly, the indivisible point within the line was not divided into two. And if they are in the process of generation or destruction, from what are they generated?[105]

For the purposes of the objection, the most important words in this passage are the two adverbs of time; namely, ὅταν and ἅμα. What they indicate is that the change from one limit to two is instantaneous in this special case of 'becoming,' as contrasted with the generation of substances which happens over a period of time. But the passage also implies that the 'generation' of mathematical entities cannot be like ordinary generation because there seems to be nothing 'out of which' (ἐξ οὗ) they come to be. Even though Aristotle does not use his usual verbal formula for the material cause, I think it is implicit in the final rhetorical question: 'from what are they generated?' (ἐκ τίνος γίγνονται;).[106] When a body is divided, two limits immediately appear where previously there was just one.

Thus, when one adopts a natural perspective on substance, the obvious question is: whence did these limits come to be? If we take the case of dividing a line at a single point, we can see that two

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[105] *Met.* 1002a34–b5: tr. Apostle (1966).

[106] In another sense, of course, this terminology also refers to the privation in some material subject 'out of which' the change to the opposite and positive state comes to be; cf. *Physics* I.7.

separate points emerge as boundaries for each segment from an indivisible point. But this is absurd because the point is indivisible by its very nature. Similarly, the line is without width and hence cannot be divided along its width to produce two lines as limits for the two parts of a bisected body. The same is true for the plane, since it is without depth. In all of these cases it does not seem that there is any material which can be divided so as to generate two limits where previously there was only one.

If there were any such physical material then the generation by division might be conceivable as a process over time. But a temporal process is precisely what is excluded by the strange fact that the limits of two bodies immediately (ἅμα) become unitary upon the contact of bodies, while the limits of a single body instantaneously become doubled upon division. When the limits of two bodies are put together (συγκειμένων) they become a single boundary for parts of the composite body, with the result that at least one set of limits must immediately perish without going through any temporal process of corruption. Conversely, when a single body is divided (διῃρημένων), there will immediately be other boundaries which did not previously exist. Once again, we cannot explain this in terms of any temporal process such as we associate with the generation of substances; nor can we say that the double set of boundaries is 'generated' by division from what was previously a single boundary.[107]

In concluding these difficulties concerning generation and corruption, Aristotle draws a very revealing comparison between the status of mathematical entities and that of the 'now' in time:

> It is likewise with the moment in time; for neither can this be in the process of generation or destruction, but, not being a substance, it always seems to be distinct. Clearly, it is likewise with points and lines and planes, and for the same reason; for all are alike either limits or divisions.[108]

The whole passage emphasizes the similarity of the two cases with respect to generation and corruption. For instance, παραπλησίως ('likewise') is correlated with ὁμοίως ('likewise' & 'alike'), while αὐτὸς λόγος confirms the similarity of the arguments which can be applied in

[107] Robin (1908) 233 ff. points out the purpose of this difficulty is to show that the 'generation' of points, lines, and planes completely defeats our ordinary intuitions about generation in the case of sensible substances.

[108] *Met.* 1002b5–11: tr. Apostle (1966).

both cases. But the argument concerning the 'now' is rather truncated and demands some reconstruction.

What Aristotle says (1002b7–8) is that the 'now' cannot be undergoing generation or corruption yet still it appears to be always different, though not as some kind of substance (οὐκ οὐσία τις οὖσα). It is not easy to make sense of this as an argument without supplying some hidden premisses or assumptions. For instance, the word γάρ in the first clause suggests that some explanation is forthcoming, but we are not told explicitly why the 'now' cannot be in the process of coming into being or ceasing to be. We are not really helped by the subsequent clause because the introductory words ἀλλ' ὅμως indicate that this is making a new and contrasting statement. Thus, in order to make complete sense of the first clause, we must look back to the previous argument which it imitates. The reason given there as to why mathematical entities could not be undergoing the temporal processes of generation and corruption was the sudden transition from being to non-being (or *vice versa*) for the limits of bodies when those bodies are compounded or divided.

This explanation fits the character of a temporal moment, according to the conception outlined in *Physics* IV.13 (222a10 ff.), where Aristotle says that the 'now' (τὸ νῦν) is a limit of time (πέρας χρόνου) which both divides past from future and connects them. As a potential division between past and future, the present moment is always different but, as the actual connection between them, it is always the same. In fact, at *Physics* 222a14–20 an explicit comparison is drawn between the 'now' and a point in a mathematical line. Just as the point is a potential division of the line, so also the 'now' is a potential division of time and, as such, it is always different. But it is always the same insofar as it is the actual link between past and future, just as is the connecting point between two line segments. Thus, when time is treated as a continuum, there is a clear analogy between the point on a line and a moment in time. Yet they are also disanalogous inasmuch as the point persists after the division of the line, whereas the moment that divides past from future is always changing and different.

In the passage from *Metaphysics* III under discussion, however, I think that Aristotle focuses exclusively upon the analogy between moments and points as potential limits of similar continua. That is why he can extend the argument so easily to lines and planes, which are also potential limits of different but analogous continua. So,

building on the analogy with the 'now' in time, we should be in a better position to reconstruct the details of his truncated argument. When taken as a division between past and future, the 'now' cannot be considered to have undergone a process of generation and corruption because the change from one moment to a different moment is instantaneous. But instantaneous change is incompatible with our ordinary experience of generation and corruption for substances; i.e. a temporal process with some subject remaining the same throughout the change.

Thus it is obvious that the 'now' cannot be some substance because, as a limit between past and future, it always seems to be different and nothing appears to remain from one moment to the next. In fact, the 'now' fails to qualify as a substance on two distinct criteria: (i) the criterion of permanence and (ii) the criterion based on a characteristic process of generation. Aristotle sticks to the second criterion here, possibly because the first would bring out the disanalogy between the 'now' and mathematical entities. I find evidence for this in the words ἀλλ' ὅμως, which make it clear that he is drawing a contrast between the statement that the 'now' appears to be always different and the previous statement that it cannot have undergone a temporal process of generation and corruption. This leaves only the instantaneous transition from being to non-being, which we have previously seen to be characteristic of the limits of divided continua. And this is precisely the basis upon which Aristotle extends the argument to points, lines, or planes, since they are all alike either limits or divisions.[109] While this is sufficient for understanding the dialectical difficulty here, we should keep in mind the analogy between the 'now' and mathematical entities as another of the 'phenomena' to be saved by Aristotle's account.

V. *The difficulties about Forms and Intermediates*

Still following the classical chiastic order, let us return to the aporia about supersensible substances that was listed first:

> Again, are we to say that only sensible substances exist, or besides these also others? And if others also, are they of one kind or are there many genera of substances, such as, for example, the Forms and the

[109] ἅπαντα γὰρ ὁμοίως ἢ πέρατα ἢ διαιρέσεις εἰσίν, *Met.* 1002b10.

> Intermediate Objects as some say, the Intermediate Objects being the subject of the mathematical sciences?[110]

Compared with the original formulation, the only significant difference here is the claim that the so-called Intermediates (τὰ μεταξύ) are studied by the mathematical sciences.[111] This confirms the link between the positing of Intermediates and some version of the argument 'from the sciences' in which mathematics plays a prominent role. But the reason for the distinction between Intermediates and mathematical Forms is that each Form is unique, whereas there are many mathematical objects of the same kind; cf. *Met.* 987b14–18.[112] In the present passage, however, the reason given for the distinction is that Intermediates constitute the specific subject-matter for the mathematical sciences.

It would be very helpful, however, if we could establish the motivation for positing the existence of mathematical Intermediates, as distinct from mathematical Forms. This would also be important for understanding the difficulties which Aristotle brings forward against the existence of such entities. Annas (1975) has claimed that Intermediates were postulated in response to what she calls the 'uniqueness problem,' i.e. when we make a simple arithmetical statement like $2 + 2 = 4$, we cannot be talking about the Form Twoness because this is unique. But to explain why we can't be talking about sensible things or counting them, we must supplement the uniqueness argument with some version of the argument from objectivity as found in *Metaphysics* XIV.3:

> Those who posit separate Numbers, in view of their belief that the axioms are not true of sensible things and also in view of the fact that mathematical statements are true and please the soul, believe that such Numbers exist and exist separately; and similar remarks may be made concerning Mathematical Magnitudes.[113]

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[110] *Met.* 997a34–b3: tr. Apostle (1966).

[111] Comparison with a similar *aporia* in *Metaphysics* XI.1 (1059a38ff.) confirms the function of Intermediates as theoretical objects for mathematical sciences which cannot be about things in this sensible world.

[112] *Metaphysics* III.6 discusses another relevant aporia, whose leading question is why one has to seek another class of things called Forms besides perceptible things and Intermediates. One reason given is that mathematical objects are many of the same kind, just like sensible things, so that their first principles can only be one in kind but not one in number. Thus, according to this argument, if one does not posit Platonic Forms, there will be no substances which are one in number as well as in kind; nor will the first principles of things be determinate in number but only in kind; cf. 1002b22–25.

[113] *Met.* 1090a35–b1: tr. Apostle (1966).

In a footnote to his translation, Apostle identifies these thinkers as Plato, Speusippus, Xenocrates, and some minor Platonists. Although these thinkers reportedly held quite different positions, it is possible that they agreed on the separation of mathematical entities as a condition for truth and objectivity in the sciences.[114] We should notice that within XIV.3 they are being contrasted with the Pythagoreans who did not separate mathematical objects presumably because they saw many of the attributes of number as belonging to sensible bodies, and hence they claimed that all things are essentially constituted by numbers and their relationships; cf. *Met.* 1090a21–23, a29–32.

On the other hand, those Platonists who separated numbers seem to have been convinced that the so-called axioms of mathematics could not be true of sensible things. Since Euclid does not record any specific axioms for arithmetic in *Elements* VII–IX, we should take the term 'axioms' here in the broad sense of basic assumptions or propositions of a science, as in *Posterior Analytics* I.10 where Aristotle says that common principles such as the axioms of equality are assumed in the science of arithmetic.[115] But to understand the argument from objectivity fully, we must make sense of the Platonic claim that axioms such as 'equals from equals leaves equals' cannot be true of sensible things, and for this purpose we can refer to *On Forms* where the argument from relatives is given for the existence of Forms. Part of this argument depends on the claim that mathematical predicates cannot be applied without qualification (ἁπλῶς) to sensible things because the quantity in them is changing continuously and hence is not determinate.[116] Some version of the flux argument is here being used to support the claim that sensible things cannot be truly equal.[117] If Plato accepted this claim then it seems that the only way to save the truth and objectivity of the mathematical sciences is to separate their objects from the flux of sensible things. But to justify the dis-

[114] By contrast, Tarán (1981) thinks that what is here described is a view of mathematical objects unique to Speusippus, since he takes Plato to have posited only Forms and sensible numbers, while Xenocrates is taken to have identified Mathematical Numbers with Form Numbers. But, if that is the case, Aristotle's use of a plural verb is puzzling, unless he is talking about Speusippus and his followers.

[115] Szabo (1978) provides a comprehensive outline of alternative meanings of 'axiom.'

[116] Alexander, *in Metaph.* 83.8–10.

[117] As I noted in chapter 3, both Irwin (1977b) and Fine (1993) interpret the flux argument when applied to sensible things as involving only the compresence of opposites, whereas I follow Bolton (1975) in taking it to involve successive change in time.

tinction between Intermediates and mathematical Forms, one must appeal to the uniqueness problem.[118]

Immediately after his reformulation of the aporia in *Metaphysics* III.2, Aristotle refers to a discussion of Forms which explained how "we" call them causes and substances by themselves.[119] Most commentators justifiably treat this as a reference back to his discussion of Platonism in I.6 & 9.[120] We can hardly accept Jaeger's hypothesis that Aristotle was still a Platonist at the time that Book I was written, since his use of the 'we' locution is common to many passages in Book III. In fact, however, the dialectical character of this book provides a different context for understanding Aristotle's use of the 'we' locution. Given his methodological stance as that of a neutral arbitrator, he cannot commit himself either to a Platonic or to an anti-Platonic position with respect to Forms. However, as an impartial judge reviewing the evidence in an even-handed manner, he may adopt one position or another for the sake of teasing out its inherent difficulties.

In this way I think it is possible to take Aristotle's statement of the Platonic position as the implicit protasis of the following conditional (997b5): If we say the Forms are causes and also separate substances, then many difficulties follow. Although this conditional structure is merely implicit in the Greek, it is clear that Aristotle is referring back to a set of difficulties which arose previously from treating Forms as causes (αἴτια) and also as independent substances (οὐσίας καθ' ἑαυτάς). In his more critical treatment of Platonic Forms in *Metaphysics* I.9, there is a parallel in the group of difficulties introduced as follows:

> Above all, one might go over the difficulties raised by this question: What do the Forms contribute to the eternal things among the sensibles or to those which are generated and destroyed? For, they are not the causes of motion or of any other change in them. And they do not in

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[118] Alexander (*in Metaph.* 196.8 ff.) appears to think that it was Plato's followers who first assigned an intermediate place to Mathematicals between Forms and sensibles as substances in their own right. This may be historically correct, since it is consistent with the fact that Alexander's report of arguments from *On Forms* contains no mention of Intermediates.

[119] ὡς . . . λέγομεν τὰ εἴδη αἴτιά τε καὶ οὐσίας εἶναι καθ' ἑαυτὰς, *Met.* 997b3–4.

[120] See especially *Met.* 987b18–20 & 990b22–29. Ross (1924 i) xv provides further textual evidence for the connection between Books I and III. Presumably in opposition to another tradition, Alexander (*in Metaph.* 196.21) takes this reference to show that Book I is an integral part of the whole *Metaphysics.*

> any way help either towards the knowledge of the other things (for, they are not the substances of them, otherwise they would be in them) or towards their existence (for they are not constituents of the things which share in them).[121]

The use of the verb διαπορεῖν suggests that I.9 reviews the difficulties about Forms, and so Aristotle can refer back to it in Book III as a task already completed. Secondly, the question in the above passage is prompted by the tension between treating Forms as causes and as separate substances. If one assumes (like the Platonists) that Forms are independent substances separate from sensible things, then it makes sense to ask about what they contribute (συμβάλλεται) either to incorruptible or corruptible sensible things.[122]

This seems to be confirmed by Aristotle's negative assessment of Forms as candidates for the role of causes. In the first place, he says, they are not the causes of motion or of any other kind of change in them. Why not? The reason is not given explicitly here but it may lie in his belief that the Platonic Forms are outside place and time, with the result that they cannot be the efficient causes of motion and change for sensible things that are in place and time. One of the defining conditions for such change, as we have seen, is that it be a process over time and a second condition is that there be some contact between the cause and the thing being changed.[123] Aristotle must deny that either of these conditions is fulfilled in the case of Forms, since he dismisses all talk of patterns and participation as empty metaphors; cf. *Met.* I.9, 991a20–22.

For similar reasons he denies Forms any role as formal causes of sensible things and hence he says that Forms do not help us towards scientific knowledge (πρὸς τὴν ἐπιστήμην) of sensible things. His explanation (991a13) is that the Forms do not constitute the substance (οὐσία) of these things, otherwise they would be in them. Thus an essential criterion of substantial form is that it be immanent in the thing of which it is the substance and, in view of Aristotle's assumption that Forms are separate substances, the criterion rules them out as the formal causes of sensible things. Finally, he seems to dismiss

[121] *Met.* 991a8–14: tr. Apostle (1966).

[122] Here, I think, the middle voice of the verb συμβάλλειν is given a very broad meaning to cover the contribution which any of the four causes could make to a sensible thing.

[123] See Fine (1983) for a good analysis of the different criteria for substantial form used by Aristotle in his criticism of Platonic Forms as explanatory causes.

Forms as (material) causes because they are not constituents (ἐνυπάρχοντα) of the things which participate in them (τοῖς μετέχουσιν), and so do not help them with respect to being (εἰς τὸ εἶναι) any more than they help us towards knowledge of sensible things. It is worth noting that none of these arguments has ruled out the possibility that Forms may function as final causes, since this type of cause is not required either to be in the thing caused or even to be in contact with it.[124]

Be that as it may, it is plausible to identify these as some of the difficulties to which Aristotle refers at III.2 when he says that many troubling consequences follow from the view that Forms are causes and separate substances, while singling out one such consequence for special attention:

> There are many difficulties in this doctrine but none is less absurd than to say, on the one hand, that there exist certain natures apart from those in the heavens, and on the other hand, that these are the same as the sensible things except that they are eternal while sensible things are destructible. For they say just this, that there exists Man Himself and Horse Itself and Health Itself, and in this they resemble those who say that there exist Gods but that they are like men. But just as the latter were positing none other than eternal men, so these thinkers are positing the Forms as being none other than eternal sensible things.[125]

The present difficulty emerges from conflicting statements attributed to the Platonists: on the one hand (μέν), 'they' say there are some natures apart from those in the sensible world (ἐν τῷ οὐρανῷ) but, on the other hand (δέ), they say these are the same as sensible things, except that the former are eternal whereas the latter are perishable.[126] As I see it, the underlying purpose of Aristotle's syntactical contrast

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[124] In *Metaphysics* I.7, 988b7–10, Aristotle admits that the Platonists do have the notion of a final cause in a way, but he insists that they do not treat it in the way in which it is naturally a cause; i.e. as the end towards which actions, changes and motions tend. Although he does not clarify the other way in which the Platonists conceive of the final cause, I think that he must have in mind something like a mathematical notion of the Good as an ordering principle in the universe but not as a source of motion.

[125] *Met.* 997b5–12: tr. Apostle (1966).

[126] In *Metaphysics* XI.2 (1060a17–18) the construction (κατασκευάζειν) of eternal substances corresponding to perishable sensibles is criticized as something that appears to go beyond all that is reasonable (ἐκτὸς τῶν εὐλόγων δόξειεν ἂν πίπτειν). The absurdity seems to lie in the duplication of the sensible world by eternal sensibles like Man Itself or Horse Itself, which appear to differ from their perishable counterparts only in being eternal.

is to emphasize the basic differences between sensible perishable things and non-sensible imperishable things which are separated from the sensible world. Indeed, it is essential to the construction of Aristotle's argument that these differences be implied by the first statement attributed to the proponents of Forms: namely, that there are some natures besides (παρά) those things in the sensible universe. Otherwise, when they are also credited with claiming that these natures are the same as sensible things except for being eternal, the absurdity of their position would not be obvious.

Now we can see just how much the whole difficulty depends on the implications which Aristotle draws from the terminology of the Platonists. The grounds for attributing this latter claim to them is that they spoke of the existence of Man Itself, as well as Horse Itself, Health Itself, and other such things. Just like those who treat the gods as 'eternal men' (ἀνθρώπους ἀϊδίους), Aristotle says, those who talk about Forms are doing nothing more than positing eternal sensibles (αἰσθητὰ ἀΐδια). It is a rather unflattering reflection upon the mental acuity of the Platonists to accuse them of falling prey to the anthropomorphism of the early 'theologians,' who were criticised so harshly by Xenophanes (Fr. 14–16 DK). Indeed, many commentators have raised doubts about the justice of such an accusation, given that Plato's dialogues show him to be a highly sophisticated theologian in the Greek sense. Alexander,[127] however, tries to defend Aristotle's interpretation by saying that the addition of 'Itself' shows that sensible things are being posited as eternal entities, and by pointing to the fact that the Platonists hold the definitions of Forms to apply also to sensibles. But, he argues, if that is the case then these must be eternal sensibles because the definition of a horse, for instance, should cover the essential fact that it is capable of sensation and so must have some material embodiment.

I think that Alexander grasps quite clearly the central point at which Aristotle is driving in his construction of this difficulty against the Platonic postulation of separate Forms. The difficulty is being prepared already in *On Forms* where his use of the compound term 'Man-Itself' (αὐτοάνθρωπος) can be seen to facilitate the complaint that the Platonists merely create an eternal duplicate of the perishable sensible thing.[128] Aristotle's basic objection is that, if Forms exist

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[127] Cf. *in Metaph.* 197.10–20.

[128] Cf. *in Metaph.* 84.4, *Met.* 990a34–b8, 1040b30–34, 1060a16–18, 1086b10–11.

apart from sensibles then they must be radically different in kind, just as gods are completely different from men. Hence the Platonists are simply confused when they postulate separated Forms for natural things like man and horse because the definitions of these things must contain at least an implicit reference to their material embodiment as living things. But the same definition must cover both the Form and what participates in it, otherwise scientific knowledge of sensible things would not be possible. Therefore, Aristotle surmises that the Platonists feel compelled to separate Forms as eternal objects of knowledge, while also allowing them many of the characteristics of sensible things. This is probably why he compares them with those early thinkers who adopted anthropomorphic views of the gods.

It is not part of my project here to adjudicate on whether or not such a comparison is fair to Plato, but I am interested in what light it throws on Aristotle's concept of separation,[129] which is very much in evidence later in his discussion of the ontological status of mathematical objects. For him the separation of Forms or Mathematicals obviously implies that they are independent substances, which are numerically different from sensible things, and which do not have any clear causal relation to things in the sensible world. This is the root of the difficulty about treating Forms (or Mathematicals) as causes and also as separate substances. Of course, a similar difficulty should arise for Aristotle's own prime mover, but that is not a problem that I need to discuss until my sixth chapter.

In his review of difficulties associated with this aporia in *Metaphysics* III.2, Aristotle gives very little space to those which follow from the postulation of Forms. If one is to judge by his references, he seems to think that these difficulties have already been adequately dealt with, and so he merely selects a sample of the most absurd consequences flowing from the separation of Forms. As a result, he quickly moves on to consider the peculiar difficulties that arise from positing Intermediates:

> Moreover, if besides the Forms and the sensible things one posits the Intermediate Objects, he will be faced with many difficulties. For clearly, just as there will be Lines besides Lines Themselves and the sensible lines, so the situation with each of the other genera will be similar. Accordingly, since astronomy is one of the mathematical sciences, there will be also a Heaven besides the sensible heaven, and similarly a Sun

[129] Cf. Fine (1984), Morrison (1985a–b).

> and a Moon and the other heavenly objects. Yet how are we to be convinced of all this? It is not reasonable that this Heaven among the Intermediates should be immovable, and it is utterly impossible that it should be in motion.[130]

In this context we should recall the previous description of Intermediates as the objects with which the mathematical sciences concern themselves. Against this background, it is plausible to assume that some version of the argument from objectivity is being used to resolve the uniqueness problem by positing intermediate objects of mathematics which are many alike, though still eternal and unchangable; cf. *Met.* 987b14–18. This gives some sense to his statement that, according to the same argument, there will be (mathematical) lines between lines-in-themselves and sensible lines, just as there will also be intermediates for the other genera.

Presumably Aristotle means that the lines dealt with by the science of geometry must be intermediate between Form lines and sensible lines; and similarly for geometrical planes and solids. Here the referent of 'kinds' (γένη) is ambiguous in that it may be taken to mean other kinds of mathematical objects, like planes and solids, or it may refer more widely to all the subject-genera of the mathematical sciences which would include astronomy. The latter interpretation is very tempting because it would strengthen the continuity of his argument without changing it substantially. Thus, in the subsequent line, the word τούτων could refer back to γενῶν and the argument would run on in the following way: since astronomy is one of these (genera), there will also be some heaven besides the sensible heaven. But, Aristotle exclaims, how are we to believe these things? This rhetorical question introduces what he sees as an impossible dilemma for Platonic astronomers. On the one hand, it is not reasonable for the heavens to be immovable (ἀκίνητον) because, as Alexander (198.10–11) explains, it belong to the nature of heavenly bodies to have a certain kind of motion. On the other hand, it is completely impossible for the ideal heaven to be moved (κινούμενον); cf. 997b19–20.[131] By way of explaining the second horn of the dilemma, Alexander says it is impossible for something to be in motion which is not material and by its own nature sensible.[132]

[130] *Met.* 997b12–20: tr. Apostle (1966).

[131] The dilemma is what modern scholars call a two-level paradox; cf. Vlastos (1973b).

[132] ἀδύνατον γὰρ κινεῖσθαι τὸ μὴ ὂν ἔνυλον καὶ τῇ αὑτοῦ φύσει αἰσθητόν, *in Metaph.* 198.13–14.

Thus, according to Aristotle, the mathematical astronomer who posits an ideal heaven (whether it be an Intermediate or a Form) is in a double bind. As a mathematician, he must regard such a heaven as eternal and immobile; whereas, as an astronomer, he must also study the apparent motions of the heavenly bodies. Thus it would appear that the two roles lead to conflict and that a mathematical astronomy such as Plato recommends in the *Republic* (529C–D) is impossible. The difficulty is more acute for 'mixed' sciences like astronomy, of course, because they seem to study sensible objects like the sun and the moon. The source of the difficulty appears to be that the Platonic conditions for scientific objectivity demand an ideal object which is quite separate from the sensible heavenly bodies that astromomers themselves treat as objects of their inquiry.

This also seems to be what Aristotle has in mind when he insists that the same difficulty arises for other applied mathematical sciences, such as optics and harmonics:

> Concerning the objects discussed by optics and mathematical harmonics, the case is similar; for they, too, cannot exist apart from the sensible objects for the same *reasons;* for if there exist Intermediate sensible objects and sensations, clearly there will also exist Animals between Animals Themselves and destructible animals.[133]

The 'reasons' (αἰτίαι) mentioned here seem to depend on the previous objections against positing Intermediates apart from sensible things. Since the only objection specifically directed against Intermediates rests upon astronomy, this buttresses the contextual evidence for assuming that Aristotle is talking about intermediate astronomical objects. However, I think the objection works equally well against any ideal object that is separated from the realm of change and motion, especially in the case of the applied mathematical sciences.

Thus it is the *separation* of the objects of optics and harmonics, rather than the fact of their being Intermediates, which is the real target of the objection. This is confirmed by the first part of the above passage where Aristotle says that the objects of optics and harmonics cannot exist apart from sensible things (παρὰ τὰ αἰσθητά) for the same reasons. The second part contains an additional objection that is not fully elaborated but which depends on the issue of separation. This objection is formulated in the following conditional manner: if there

[133] *Met.* 997b20–24: tr. Apostle (1966).

are intermediate sensibles (αἰσθητὰ μεταξύ) and sensations (αἰσθήσεις), then it is clear that there will be intermediate animals between Animals Themselves and perishable animals. Aristotle clearly thinks that the *apodosis* contains an absurd consequence that undermines the position of those who posit such intermediate objects of science.[134] It is not clear, however, why they should be committed to the statement contained in the *protasis*. Why should the Platonists posit such absurd entities as intermediate sensibles (or sensations), when the whole purpose of separating objects of science was to remove them from the flux of sensible things?

Here there is a considerable lacuna in Aristotle's argument which Alexander, in his own inimitable fashion, tries to fill. First, he carefully distinguishes between empirical and mathematical harmonics, presumably because this is a distinction that is well-attested in Platonic dialogues like the *Republic* and the *Philebus*. While empirical harmonics deals with sensible things, insofar as it is concerned with the fitting together of sounds, mathematical harmonics is concerned rather with proving that each concord consists in some ratio of numbers.[135] The same distinction presumably could be made for the applied science of optics which is concerned both with concrete seeing and also with the geometry of vision. Thus, as Alexander (*in Metaph.* 198.21–23) explains, insofar as these sciences have a mathematical content, their objects will be mathematical substances that are intermediate between Forms and sensibles. At this point, the difficulty which Aristotle wants to raise becomes a little clearer, since mathematical objects are not by their nature sensible. Yet the applied sciences of optics and harmonics are essentially concerned with the senses of seeing and hearing. That is why Aristotle is able to justify the assumption (in the *protasis* of the conditional) that there must be intermediate sensibles (and corresponding sensations) as objects of inquiry for the mathematical sciences of optics and harmonics. Given that sensations belong essentially to animals, this assumption leads to the even more absurd conclusion that there are intermediate animals between ideal and corruptible animals.[136]

[134] In *Metaphysics* XI.1 (1059b3 ff.), with reference to intermediate mathematical objects, one difficulty raised is that the Platonists give no good reason for refusing to posit similar Intermediates for other kinds like man and horse. There seems to be a clear parallel with the present difficulty.

[135] ἡ δεικνῦσα ἐν τίνι λόγῳ ἀριθμῶν ἐστιν ἑκάστη συμφωνία, *in Metaph.* 198.20.

[136] Syrianus (*in Metaph.* 24.35–25.29) does not seem to regard this conclusion as

The fact that the same argument is recalled in Aristotle's explicit *elenchus* at *Metaphysics* XIII.2 seems to indicate that one of the primary purposes of a review of difficulties is to refute his opponents. But this does not fit very well with his declared purpose at III.1 of exploring the problem by giving an even-handed review of difficulties on both sides of the question. In order to eliminate such apparent inconsistency, I will argue in my next chapter that the *elenchus* represents a further stage in Aristotle's dialectical process, which is subsequent to the review of difficulties rather than being simultaneous with it. In *Metaphysics* III there is a clear distinction between these two stages in the dialectical process, since Aristotle does not press his objections to the point where his opponents would be explicitly refuted.[137] Indeed, that would contravene the methodological principle laid down at the beginning; namely, that one must first go through the difficulties and identify the problem, just as an arbitrator would sift through the conflicting evidence on both sides of a lawsuit. If Aristotle had already taken sides then he would have broken the fundamental rule of fair play which a good judge must follow.

But I am inclined to give him the benefit of the doubt, in view of the evidence that he made a conscious effort to follow his own methodological maxims. For instance, in a subsequent passage, he raises another difficulty about the applied sciences, again without pushing for a refutation:

> One might also raise this question: What kinds of things should be sought by these sciences? If geometry is to differ from geodesy only in this, that the latter is concerned with things we sense but the former with non-sensible things, clearly in the case of the medical science, too, there will be a science between Medical Science Itself and medical science which is concerned with sensible health, and similarly with each of the other sciences. Yet how is this possible? For there will have to be also Healthy Objects besides the sensible healthy objects and Health Itself.[138]

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absurd but he may be thinking of some Neoplatonist schema of emanation which is quite foreign to Plato.

[137] With reference to Aristotle's method in ethics, Barnes (1980) finds three clearly marked stages: (i) setting down (τιθέναι) the endoxa; (ii) puzzling through (διαπορεῖν) the difficulties; (iii) proving (δεικνύναι). The difference between the second and third stages roughly corresponds to the distinction I am making here, if we take the *elenchus* to be a form of proof.

[138] *Met.* 997b25–32: tr. Apostle (1966).

The emphatic position of the verb ἀπορήσειε in this passage suggests that Aristotle simply intends to compound the impasse or aporia about the objects of the mathematical sciences. In this particular case, I think that he has the applied sciences specifically in mind when he asks: Concerning what kinds of beings should these sciences inquire?[139] According to Alexander (198.33 ff.), Aristotle is asking about which of the sensibles have corresponding Intermediates with which the mathematical sciences are concerned. In other words, which kinds of sensibles have Intermediates corresponding to them and which do not? Thus, by extending the question to the applied sciences, he is going beyond the intentions of those who posited intermediate mathematical objects; cf. *Met.* 1090a25–28.

It is quite likely that the Platonists based their argument for the separation of mathematical objects upon such 'pure' sciences as arithmetic and geometry; cf. *Phil.* 55–59. Thus they may have said that the corresponding applied sciences, such as harmonics and optics, deal with sensible things. In fact, the present difficulty does not arise for their position unless they have either explicitly or implicitly committed themselves to some such statement. The difficulty is developed through the extrapolation of that statement to the applied science of medicine so as to generate an intermediate science of medicine with its own intermediate objects. Since the structure of the whole argument is that of a conditional, it is clear that it will be successful against the Platonists only if the *protasis* truly represents their position. What the *protasis* assumes is that the only difference between land-surveying (γεωδαισία) and geometry (γεωμετρία) is that the former is concerned with things we perceive (αἰσθανόμεθα), while the latter deals with non-perceptible things. Such an assumption appears to be plausible in the light of what Plato says about the crafts in the *Philebus* (56Aff.). For instance, it would seem that he distinguishes (57B5–7) between empirical crafts like measurement (μετρητική) and 'philosophical' sciences like geometry on the grounds that they deal with different things and hence possess different degrees of clarity or precision. However, except for some obscure hints in the *Republic,* Plato does not talk about the objects of the mathematical sciences being intermediate between Forms and sensible things.

This is important to keep in mind, especially with regard to the present difficulty where Aristotle extrapolates from the mathematics

[139] περὶ ποῖα τῶν ὄντων δεῖ ζητεῖν ταύτας τὰς ἐπιστήμας, *Met.* 997b25–26.

to medicine as follows: If mensuration differs from geometry only in being about a sensible object then it is clear that there will be an intermediate science of medicine between Medicine Itself and the medicine which is concerned with sensible health. But, as it stands, this argument seems to be a complete *non sequitur*. If the analogy between mathematics and medicine holds (which seems to be supported in *On Forms*), the only thing that seems to follow is that the empirical science of medicine will have a different object from the theoretical science of medicine. There are no clear grounds for positing an intermediate science of medicine, unless the uniqueness problem arises also for medicine. In his commentary on this passage, Alexander (198.33 ff.) defends the analogy between mathematics and medicine without giving any good reason to believe that it holds. He simply asserts (199.6–11) that there will be a kind of medicine analogous to geometry, which is intermediate between the science that deals with Health-Itself and that which deals with health in sensible things.

Since the conclusion that there is an intermediate science of medicine would probably seem absurd even to the Platonists, the grounds for asserting it can hardly be that they are explicitly committed to it. Thus the argument must rest on the claim that they are implicitly committed to this result if they posit Intermediates as objects of mathematics. In his acute commentary on this book, Aquinas confirms that this is the basis for the argument but the only justification which he offers is that "the intelligible structure of all sensible things and Forms seems to be the same."[140] But I find this explanation unsatisfactory because it assumes what has to be shown; i.e. that there is an exact analogy between mathematics and medicine. Yet perhaps some light can be thrown on this matter by the third version of the argument 'from the sciences' in *On Forms*, where geometry and medicine are used as parallel sciences to show that Forms exist.

If those arguments are genuinely Platonic in origin, then Aristotle might also feel justified in extrapolating from mathematics to medicine with respect to the argument for Intermediates. I think that he is here pushing the argument 'from the sciences' beyond its normal range of application in order to generate the difficulty about intermediate healthy objects between Health Itself and sensible healthy

[140] Cum de omnibus speciebus et omnibus sensibilibus videatur esse similis ratio, *in Metaph.* III, L.7, c. 413.

things. In the above passage, the absurdity of such a conclusion is underlined by his rhetorical question: Yet how is this possible? When it is taken along with the subsequent statement about intermediate healthy things, this question seems equivalent to an assertion that the existence of such entities is absurd and impossible. Since medicine and mathematics are assumed to be parallel sciences, the implication of this conclusion is that it is just as absurd to posit intermediate mathematical objects. It should be noted, however, that Aristotle refrains from drawing this conclusion, which is consistent with his purpose here of raising difficulties rather than of refuting opponents.

If this is to be a genuine aporia, however, there should be arguments on both sides. In a subsequent passage, he lists difficulties that indirectly support the Platonic separation of objects of science:

> At the same time, it is not even true that geodesy is concerned with sensible and destructible magnitudes; for it would be destroyed if these are destroyed. Nor would astronomy be concerned with sensible magnitudes or with this heaven. For neither are sensible lines such as the geometrician speaks of (for no sensible thing is straight or round in the sense in which he uses the terms "straight" and "round"; for the circle touches the ruler not at a point, but in the way in which Protagoras used to say in refuting the geometricians), nor are the motions and orbits of the heavenly bodies similar to those discussed by astronomy, nor do points have the same nature as the stars.[141]

Against the view that applied sciences like mensuration are about sensible things, he argues that it involves the destruction of the science of mensuration when its objects of study perish. But no art or science perishes along with individual sensibles, because it is really about universals which are eternal and it deals with particulars only in so far as they belong to some kind which is common. Thus surveying is not about sensible magnitudes and cannot be distinguished from geometry on that basis, since they are both about intelligibles.

In his commentary on the passage, Aquinas treats this difficulty as being directed against the thesis that there are Intermediates.[142] Then he proceeds to list the subsequent objection concerning astronomy under the so-called antithesis, as if it were a difficulty that supports the Platonic position. Although Aquinas never appeals directly to the

[141] *Met.* 997b32–998a6: tr. Apostle (1966).
[142] Cf. *in Metaph.* III. L.7, c. 414–415 & c. 416.

Greek, he could have cited the fact that the second objection is introduced by the particles ἀλλὰ μὴν, which often have a strong adversative sense when combined in this way.[143] Thus, in their editions of the text of the *Metaphysics*, both Ross and Jaeger indicate a sharp transition at this point. But, in spite of this evidence, I think that we cannot ignore the parallels in structure and content between the first and second objections in the above passage. The first objection begins with the words ἅμα δὲ οὐδέ which can be seen to parallel the words ἀλλὰ μὴν οὐδέ that introduce the second. Just as the first denies that the applied science of mensuration is about sensible magnitudes, so also the second objection denies that astronomy is about such magnitudes.

But here the parallel begins to break down because the same reasons cannot be given for the denial in both cases.[144] In the case of mensuration, the reason given (997b33–34) as to why it cannot be about sensible magnitudes is that these are corruptible (φθαρτῶν); so that when these actually cease to exist, the science itself would also be destroyed. Yet Aristotle can hardly apply the same argument to the sensible bodies in the heavens, since none of the Greeks (including himself) believed them to be corruptible. But in the second objection he does not spell out the reasons why astronomy cannot be about 'this world (περὶ τὸν οὐρανὸν . . . τόνδε), though at the end of the passage he does provide a clue when he says (998a4–5) that the motions of the heavens are not such as the science of astronomy describes in its accounts. Presumably he means that the apparent orbits of the planets do not correspond with the perfect circles postulated by Greek astronomers like Eudoxus and Callippus. In the same way, when he denies that points have the same nature as the stars, he must have in mind some conventional practice among these astronomers of treating the stars *as if* they were points in a geometrical construction.[145]

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[143] Cf. Denniston (1934) 341–7.

[144] Alexander (*in Metaph.* 200.5 ff.) notes the breakdown in the parallel and hastens to explain that, because the heavens are imperishable, Aristotle makes a separate argument for astronomy not being about the sensible heavens, since he could no longer argue as in the case of mensuration that the science would perish when its objects perished.

[145] Alexander (200.22 ff.) thinks that the circular or spiral motion of the heavenly bodies is not such as the astronomer assumes, since the spiral or circular lines that he posits are lengths without breadth; whereas there is no such thing among sensibles. This is consistent with the objection against assuming that stars are geometrical

In this light we can see the point of the parallel between the astronomer and the geometer, which was introduced earlier in the passage. Even though he may appeal to a sensible diagram, the geometer cannot be talking about sensible lines because they do not correspond to his definition of 'straight' and 'round.' For instance, as Protagoras had pointed out, the contact of a sensible circle with a ruler does not correspond with the definition of a tangent. Here Aristotle seems to accept for the sake of argument that Protagoras is correctly describing the situation in the case of sensible lines and circles. But, as I have already argued, the postulation of ideal mathematical entities can be taken as Plato's response to the challenge of Protagoras, and so the whole set of difficulties outlined in the above passage may be placed under the antithesis, which indirectly supports Platonism in mathematics. Even the ambiguous first objection, for instance, is motivated by the fundamental assumption that the empirical science of mensuration must have a stable and indestructible object. This is clearly analogous to the Platonic assumption that one of the conditions of any science worthy of the name is that it be about an eternal paradigm. Hence, despite textual evidence to the contrary, I think that Aquinas is correct to read this objection as a possible proof for the thesis that all sciences are about either Intermediates or Forms.[146]

Therefore the net result of this latter set of difficulties is to deepen the impasse rather than to refute one side or the other. Although neither Plato nor Aristotle accept the Protagorean objection as refuting the geometer, still it raises an issue about the truth of the mathematical sciences which is especially pressing in the case of applied sciences that appear to deal with sensible objects. If we are to judge from the dialogues and from Aristotle's reports, it seems that Plato tried to guarantee the truth of mathematics by positing the existence of separate intelligible objects which conform exactly to the definitions given by mathematicians.

However, as Aristotle pointed out in previous passages from *Metaphysics* III.2, those who posit such separated objects (whether these be Forms or Intermediates) face a number of difficulties. In the case of astronomy, for instance, they will have to say that the scientist

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points, and so it appears to depend on the distinction between mathematics and physics.

[146] .... vel sint de rebus mediis, vel sint de speciebus, *in Metaph.* III. L.7, c. 415.

must study an ideal mathematical object that is immobile by virtue of being mathematical. On the other hand, the subject-matter of the science of astronomy is usually understood to be the heavenly bodies which are mobile by their very natures, as even Aristotle himself assumed. Furthermore, applied mathematical sciences like optics and harmonics seem to have an integral connection with the senses, but the Platonists cannot accept this without positing such absurdities as intermediate sensations and intermediate animals to which they belong. When the difficulties on both sides are thus multiplied, the mind is plunged into a state of complete puzzlement about what kind of objects are being considered by these sciences. Yet, as Aristotle's remarks at the beginning of Book III indicate, this is a necessary stage in philosophical inquiry. Indeed such puzzlement may be essential to the wonderment (θαυμάζειν) that is the starting-point of philosophy, since wonder results from encountering difficulties; cf. *Met.* I.2.

Having discussed the difficulties encountered by those who posit intermediate mathematical objects as *separate* from sensible things, Aristotle adds other difficulties which arise from the peculiar view that these so-called Intermediates are *in* sensibles:

> There are some thinkers who say that the so-called Intermediate Objects between the Forms and the sensible things do exist, but they exist in the sensible things and not apart from them. It would take too long to go through all the impossible consequences of this doctrine, but it is enough to consider even such as the following: It is not reasonable that this should be so only in the case of the Intermediate Objects, but clearly it would be possible also for the Forms to be in the sensible things; for both come under the same argument. Moreover, two solids will necessarily be in the same place, and the Intermediate Objects will not be immovable while they are in moving sensible things. In general, for what purpose would one posit the Intermediate Objects to exist, but to exist in sensible things? The same absurdities will result as those previously mentioned; for there will be a Heaven besides this heaven, except that it will not exist separately but in the same place, and this is indeed more impossible.[147]

From the manner of its introduction, this would appear to be a view espoused by some people, perhaps to resolve difficulties arising from the separation of mathematical objects. Such thinkers denied that Intermediates are separate from sensibles (χωρίς . . . τῶν αἰσθητῶν) and

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[147] *Met.* 998a7–19: tr. Apostle (1966).

instead asserted that they are in them (ἐν τούτοις). If this view were held by some thinkers within the Academy, then a brief review of the difficulties associated with it forms a natural progression from the previous review. So Aristotle is simultaneously covering all the logical possibilities for the postulation of independent mathematical objects, while reviewing the opinions of his predecessors.

I will argue in my next chapter that such a complete review of the logical possibilities constitutes a preliminary stage in his method of dialectical refutation. In fact, in his refutation of opponents at *Metaphysics* XIII.2, Aristotle explicitly refers back to the difficulties associated with this particular position in III. Furthermore, the view that mathematical objects are *in* sensible things has an intrinsic bearing on Aristotle's own position, which has not yet emerged but which is being prepared through this exploration of difficulties associated with previous views. In his commentary on this passage, therefore, Alexander takes great pains to distinguish this opinion from the view that mathematical objects are grasped through abstraction.[148]

The only possible justification for Alexander's introduction of what he regards as Aristotle's own position is that it may help to clarify the view of those who say that mathematical objects are *in* sensible things. If one conjectures that the latter view was propounded by Eudoxus, then the need to differentiate Aristotle's position as distinct and independent becomes more pressing. Alexander (201.2–4) claims that the thinkers who hold the view being criticized give to the Mathematicals their own peculiar nature that is different from and beside (παρά) that of the sensibles, while insisting that they are *in* sensible things. By contrast, he thinks (201.4–8) that those who conceive of mathematical objects as abstractions merely introduce a logical separation (τῷ λόγῳ . . . χωρίσαντες) of some aspects of sensible things, while allowing these aspects to be completely sensible together with the whole.

Aristotle himself speaks about the many difficulties which arise for the view in question, though he gives only one representative sample of such impossible consequences (συμβαίνοντα ἀδύνατα) presumably because it would take too long to review them all. Thus he thinks it sufficient to consider the following difficulty (998a11–13): If the Intermediates can be *in* sensible things then it is reasonable that Forms

[148] ἡ δόξα αὕτη τῆς λεγούσης ἐξ ἀφαιρέσεως τὰ μαθηματικὰ λαμβάνεσθαί τε καὶ νοεῖσθαι, *in Metaph.* 201.1–2; cf. also 199.20 ff.

should be *in* them also, since the same argument applies to both kinds of entity. Obviously, this objection is *ad hominem* to the extent that it only works against those people who deny that the Forms are *in* sensible things, while asserting that the so-called Intermediates are *in* them.[149] But, as Alexander (201.11–21) points out, this position might be linked with Eudoxus who is mentioned at *Metaphysics* I.9 and XIII.5 in connection with a theory of mixture (μεμιγμένον); cf. 991a8–19 & 1079b12–23. In fact, these two passages are exact doublets which both begin with the question of what the Forms contribute (συμβάλλεται) either to eternal sensibles or to destructible sensibles.

As we have already seen, this is a characteristic aporia which arises for those who separate the Forms from sensible things. When they are thus separated, Aristotle objects, they cannot be the causes of motion and change in sensibles nor can they help towards knowledge of these sensibles because they are not the substance of the latter, given that they are not in them (ἐν τούτοις); cf. 991a13 & 1079b16–17. Furthermore, the Forms cannot help towards the being (εἰς τὸ εἶναι) of sensibles, since they are not internal constituents in the things which participate in them. The passage now goes on to concede that if forms were to be posited as constituents in sensible things, they might perhaps qualify as causes. For instance, they may be causes in the way in which whiteness (τὸ λευκόν) is a cause when it has been mixed with the white thing (μεμιγμένον τῷ λευκῷ), as Anaxagoras and Eudoxus argued in turn. What these duplicate passages tend to suggest, therefore, is that Eudoxus may have been trying to avoid the difficulties arising out of the separation of Forms when he posited them as constituents blended with sensible things. If that is the case, however, then Aristotle's *ad hominem* objection at III.2 can be seen to fail against Eudoxus because, so far as we know, the latter postulated *both* Forms and Mathematicals as being in sensible things. But perhaps Aristotle is using Eudoxus to show that it is not reasonable for others to posit Intermediates as being in sensible things, without also accepting that Forms are in them, according to the same argument.

The subsequent difficulties, which are mentioned briefly in the rest of the passage quoted above, depend on the assumption that Intermediates are being posited as independent substances *in* sensible things.

[149] Arthur Madigan (1992) has offered the plausible suggestion that this doctrine reflects a later Platonist attempt to mediate between Forms and sensible things.

Only in the light of such an assumption can one make sense of Aristotle's claim (998a13–14) that, as a consequence of this position, there must be two solids *in* the same place (ἐν τῷ αὐτῷ τόπῳ). This claim is not explained further nor is the difficulty elucidated, presumably because he thinks that the consequence is patently absurd. The two solids in question must be the mathematical and the sensible, each of which is thought of as competing for the same place. If each of them is regarded as having its own peculiar nature (κατ' οἰκείαν φύσιν), as Alexander (*in Metaph.* 201.23–24) explains, then there will be a body going right through a body.

What Alexander fails to notice, however, is that the difficulty trades upon a familiar ambiguity between the words στερεόν and σῶμα. Even if the science of stereometry is concerned with the mathematical solid, it is not clear that this can be regarded as a body of the same kind as the sensible solid. Neither is it clear that the solids studied by the geometer will be competing for space with sensible bodies, even though these so-called Intermediates are posited as being *in* sensibles. Perhaps the only way to clarify the objection, therefore, is to take it that the postulation of Intermediates as independent substances implies for Aristotle that mathematical solids, for instance, are *in* some place just like sensible bodies. We should notice that such an implication contravenes the explicit Platonic view that intelligible objects like Forms (and perhaps Mathematicals) do not occupy a place as do sensible things. Hence those people who postulate Intermediates as being *in* sensible things are breaking with strict Platonism, whether they realize it or not.[150] While they may be responding to difficulties which arise out of the separation of Forms and Mathematicals, they generate other difficulties by insisting upon treating mathematical objects as independent substances.

Another difficulty (998a14–15) is that Mathematicals cannot be unchangable (ἀκίνητα) if they are real entities in changing sensible things. This difficulty is not elucidated further by Aristotle but it would seem to depend on the assumption that entities which are really *in* sensible things are themselves subject to change, either essentially or accidentally. This is related to the previous assumption that Mathematicals occupy some place, just like sensible bodies. As Alexander

[150] Cf. Konstan (1988) for a good discussion of how later Hellenistic thinkers found ways of breaking this Academic impasse about two bodies being in the same place.

(*in Metaph.* 201.24–29) points out, the Mathematicals will not be moved accidentally but rather in the manner of those things whose very nature it is to occupy some place. More specifically, the Intermediates will not be moved by some internal impulse but by external force. Insofar as they have magnitude and extension, Aristotle seems to think that they will be subject to change through the force of an external mover just like inanimate physical things. Here the lack of a clear distinction between mathematical and physical entities suggests that some Pythagorean thinkers may have been the target for these objections against positing Intermediates as being in sensibles.

Whether or not this is the case, it is clear that Aristotle does not think much of this expedient for holding on to mathematical objects as independent substances, since at *Metaphysics* XIII.2 he calls it a 'concoction' (πλασματία). But I will not comment further on his dismissive attitude until the next chapter where I will be dealing with this passage as part of the refutation of opponents. Such an elenctic step in his dialectical procedure represents a new stage that goes beyond his express purpose here of reviewing the difficulties. Thus at the end of III.2 he is content to summarize the previous review of difficulties about Intermediates with these aporetic questions: "To what purpose should one posit Intermediates to exist and (even more so) to what purpose would they be posited in sensible things?"[151] These questions appear to make the point that the postulation of Intermediates serves no purpose and also leads to difficulties. For instance, they cannot function as the objects of knowledge unless they are separated from sensibles but, as a result of the separation, they cannot give us knowledge of this world because they are not the substances of sensible things when they are not in them.

On the other hand, if Intermediates are not separate from but are *in* sensibles, there is a multitude of difficulties arising from their mode of being as independent substances *in* sensible things. For instance, he says (998a17–19), there will be a heaven alongside this heaven (παρὰ τὸν οὐρανόν), not as separate (οὐ χωρίς) but as being *in* the same place. From Aristotle's point of view, this latter consequence of positing Intermediates *in* sensibles is even more absurd and impossible (ἀδυνατώτερον) than the consequences of separating these mathematical objects from sensible things. It would appear that he has some sympathy for

[151] ὅλως δὲ τίνος ἕνεκ' ἄν τις θείη εἶναι μὲν αὐτά, εἶναι δ' ἐν τοῖς αἰσθητοῖς; *Met.* 998a15–16.

those who separated Mathematicals in response to the demands of the mathematical sciences with respect to truth and accuracy. Such a project has a clear purpose, even though it is fraught with difficulties. But he sees no point in positing independent substances while placing them *in* sensible things, since this latter move brings on additional difficulties.

VI. *Aporetic conclusions*

By following a particular set of aporiae through *Metaphysics* III, I have tried to establish that the central purpose of this fascinating book is to raise difficulties on both sides of a question without coming down on one side or the other. This needed to be shown in detail because the aporetic character of Aristotle's method has been neglected by modern commentators and, even when it has been noticed, it has been misunderstood even by the ancient and medieval commentators. For instance, Aquinas claims that "the Philosopher . . . first proceeds in a disputatious manner, showing those things which are doubtful from the point of view of the truth of things."[152] Despite his conviction that Aristotle already has the truth, Aquinas is quite sensitive to the dialectical procedure of *Metaphysics* III. For example, while considering Aristotle's disputatious treatment of the aporiae, he concedes (*in Metaph.* III, L.4, c. 371) that 'the Philosopher' not only uses probable arguments but sometimes ones that are sophistical, especially when he gives arguments that are credited to others.

But Aquinas finds it hard to accept that such a great thinker would use trifling and insignificant arguments about important matters and so, wherever Aristotle's argument looks superficial, Aquinas searches for a deeper meaning. While I am sympathetic to such a principle of charity, I think that Aquinas takes it too far in the direction of making Aristotle into a systematic thinker. Witness his initial attempt to explain why all of the problems are gathered together in a single book of the *Metaphysics,* whereas in other works the problems are set down one at a time so as to establish the truth about each one. The reason for this, according to Aquinas (*in Metaph.* III, L.1, c. 343), is that the other sciences deal with the truth in a particularized way and

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[152] primo procedit mode disputativo, ostendens ea quae sunt dubitabilia circa rerum veritatem, *in Metaph.* III, L.1, c. 338

therefore it is appropriate for them to raise particular problems about individual truths. By contrast, he claims, it is proper for the science of metaphysics to discuss all of the problems pertaining to the truth, since it is the task of this science to deal with the truth in a universal manner. But even Aquinas himself was aware of the conjectural nature of this explanation, since he goes on (c. 344–345) to offer two other possible reasons for collecting aporiae in *Metaphysics* III. For instance, he reports an explanation given by Averroes (*in III Metaph.,* com. 1) in terms of the relationship of metaphysics to logic, as exemplified in *Metaphysics* IV. On account of this integral relation, Averroes thinks that Aristotle made dialectical discussion (dialecticam disputationem) a principal part of the science of metaphysics.

What these conjectural explanations show is that there was no clear consensus among medieval commentators concerning the function of the aporiae in Aristotle's science of metaphysics. I think that this was due mainly to their refusal to take seriously the possibility that Aristotle might be a genuinely aporetic thinker. In the case of Aquinas, for instance, it is clear that he regards *Metaphysics* III as containing merely a dialectical procedure for establishing what is already in Aristotle's mind. This is evident from the second conjectural explanation which Aquinas offers (c. 344) for the collection of problems; namely, that those problems are mainly those about which philosophers have held different opinions. He goes on to explain that Aristotle does not proceed to investigate the truth in the same order as the other philosophers. Whereas 'the Philosopher' begins from sensible and evident things in order to move towards separated things, as is clear from *Metaphysics* VII; the other philosophers wish to apply intelligible and abstract things (or principles) to sensible things. Hence, Aquinas claims, Aristotle decided to give all of these problems first in a separate section and, subsequently, to solve the problems in their proper order. He does this (according to Aquinas) because he does not want to establish the truth in the same order as that followed by the other philosophers from whose opinions the problems arise.

Thus the general impression created by Aquinas' explanation is that Aristotle gathered all of these problems together in one book because they do not really fit into his own order of inquiry; i.e. starting from sensible things and moving to the supersensible. But this makes nonsense of everything that Aristotle says at the beginning of *Metaphysics* III about the necessity of first getting to know a problem and its cluster of difficulties before going on to propose a solution.

Aquinas duly notes what Aristotle says, and then proposes two outlandish explanations by which he shows that he has not taken the words seriously. A possible reason for this blind spot on the part of Aquinas is his obvious assumption that Aristotle is a systematic thinker who establishes a clear order of inquiry in the *Metaphysics,* starting with sensibles and moving to supersensibles, just as his theory of knowledge dictates. Since Aquinas takes most of the aporiae of Book III to arise from an opposite order of inquiry, he simply assumes that they do not fit into Aristotle's systematic framework, and so he treats it as a source book where diverse problems are collected rather than as an integral part of the science of metaphysics.

By contrast, I have argued that the review of difficulties associated with each aporia is an essential first step for Aristotle in the treatment of these fundamental metaphysical problems. Since these are problems about 'first things' (ἀρχαί), there is no other way of approaching them except through the opinions of the many and the wise who have previously struggled with the puzzles concerning substance and being. According to the *Topics,* it is the characteristic task of dialectic to make such a review of *doxa* in order to reach the principles of the 'philosophical sciences.' Even though the science of metaphysics may not have been within the purview of that early work, it is arguably covered by such a description. Furthermore, the necessity of starting with common opinions (i.e. what is more familiar to us) if we are to reach first principles (i.e. what is more familiar by nature) is consistent with Aristotle's methodological remarks in the *Posterior Analytics* and elsewhere. According to him, it is impossible to demonstrate the first principles within any of the special sciences and thus some other way of making them known must be available, since the 'firsts' are not inaccessible to our knowledge.

But, apart from calling this other way 'induction' (ἐπαγωγή), Aristotle gives no clear and systematic account of how we grasp the first principles of the particular sciences.[153] Perhaps he thought that no such account could be given beyond saying that we must become familiar with the so-called phenomena, whether these be sensible things or mathematical objects or even the opinions of the many and the wise. The problem of reaching first principles is further exacerbated in the case of first philosophy, whether this be regarded as a particular or

[153] Cf. Hintikka (1980) for a concise analysis of the difficulties in Aristotle's account of induction.

a general science. Even if Aristotle has given up the Platonic concept of dialectic as a science that somehow grounds the hypotheses of the special sciences, he still faces problems about the foundations of these sciences. For instance, if the truth of mathematics is challenged in a foundational way, it is the task of first philosophy (and not of mathematics) to deal with the problem. Thus, part of the old agenda of Platonic dialectic has been taken over by Aristotle's science of metaphysics whose subject-matter is defined in terms of being qua being. But the problem of reaching the first principles of this most universal science is insoluble unless another part of this agenda can be taken up again; i.e. the dialectical review of opposing opinions on a leading question such as: 'What is being?' or 'What is substance?.' At the beginning of *Metaphysics* VII, Aristotle tells us that these are the questions which have always been asked and which continue to puzzle his contemporaries. From my analysis in this chapter, I think it is clear that Book III rehearses that puzzlement as an essential stimulant for the activity of philosophy.

CHAPTER FIVE

THE ONTOLOGICAL STATUS OF MATHEMATICAL OBJECTS

For two related reasons, this chapter focuses on a self-contained treatise about the mode of being of mathematical objects. First, at the methodological level, it illustrates Aristotle's dialectical method of exploring and resolving a metaphysical problem. Second, at the philosophical level, this treatise is important because it contains his most systematic attempt to deal with metaphysical problems about the objects of mathematics. I have already argued that Aristotle's break with Plato's mathematical cosmology gives rise to such problems, and *Metaphysics* XIII.1–3 shows how he tries to resolve some of them.

I will argue that he arrives at solutions by moving beyond the aporetic strategy of *Metaphysics* III, where he simply reviewed the difficulties on both sides of each question. In XIII.2, by contrast, his method is elenctic as he tries to refute the views of his opponents; so that in forensic terms he is no longer an impartial judge but is rather a plaintiff in the case. This represents progress in the dialectical search for truth because one should not remain in puzzlement, though this is essential for philosophy.[1] So in XIII.3 he resolves the puzzle by proposing an alternative mode of being for mathematical objects.

I. *The program of* Metaphysics *XIII*

As regards the project of Book XIII, Aristotle's introduction is important for identifying its central problems and perhaps some of its presuppositions:

> We have stated what is the substance of sensible things in two places, first in the inquiry concerning physical things, where we dealt with substance in the sense of matter, and later with substance with respect to actuality. Since our concern is whether, besides the sensible sub-

[1] In this respect Aristotle's methodological attitude differs fundamentally from that of the ancient Sceptics who used the aporetic method as an end in itself for their philosophical inquiries.

> stances, there exists an immovable and eternal substance or not, and if it does exist, what it is, first we must examine what is said by others, so that, if there is anything which they do not state well, we may not be liable to the same error, and, if there is any view common to them and us, we too may not be privately dissatisfied with ourselves; for one should be content to state some things better than one's predecessors and the rest not worse.[2]

Aristotle's reference to previous treatises suggests that some consensus about sensible substance provides the basis here for discussing claims about other kinds of substance.

But commentators disagree about which treatises are being presupposed. For instance, Ps.-Alexander[3] thinks the first reference is to the discussion of sensible substance as matter at *Physics* I, while the 'subsequent' discussion of substance as actuality he takes to be *Physics* II. But I think Ross[4] is right in taking ὕστερον as referring to *Metaphysics* VII–IX because the *Physics* could hardly be described as an inquiry into substance 'according to actuality' (κατ' ἐνέργειαν), i.e. substance in the sense of form.[5] So I will assume *Metaphysics* VII as the background for his discussion of candidates for supersensible substance, and in fact there is some textual evidence for such an order of inquiry.[6]

After giving these references, Aristotle introduces the leading question of the present inquiry; i.e., whether or not there is anything immobile and eternal apart from sensible substances.[7] This is similar to the question about whether or not there are other kinds of substance besides (παρά) sensible substances. In fact, Aristotle uses the aporiae about Forms and Intermediates to guide his subsequent inquiry in XIII, though it remains unclear how this inquiry is related to *Metaphysics* XII.

This unclarity constitutes an interpretative problem for us because the leading question in XIII.1 could apply to the Prime Mover, which is described in Book XII as "some eternal and unchanging substance

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[2] *Met.* 1076a8–16: tr. Apostle (1966)

[3] Cf. *in Metaph.* 722.14–16. I accept the current consensus that the commentator on *Metaphyics* XIII is not Alexander of Aphrodisias, who commented on Book III.

[4] Cf. Ross (1924) ii, 407–8.

[5] Apostle (1966) seems to agree with Ross, in giving references to the *Physics* (188a19–193b21) and to the *Metaphysics* (1028a10–1052a11) as locations for these two different inquiries into substance.

[6] Cf. *Met.* VII.2, 1028b27–32; VIII.1, 1042a22–4.

[7] πότερον ἔστι τις παρὰ τὰς αἰσθητὰς οὐσίας ἀκίνητος καὶ ἀΐδιος ἢ οὐκ ἔστι, *Met.* 1076a10–12.

that is separated from sensibles."[8] Yet there is no reference in XIII.1 to any treatise that has established the existence of one kind of eternal and unchanging substance apart from sensible substances. Whatever one may think of Jaeger's conjecture about Aristotle's philosophical development, one must ask whether or not Book XII is being presupposed here.[9] Despite the traditional order of the *Metaphysics,* however, there is no sign that Aristotle drew on XII in discussing Forms and Mathematicals.

Instead the appropriate framework for understanding that discussion is provided in VII.1 where Aristotle briefly outlines the different categorial senses of being, while indicating that substance (οὐσία) is being in the primary sense. There (1028b2–6) he argues that one must give special, primary and exclusive attention to being in the sense of substance. After stating the subject of inquiry, Aristotle records some of the common opinions about substance among predecessors and contemporaries.

In *Metaphysics* VII.2, for instance, he begins the review of opinions by saying: "Substance is thought to belong most evidently to bodies."[10] Aristotle cannot be giving his own considered view here, since that would preempt the whole inquiry. Yet he does not regard it as a misguided opinion about substance because he explains that 'we say' (φάμεν) animals and plants and their parts are substances. This may be an appeal to common usage, which supports those who claim that natural bodies like fire and earth, are substances.[11] Along with elemental bodies, heavenly bodies like the Sun and the Moon are also thought to be substances. We can see how seriously Aris-

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[8] οὐσία τις ἀΐδιος καὶ ἀκίνητος καὶ κεχωρισμένη τῶν αἰσθητῶν, *Met.* 1073a4–5.

[9] Schwegler (1847/48) iv, 297 raises questions about the order of books, while Ross (1924) ii, 408 thinks that some other systematic treatment of the Prime Mover is being referred to. More recently, Patzig (1987) 114–5 and Berti (1987) have paid some attention to this question, without reaching any convincing conclusions. If Aristotle has Book XII in mind when he refers to an inquiry into substance 'according to actuality,' then we might expect him to use different criteria to test Forms and Mathematical Objects as candidates for supersensible substance. However, if Books VII–IX have already set out the criteria for substance, which even supersensible substances are expected to fulfil, then it is not necessary to presuppose Book XII for his inquiry in Book XIII.

[10] Δοκεῖ δ' ἡ οὐσία ὑπάρχειν φανερώτατα . . . τοῖς σώμασιν, *Met.* 1028b8–9.

[11] Cf. *Met.* 1028b10–11. In a parallel passage at *Metaphysics* VIII.1, Aristotle records a list of things that are generally agreed (ὁμολογούμεναι) to be substances: i.e. physical bodies like earth, air, fire and water; plants, and their parts; animals, and their parts; the universe, and its parts. But he also records (1042a6 ff.) some peculiar (ἴδια) views about substance: i.e. that Forms and Mathematical Objects are substances.

totle takes such opinions when he says (1028b13–15) that one must inquire whether these *alone* are substances or whether there are also others.

By contrast, he appears (1028b16–18) rather dubious about the opinion that the limits of body such as planes, lines, and points, are substances to a higher degree than the body and/or the solid (τὸ σῶμα καὶ τὸ στερεόν). Such a view about substance is also listed in *Metaphysics* III where body is argued to be less substantial (ἧττον οὐσία) than a plane because the latter defines it and can exist without it. This view depends on a schema of priority which follows the logical order of definition in mathematics rather than the ontological order of Aristotle's categories. Thus Plato is said (1028b19–21) to hold the view that apart from sensible substances there are at least two kinds of entity (i.e. Forms and Mathematicals) which are eternal and more real (μᾶλλον ὄντα).

We should notice, however, that Plato is represented as positing three kinds of substance (i.e. sensibles, Forms and Mathematicals), and that this results from asking whether there are supersensibles apart from sensible substances. By putting the question in this way, Aristotle ignores the Platonic view that sensible things are not substances because they are changing objects of opinion. In his leading aporia, therefore, he takes the substantiality of sensible things for granted, so that the only question is how many other kinds of substance there are.

Along the same lines he reports (1028b21–24) that Speusippus posited many more kinds of substance (than Plato), with distinct principles for each kind. For instance, beginning with the One as the principle of numbers, he posited another principle for magnitudes and another for soul and, in this way, Speusippus 'strings out' (ἐπεκτείνει) the different kinds of substance. Aristotle's rather disparaging remark should be understood in the light of his philosophical objections to the reputedly disjointed ontology of Speusippus; cf. *Met.* XII.10, 1075b35–1076a4. In VII.2, however, he does not mention these objections but rather reports (1028b25–27) opinions about different kinds of substance and their number. For instance, other thinkers say that the Forms and the Numbers have the same nature and that from these follow all the others, such as lines and planes, until one reaches the substance of the universe and sensible things. This refers to the peculiar theory of Xenocrates; cf. *Met.* 1086a5–10, 1090b20–32.

In any case, it is clear that Aristotle's dialectical procedure has shifted into a new phase, as compared with *Metaphysics* III where no special effort was made to distinguish the individual proponents of different views. The significance of this shift is that Aristotle intends to refute his opponents and this requires that he identify their positions more specifically.[12] Furthermore, the use of elenchus for testing opinions fits nicely with his explicit statements in VII.2 about the purpose which is served by this review of opinions:

> Concerning these matters we must examine what is stated well and what is not, which are the substances, whether there are other substances besides the sensible or not and how these exist, whether there exist separate substances other than sensible substances or not, and if yes, then why such exist and how. But first, we must sketch out what a substance is.[13]

It is clear that this set of questions outlines a larger project than that carried out in the central books (VII–IX) on substance. For instance, the leading question of Book XIII is whether or not there are other substances besides sensibles and how these exist.[14]

But VII.2 also lists the question of whether or not besides sensibles there is some separate substance and why and how.[15] This could be taken as a leading question for *Metaphysics* XII which tries to show that there is such a substance apart from sensible substances. Yet Aristotle says (1028b2–4) that neither question can be answered without first dealing with an issue which has always puzzled philosophers; namely, what is being or (in its primary sense) what is substance.

What role does the review of opinions play in an inquiry either about substance itself or about the existence of separated substances? With reference to the opinions about substance which are outlined in VII.2, Aristotle says that one must consider what is stated well (καλῶς) and badly. Although the proposed method is not explicitly mentioned, his language suggests that some kind of elenchus is in-

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[12] Robert Bolton (1987) 123n9 makes a distinction between *ad hominem* dialectical testing where the relevant endoxa are restricted to the views of the person or group being tested (*Top.* 159b27–35), and dialectic *simpliciter,* where the relevant endoxa are simply reputable opinions accepted by everyone (*Top.* 159a38 ff.). But Bolton's distinction does not cover the aporetic review of opinions, which I take to be a distinctive stage in dialectical inquiry.

[13] *Met.* 1028b27–32: tr. Apostle (1966).

[14] πότερον εἰσί τινες παρὰ τὰς αἰσθητὰς ἢ οὐκ εἰσί, *Met.* 1028b28–29.

[15] πότερον ἔστι τις χωριστὴ οὐσία, καὶ διὰ τί καὶ πῶς, ἢ οὐδεμία, παρὰ τὰς αἰσθητάς, *Met.* 1028b30–31.

tended. At VII.2 we are not told the purpose of such an elenctic survey but XIII.1 (1076a12 ff.) sketches a dual purpose in testing opinions, roughly corresponding to two possible outcomes of an elenchus. If, on the one hand, there is something they do not say well (μὴ καλῶς λέγουσι) then (having refuted them) we would not also be liable to error (ἔνοχοι). This suggests that the elenchus may help us to avoid any snares in which we might become entangled by accepting previous opinions.

After becoming familiar with our intellectual bonds as aporiae, a different procedure is needed to free us from them. The negative stage of that procedure seems to be an elenctic test of the opinions which have created the impasse. If some opinions survive the elenchus then they should be accepted as true. This seems to be the main point of the rather puzzling conditional statement which Apostle translates as: .“. if there is any view common to them and us, we too may not be privately dissatisfied with ourselves.”[16] Such a translation suggests that he is anxious to assert his originality and is disgruntled to find that others have anticipated his views.[17] But this suggestion is inconsistent with his general respect for predecessors, and it sits badly with his usual expressions of gratitude to anyone who makes a good start on some problem of interest; cf. *Met.* II.1, 993b12–14; *Cael.* II.5, 288a1–2.

Therefore we must look for some alternative to the usual translations.[18] The nub of the problem is the meaning of the verb δυσχεραίνω which most translators take in its primary sense as ‘to bear with a bad grace’ or ‘to be discontented’ or ‘to be displeased.’[19] It is quite strange that no attention is paid to a second sense; i.e. ‘to make things hard’ or ‘to make difficulties,’ given its linguistic connection with the ‘difficulties’ (δυσχερείαι) that arise from a review of the opinions of predecessors in Aristotle’s method. But how can this help

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[16] εἴ τι δόγμα κοινὸν ἡμῖν κἀκείνοις, τοῦτ’ ἰδίᾳ μὴ καθ’ ἡμῶν δυσχεραίνωμεν, *Met.* 1076a14–15: tr. Apostle (1966).

[17] If this were the case then we have here an interesting confession on the part of Aristotle, which might provide some psychological insight into a thinker who lays some claim to the gratitude of posterity for his discovery of the science of logic; cf. *Soph. El.* 183b15–184b8.

[18] Cf. Ross (1908) who translates as follows: “.. if there is any opinion common to them and us, we shall not quarrel with ourselves on that account.” But this leaves the problem unsolved, just as does the translation of Annas (1976): “.. and also so as not to be secretly annoyed with ourselves if there is some opinion common to us and them.”

[19] Cf. LSJ. δυσχεραίνω.

to clarify Aristotle's methodological statements here?

After stating the leading question for the proposed inquiry, Aristotle emphasizes the necessity of first considering what others have said about this question, and then he explains the purpose (ὅπως) of reviewing these opinions. If there is something which is stated badly (μὴ καλῶς), or which is refutable, then the purpose served is that one will avoid that particular error. While refutation is the negative outcome of an elenchus, the positive outcome remains unclear, even though the *protasis* of the second conditional statement implies that the shared view (δόγμα κοινόν) is stated well if it has survived attempts to refute it.

Fortunately, there is a parallel passage in *De Anima* I.2 (403b20–24) where Aristotle specifies two purposes in making a collection of opinions. On the one hand, he says, we should accept what has been stated well (καλῶς) while, on the other hand, we may guard against what has not been stated well if there are such opinions.[20] The latter part of the statement yields an exact parallel with *Metaphysics* XIII.1 because it corresponds to that part of the statement of purpose which envisages a negative outcome of an elenchus: If there is something which is not stated well (as shown by the success of an elenchus) then we should beware of it lest we be bound by the same error. On the other hand, if there are things which have been stated well (as shown by a failed elenchus) then we should accept them, otherwise one possible outcome of an elenctic test will not further the inquiry; cf. *Pol.* 1260b32 ff.

Such a clear statement of purpose in *DA* I.2 provides a vital clue for interpreting the obscure statement in *Met.* XIII.1. Since he holds that one should accept any view which survives elenctic testing, it makes sense for Aristotle to begin his conditional statement as follows: "If there is some opinion common to us and to them . . ." This is not just an accidental coincidence of opinion but rather a consensus reached through the elenctic process. Therefore, he says, we should not make difficulties for ourselves, as if this were a private opinion (τοῦτ' ἰδίᾳ). The point seems to be that there is less doubt about a shared opinion, which has presumably undergone critical scrutiny, than about an opinion which is peculiar (ἰδίον) to one person. This reflects a characteristic Greek preference for the common (τὸ κοινόν) which is explicit even in elitists like Heraclitus and Plato. Such a

[20] εἰ δέ τι μὴ καλῶς, τοῦτ' εὐλαβηθῶμεν, *DA* 403b24.

contrast between the common and the peculiar (or private) dominates Aristotle's statement of purpose in XIII.1, where he sees philosophy as a shared enterprise whose goal is to extract truth from common opinions.

In the above passage from XIII.1, for instance, this vision of philosophy is reflected in Aristotle's remark (1076a15–16) that one should be content (ἀγαπητόν) if one states some things better and other things no worse than others. The comparatives here depend on an elenctic test for deciding whether things are said well or badly. Thus, espousing a rather modest ideal for philosophical inquiry, Aristotle claims that one has done an adequate job if one formulates some theories that avoid the mistakes of previous thinkers (as exposed through a successful elenchus), while accepting those views which have survived the critical scrutiny involved even in a failed elenchus. That is why one must begin with the opinions of predecessors and pursue the truth by attempting to refute them.

The importance of attending to Aristotle's own method of inquiry will become clear as we try to make sense of his self-contained treatise (XIII.1–3) on mathematical objects. After introducing the leading question for the whole inquiry in XIII (i.e. whether there is some immovable and eternal substance apart from sensibles), he makes a brief review of opinions that are relevant to this question. Along with this review, Aristotle outlines the whole project of *Metaphysics* XIII as if it were somehow dictated by the differences of opinion among predecessors. For instance, he begins with two apparently different opinions about these supersensible substances; i.e. that they are Mathematicals and that they are Forms. However, the parallel review at *Metaphysics* VII.2 (1028b18–20) shows that the first sentence of the review in XIII.1 (1076a16–19) states the opinion of Plato, as distinct from other thinkers who posited just one kind of supersensible substance.

This emerges clearly when Aristotle contrasts (1076a19–22) those who posited two distinct kinds, i.e. Forms and Mathematical Numbers, with those who gave them both the same nature. Yet a third group claim that only mathematical objects are supersensible substances.[21] Although these views are usually attributed to Speusippus and Xenocrates, respectively, the details of their theories remain obscure. Aristotle says that one view (of Xenocrates) is that Forms

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[21] ἕτεροι δέ τινες τὰς μαθηματικὰς μόνον οὐσίας εἶναί φασι, *Met.* 1076a21–22.

and Mathematical Objects have a single nature; whereas another view (of Speusippus) denies that there is anything except Mathematical Objects. Assuming that both admit only one kind of supersensible substance, the difference might be that one calls them Forms and the other Mathematicals.[22] Thus Xenocrates may have called such supersensible substances Forms, while giving them the same nature as Mathematical Numbers.

Having distinguished these Academic views from each other in this sketchy manner, we must consider how differences of opinion determine Aristotle's subsequent procedure. His use of the connective words ἐπεὶ δέ suggests that it is precisely because of such disputes that Aristotle thinks (1076a22–23) one should first consider mathematical objects as such, without attributing to them any other nature. For instance, Aristotle says, we should not (immediately) ask whether they are Forms; nor should we inquire whether they are the principles and substances of things or not. The formulation of these additional questions about mathematical objects makes it clear that they are subsequent to the aporia about whether they exist or not, taken simply as mathematical objects, and if they exist, what is their mode of being.[23]

Most modern commentators take this to be a question about existence *simpliciter* but this can be misleading if we are not careful to clarify what that means for Aristotle. The concept of existence in modern philosophy after Frege is defined in terms of quantification logic; e.g. for Quine (1960 & 1969) 'to be' is to be the value of a bound variable. So I prefer to formulate Aristotle's question in terms of being, given that his categories are different modes of being. Speaking in those terms, we can say that the question is whether or not mathematical objects are entities, and what is their mode of being.

In the case of mathematical objects, for instance, the crucial question is whether they exist as independent substances or whether they have some other mode of being. But it is a different question, both historically and logically, whether mathematical objects are identical

[22] One might differentiate their positions in terms of how they deal with the uniqueness requirement for Forms and the plurality requirement for Mathematicals. Thus Speusippus accepts only Mathematicals because he gives precedence to the plurality requirement, whereas Xenocrates identifies them with Forms because he gives priority to the uniqueness requirement. This is consistent with Aristotle's remark that Speusippus treated numbers mathematically, whereas Xenocrates did not.

[23] ἀλλ' ὡς περὶ μαθηματικῶν μόνον εἴτε μή εἰσί, καὶ εἰ εἰσὶ πῶς εἰσίν, *Met.* 1076a25–26.

with Forms or not. The latter would correspond to what Aristotle himself calls the what-is-it (τί ἐστίν) question. Thus the reason why he emphasizes the historical divergence of opinion is that he takes it to reflect a logical difference between the questions at issue. There is, first of all, the question of whether or not mathematical objects exist as independent substances distinct from sensible things. This can be logically distinguished from the questions of whether these are identical with Forms and whether they are the principles and substances of things.

If this is the case, then we may have a clue to his rationale for the order of inquiry outlined in XIII.1. On account of the logical priority of the if-it-is question, it is possible (and perhaps even necessary) to ask first about the existence or mode of being of mathematical objects, taken strictly as such.[24] Similarly, on account of historical differences of opinion, one should also ask about the existence of Forms, considered strictly in themselves. Aristotle claims, however, that this has been adequately covered in the so-called exoteric writings and hence does not need to be dealt with here, except perhaps for the sake of completeness.[25]

Finally, Aristotle sketches the program for a longer third inquiry whose leading question is whether or not Forms and Numbers are the substances and principles of things.[26] Despite differences in content and form, I think that this question is closely related to the third aporia from *Metaphysics* III which I have discussed; i.e. whether or not Being and the One are the substances of things.[27] If Being is the most universal Form and One is the principle of number, this general aporetic question might also cover the specific question raised in XIII.1. Thus there is a clear continuity between the relevant aporiae

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[24] Perhaps this approach is also facilitated by the historical fact that Plato posited Forms and Mathematicals as distinct supersensible substances because of the special requirements of mathematics.

[25] Aristotelian scholars have differed on whether the phrase οἱ ἐξωτέρικοι λόγοι refers to some works (or arguments) that were published in a fuller sense than the *Metaphysics* itself. Ross (1924) ii, 408–10 has collected the passages in the Corpus where the phrase occurs, while also giving a brief survey of the many ways in which it has been interpreted, but it is not necessary for me to adjudicate between these different interpretations, since Aristotle's criticism of Platonic Forms is a side issue from my perspective. It is sufficient to note that the phrase might be taken as a reference to such early works as *On Forms, On Philosophy* and *On the Good.* As we can see from extant fragments of these works, Aristotle criticised those who posited Forms as separate substances apart from sensible things.

[26] εἰ αἱ οὐσίαι καὶ αἱ ἀρχαὶ τῶν ὄντων ἀριθμοὶ καὶ ἰδέαι εἰσίν, 1076a30–31.

[27] πότερον . . . τὸ ὂν καὶ τὸ ἓν οὐσίαι τῶν ὄντων εἰσί, *Met.* 1001a5–6.

in *Metaphysics* III and the whole project of Book XIII as set out at the beginning.

II. *The refutation of opponents*

At the end of XIII.1 Aristotle sets out the possibilities for the mode of being of mathematical objects in the following way:

> If mathematical entities exist, then, they must be either in the sensible things, as some say, or separate from sensible things (and this is what others say). Or, if they exist in neither of these ways, either they do not exist, or they exist in some other manner; and in the latter case, the point at issue will not be with their existence but with the manner of their existence.[28]

Clearly, he wants to give an exhaustive outline of the logical possibilities, some of which correspond to the opinions of previous thinkers. For instance, the first possibility is that mathematical objects are *in* sensible things (ἐν τοῖς αἰσθητοῖς), and this is the view reported in *Metaphysics* III.2 (998a7–19).[29]

Since Aristotle does not name the proponents of this view, commentators are forced to speculate about their identity. For instance, Ps.-Alexander (*in Metaph.* 724.31–33) thinks some Pythagoreans held this view; whereas Syrianus (*in Metaph.* 84.21) denies that any such view was held by either Pythagoreans or Platonists, though he attributes (28.18) it to the Stoics in his commentary on *Metaphysics* III. As Ross[30] points out, however, the position outlined in III and refuted in XIII is clearly different from the Pythagorean view reported elsewhere (*Met.* 987b27–29) by Aristotle. But Robin[31] attributes the view to a group of platonizing Pythagoreans who posit mathematical objects as being distinct from but *in* sensible things, whereas earlier Pythagoreans say that sensible things are constituted by the numbers themselves. Perhaps the mixture doctrine of Eudoxus may have been taken to mean that mathematical objects are independent substances *in* sensible things, depending on which sense of 'in' is involved.[32]

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[28] *Met.* 1076a32–37: tr. Apostle (1966).

[29] In order to signpost this view as represented by Aristotle, I will adopt the convention of italicizing the 'in' as follows: '. . . mathematical objects *in* sensible things.'

[30] Cf. Ross (1924) ii, 411–12.

[31] Cf. Robin (1908) 205–6, 649–51.

[32] In his treatise on place in *Physics* IV, Aristotle lists at least eight different ways

Hippocrates Apostle[33] makes the interesting suggestion that, since this first view about the mode of being of mathematical objects is superficially similar to Aristotle's own view, he will be most anxious to refute it completely and to allay any suspicion that he may be borrowing from it.[34] This might explain the rather curious fact that, in his treatment of this view at III.2, he does not provide an antithesis which would typically include the arguments that might support the view. But it is more likely that Aristotle saw no merit whatsoever in positing mathematical objects as independent substances *in* sensible things, and so he did not give any arguments under the antithesis. Due to the lack of supporting arguments, however, there remains some doubt as to whether this view found any defenders.

By contrast, there is little doubt that some strict Platonists espoused the view that mathematical objects are separated from sensible things as independent substances, whether these are called Forms or Intermediates or both. This view corresponds exactly with one of the possibilities listed in *Metaphysics* III in connection with the aporia about whether or not mathematical objects are some (kinds of) substances or not; cf. 996a12–15 & 1001b26–28. In fact, the subsequent question laid out two possibilities for mathematical objects as substances; i.e. either they are separated from sensibles (κεχωρισμέναι τῶν αἰσθητῶν) or they belong *in* them (ἐνυπάρχουσαι ἐν τούτοις).[35] The whole aporia suggests that these two possibilities exhaust the answers to the πῶς εἰσί question with respect to mathematical objects as substances, just does the the summary at the end of XIII.1.

Since the first two possibilities cover the ways in which mathematical objects can exist as substances, the last two possibilities must be about alternative modes of being: (iii) either mathematical objects do not exist (ἢ οὐκ εἰσίν) or (iv) they exist in some other way (ἢ ἄλλον τρόπον

in which one thing can be said to be 'in' another, although the strictest sense involves something being in a place as in a vessel; cf. 210a14–24. It seems from his argument in *Metaphysics* XIII.2, however, that it is the latter sense of 'in' that Aristotle has in mind when he argues that two bodies will be in the same place, according to this view.

33 Cf. Apostle (1966) 410n10.

34 Thus, for instance, Moukanos (1981) 9 complains that this view cannot be distinguished clearly from Aristotle's own view that mathematical objects exist in sensibles; cf. *Met.* 1036a9–12.

35 It may be significant that the verb ἐνυπάρχειν used here is the same as that used by Aristotle for the way in which substantial material elements like earth and fire belong to sensible things; cf. *Phys.* 193a10, 194b24; *Met.* 986b7, 1014a26, b15, b18, 1040b32, 1043a21, 1059b24; *GA* 724a25, 729b3.

εἰσίν). The third possibility is included only for the sake of logical completeness, since Aristotle does not subsequently return to it. His apparent oversight can be explained in terms of the argument from the sciences, whose fundamental assumption is that any genuine science must have a real or existent object.[36] Since Aristotle shares that epistemological assumption, he cannot accept the non-existence of the objects of mathematics as that would leave his paradigmatic sciences without foundations. Hence, if the first two possibilities are to be denied and the third is ruled out, the remaining option assumes a new importance. As stated, this is the possibility that mathematical objects exist in some other manner which lies between complete non-being and being in the primary sense as substance.[37]

Having outlined all the possibilities with respect to the existence of mathematical objects, Aristotle concludes (1076a36–37) that the dispute will not be about their being (περὶ τοῦ εἶναι) but rather about their mode of being (περὶ τοῦ τρόπου). Since this implicitly excludes the possibility that mathematical objects do not exist in any way, there remains only their two possible modes of existence as substances and some other unspecified mode of being. As these are merely different ways in which mathematical objects can exist, Aristotle can conclude that the dispute is not really about whether these objects exist but rather about their mode of existence. In his own terms, it is not whether they are (εἰ εἰσί) but rather how they are (πῶς εἰσί) which is in dispute.

The fact that he makes himself a party to the dispute gives us an initial clue to the character of the subsequent discussion in XIII.2. Since he explicitly calls this a controversy (ἀμφισβήτησις), we know that the inquiry will be elenctic in character.[38] The difference be-

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[36] In addition, we might connect the argument 'from the sciences' with the Parmenidean dictum that it is impossible to think or inquire about not-being; cf. *Cael.* III.1, 298b17–25. By contrast, Ingeborg Schussler (1982) thinks that Aristotle takes non-being seriously as a possible mode of being for mathematical objects because they always appear as sensible bodies and never as pure measures; so it seems problematical whether they are beings at all.

[37] Pseudo-Alexander (*in Metaph.* 725.4) declares this mode of being to be "abstract" (ἐξ ἀφαιρέσεως), even though Aristotle appears to be deliberately vague in talking about "some other way." Indeed at this point there is no basis for Ps.-Alexander's introduction of abstraction terminology, as we have no guidance as to what it could mean within the present context.

[38] Cf. *Phys.* 253a34, *Cael.* 294b31, *EN* 1096b8, 1125b17, 1135b28, 1145b28; *EE* 1215a26, 1237b9, 1243b37; *Pol.* 1275a2, 1283a15; *Rhet.* 1374a11, 1416a9, 1391b19, 1394b10.

tween this and his treatment of similar questions in *Metaphysics* III can be expressed once again in forensic terms by saying that Aristotle is no longer acting as an arbitrator who listens to both sides, since he has now become one of the plaintiffs who argues to win his case.[39] So we can expect partisan arguments and sharp refutations, even though the ground-rules dictate that they be guided by common assumptions or shared principles. Therefore I do not accept the claim[40] that there is a discrepancy between Aristotle's explicit methodological statements and his actual dialectical practice. When he speaks of even-handedness in reviewing the opinions of others, Aristotle is describing quite accurately the procedure of going through the difficulties (διαπορῆσαι) as in *Metaphysics* III. But when he speaks of a dispute (ἀμφισβήτησις), we should take this to mean that other thinkers are now being treated as opponents in debate.[41]

II.1. *Mathematical objects* in *sensible substances*

Aristotle begins his brief refutation of the first possibility with an explicit reference back to his aporetic treatment of this view in *Metaphysics* III:

> It has been already stated in our discussion of difficulties that mathematical objects cannot exist in sensible things and that the account of these thinkers is also fictitious, for it is impossible for two solids to be in the same place; moreover, for the same reason the other powers and natures would have to be in sensible things and not exist separately.[42]

Although this passage confirms the link between III and XIII, it also seems to undermine my claim that these two books contain different dialectical ways of treating the question about mathematical objects. Here Aristotle appears to say that this view has already been refuted in III, when he refers back to what had been said in the 'discussion of difficulties' (ἐν τοῖς διαπορήμασιν). But I regard his treatment of

[39] Cf. also *EN* 1132a19, *Pol.* 1291a40, *Rhet.* 1354a27–b4, 1358b12, 1365a2, 1383a11–24,1391b26, 1399b31.

[40] See Quandt (1978) who follows Cherniss in this matter.

[41] διαπορεῖν and ἀμφισβητεῖν are treated as being practically synonymous verbs at *EE* 1215a20 but this is exceptional, since they are usually used for distinct but related activities; cf. *Cat.* 8a26, *Phys.* 253a34, *Cael.* 294b31, *Met.* 998b17, *EN* 1096b8, 1100a18, 1110a7, 1163a9, 1172a27, 1175b33; *EE* 1210a37, 1243b37, *Pol.* 1326b36, *Rhet.* 1374a11, 1416a9.

[42] *Met.* 1076a38–b4: tr. Apostle (1966).

this particular view as being exceptional, on the grounds that he introduces no antithesis in III because he thinks there is nothing to be said in support of the view in question.

Within an elenctic framework, of course, the inference would be that such a view is untenable, but the difference between III and XIII is that a clear elenctic purpose is present only in the latter book. Witness the fact that in III.2 Aristotle does not refer to this position in such dismissive terms as in XIII.2 when he calls it a concoction (πλασματία).[43] In keeping with the aporetic character of *Metaphysics* III, however, he expresses (998a15–19) puzzlement as to how anyone could hold a position that leads to all of the absurd (ἄτοπα) consequences associated with Intermediates and more besides. But puzzlement belongs to a different dialectical stage than refutation, even though the difference is less obvious in this case because Aristotle unaccountably does not give any antithetical arguments in support of the view that mathematical objects are *in* sensible things.

Although Aristotle repeats some difficulties from Book III as objections in the present context, let me focus on a new objection to this view:

> That has already been said; but in addition it is clear that (on this theory) it is impossible for any body to be divided; for it will be divided along a plane, and the plane along a line, and the line at a point, so that if the point cannot be divided, neither can the line, and if the line cannot, the rest cannot either. So what difference does it make whether perceptible bodies are objects of this kind, or whether, while they are not, objects of this kind exist in them? There will be the same result: either they will be divided when perceptible objects are divided, or perceptible objects will not be divided either.[44]

The primary argument in the above passage seems to go as follows: (A) If any body is divisible, it must be divisible along a plane and, similarly, (A1) this plane must be divided along a line and, similarly, (A2) the line must be divided at a point. But (B) if it is impossible to divide the point then one cannot divide a line and (B1) if the line cannot be divided, neither can the others; i.e. the plane and the solid. Thus (C) it is impossible for any body to be divided. The fundamental assumption here is that planes and lines and points are *in*

[43] By insisting that none of the Pythagoreans or Platonists ever held such a position, Syrianus (*in Metaph.* 84.20–3) implies that Aristotle himself may be responsible for the concoction.

[44] *Met.* 1076b3–11: tr. Annas (1976).

bodies as actual entities, so that when a body is divided all of these are divided also. Presumably Aristotle thinks that such an assumption is being made by those who hold that mathematical objects are independent substances *in* sensible bodies. If so they are caught in a dilemma because, on the one hand, they must treat the point as an indivisible substance in the line whereas, on the other hand, they must regard it as divisible if the line is to be divisible.

Julia Annas (1976) dismisses Aristotle's objection here on the grounds that he obtains his conclusions only by foisting on his opponent such 'implausibly crude' conceptions of mathematical operations as to make them analogous to physical operations. Just as the physical operation of splitting a length of some material involves dividing some extent of it, so also (she claims) the geometer's division of a line at a point is thought of as splitting the point at which the division is made, while the point itself is taken as a minimal extension. This now yields a contradiction because the mathematician's points are assumed to be indivisible. Annas notes (quite correctly) that Aristotle himself does not accept this crude way of conceiving of the geometer's division of a line at a point nor does he give any argument to show that the platonist must think of it in this way. For an alternative conception of how a line can be divided at a point, she refers to III.5 (1002a28–b11) but fails to note that this conception of division is used *against* the view that mathematical objects are substances. If the point is regarded as a substance, it would not be plausible to say that two points can be instantaneously generated (as the ends of line segments) from a single point.

According to the ordinary conception of physical substance to which Aristotle is appealing here, its generation and corruption are temporal processes that do not happen instantaneously. This difficulty is relevant for the present objection in XIII.2 because those who hold mathematical objects like points to be substances would appear to have only two ways of explaining how a line can be divided: (i) either between two consecutive and indivisible points or (ii) at a point. Only the second option is explicitly (1076b7) ruled out in the premiss (B) which assumes that it is impossible to divide a point. This assumption yields the desired conclusion (i.e. that a body is not divisible) which contradicts the initial assumption (i.e. that a body is divisible), and so the *reductio* argument is complete.

One might wonder why Aristotle never allows his opponents the benefit of the first option; namely, that the division of a line may

happen between two consecutive points. The answer may lie in Ps.-Alexander's (726.9 ff.) conjecture that these people thought of the line as a continuous magnitude and (in the light of Zeno's paradoxes) could not hold it to have any consecutive points. Even though Aristotle shares this view of continuous magnitudes, he encounters no such difficulty because he does not regard points as substances that actually exist in the line. This may also answer the question as to why he could not give his opponents the benefit of the more subtle notion of mathematical division to be found at III.5. Such a notion presupposes that points are not actually but potentially present in the line so that they can be somehow actualized in the process of division. This distinction may be important for understanding Aristotle's own position.

II.2. *Mathematical objects as separated substances*

Consistent with his outline in XIII.1, Aristotle now considers the other possible mode of being for mathematical objects as independent substances; i.e. that they are separated (κεχωρισμένα) from sensible things. This is the view attributed to the Platonists generally, though people like Speusippus and Xenocrates espoused different views of the relationship between Forms and mathematical objects. But the question about how many kinds of separated substance exist has been shelved, so Aristotle can assume that all of the Platonists agreed on the central point that mathematical objects are separated from sensible things. In addition, they accepted as self-evident some assumptions that were typical of the Academic tradition as distinct from other traditions like Ionian natural philosophy. Hence it is important to identify these assumptions, and thereby to render intelligible some of Aristotle's elenctic arguments against the so-called Platonists.

II.3. *An absurd clutter of scientific objects*

The first argument (1076b11–39) is long and complicated but its main point is clearly that the separation of mathematical objects leads to an absurd clutter of ideal objects. The general gist of the argument is that consistency requires Plato to accept that there are ideal planes and lines and points separated from their sensible counterparts, as long as he holds that there are ideal solids separated from sensible solids. So far there seems to be no obvious absurdity about such an

implication, and it conforms reasonably well to the standard Platonic view reflected in the argument from the sciences.

But, Aristotle continues, if ideal solids are separated from sensible solids, so should ideal planes be separated from sensible planes and, similarly, ideal lines and points should be separated from their sensible counterparts. This is a crucial first step in an objection which goes on to generate a multiplicity of ideal objects by applying the same Platonic principles. But the objection depends on another reason given for the separation of ideal objects; namely, that they are prior (πρότερα) to sensible objects. A little later (1076b18–19), for instance, Aristotle appeals to the criterion that incomposite things (τὰ ἀσύνθετα) are prior to composites (τῶν συγκειμένων). It would appear that the basic notion here is that the simpler thing is prior to the more complex.[45]

Thus, from the Platonic assumption that there are mathematical lines and planes apart from their sensible counterparts, Aristotle infers an absurd multiplication of ideal objects:

> If so, then again, besides the planes and lines and points of the mathematical solids there exist, respectively, other planes and lines and points which are separate; for the incomposites are prior to the composites. And if there exist non-sensible bodies which are prior to the sensible bodies, for the same reason there exist also planes which are prior to the planes in the immovable solids and which exist by themselves. Thus, these planes (and this applies also to lines) will be distinct from the planes which are in the separate solids; for the latter planes are with the mathematical solids, but the former planes are prior to the mathematical solids. Once more, in these prior planes there will be lines, and for the same reason there must be other lines and points prior to these lines; and those prior lines will have points, and prior to these points there will be other points, beyond which no other prior points exist.[46]

The general objection here is that, once we begin to separate ideal objects from sensible things, there is no reason to stop there because we can apply the same argument for separation to the ideal objects themselves. Aristotle explicitly appeals to the criterion that incomposites are prior to composites but it is not clear that this is appropriate for the separation of ideal objects from sensible things, since it is specifically applicable to the distinction between those planes, lines, and

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[45] τὸ γὰρ ἁπλούστερον πρότερον τοῦ συνθετωτέρου, Ps.-Alex. *in Metaph.* 727.2. Cf. Alex. Aphr., *in Metaph.* 55.20–25.

[46] *Met.* 1076b16–28: tr. Apostle (1966).

points which are composing a mathematical solid and those which are not. He pursues the objection further (1076b19–20) by assuming that non-sensible bodies are prior to sensibles. Since both kinds of bodies are composites, it is difficult to see how the previous criterion of priority could be applied here. Yet Aristotle goes on to say that, by the same argument, there will be planes which exist by themselves (αὐτὰ καθ' αὑτά) apart from the planes in the motionless solids. Now this is rather puzzling because it is not easy to see what argument (or ratio) provides the parallel with previous cases.

But perhaps we can throw light on the Aristotelian objection from an Academic perspective. His first move is to claim that if there are other solids apart from sensible solids, which are separate from and prior to them, then by parallel reasoning there must be other planes and lines which are separated from their sensible counterparts. The priority in question here would appear to be priority in knowledge if there is a parallel with the Platonic argument from the sciences. But, just as in the case of that argument, the priority in knowledge of ideal mathematical objects implies their ontological separation from sensible things which share the same names. So far there seems to be nothing that is either absurd or unPlatonic about the conclusions of Aristotle's argument. From this point onward, however, he begins to duplicate ideal objects while insisting that the same principles are being applied. My own conjecture is that he sees this as a permissible dialectical move because the Platonists inferred ontological separation from priority, without distinguishing different senses of priority. Thus he argues that, according to their principles, there will be other planes and lines and points which are separated from those contained in the mathematical solid.

But we have yet to make sense of his appeal to the general criterion that incomposite things (τὰ ἀσύνθετα) are prior to composites (τῶν συγκειμένων). Is he using the term 'composite' in the same sense here? The way in which he applies the criterion seems to suggest otherwise. For instance, it appears to be used primarily to justify the distinction between planes and lines which are together with (ἅμα) the mathematical solids and those which are separated from them, according to the Platonists. The first kind of planes and lines might properly be called elements, since they are in the solids, but the second kind cannot be so called because they are separate and distinct from the first kind. Thus it would appear that the sense in which the latter kind of mathematical objects are incomposite is that they exist

by themselves and are not involved in compounding other figures. Aristotle cannot be using 'incomposite' to mean that planes and lines are themselves without formal elements, since the continuation of his objection depends on them having such elements.

Thus he seems to be trading on an ambiguity in the terms 'composite' and 'incomposite' when he goes on to claim that, according to the same argument, there must be other lines and points that are prior to the lines belonging to those incomposite planes that exist by themselves. Similarly, the objection continues, there will be other points that are prior to the points which belong with the latter kind of separated lines. But the multiplication of ideal mathematical objects comes to a stop with points, presumably because they are absolutely incomposite. Still, the accumulation so far is sufficient for Aristotle's purpose of showing the absurdity of separating mathematical objects from sensible things. So his objection depends upon treating priority and 'incomposite' as univocal terms when, in fact, they are equivocal or, perhaps, analogical.

Let me summarise this whole argument (*Met.* 1076b11–39) by making some general points about its cumulative effect within Aristotle's treatise.[47] As already noted, the purpose of the argument is to refute the view attributed to the Platonists; i.e., that mathematical objects are independent substances apart from sensible things. Aristotle uses a *reductio* argument which tries to show that the view in question yields an absurd accumulation of different kinds of ideal mathematical object. Along with flouting the canon of parsimony, what is absurd about this result is that none of these kinds can be positively identified as the subject-matter of any of the mathematical sciences. If this is the case then the Platonists are refuted, since their original purpose in positing ideal objects has been defeated.

Thus, if he has not made any illegitimate assumptions, Aristotle's objection to their views on mathematical objects is an effective refutation. But the general suspicion of illegitimacy has been cast upon his dialectical technique of argumentation by scholars like Cherniss.[48] Without trying to exonerate Aristotle completely, I claim that such suspicions are not always well-founded. From the procedural point of view, for instance, we should note the use which he makes of

[47] For a detailed analysis of this argument see Cleary (1982).

[48] It is a pity that Cherniss (1944) 185–7 gives only a general survey of the arguments in *Metaphysics* XIII.1–3, since these chapters seem particularly relevant to his theme.

guiding principles and criteria which were probably accepted by all parties to the dispute. For example, the criteria governing the priority of the incomposite to the composite are used to generate an absurd multiplicity of ideal objects, when combined with the generally accepted maxim that the sciences deal with what is prior. Of course, Aristotle assumes that the same meaning of priority applies in both cases, and this seems to be a rather dubious move, since he himself distinguishes between many different senses of priority. However, if the Platonists failed to make these distinctions then it is quite legitimate for Aristotle to construct a refutation on the assumption that priority is univocal. In general, it is important to keep in mind that these dialectical objections all have a rhetorical flavor. In the present argument, for instance, the absurd piling up of ideal mathematical objects seems designed, in part, to persuade the audience that the Platonists have been refuted.

III. *Difficulties become objections*

The subsequent objection (1076b39–1077a9) refers back to *Metaphysics* III, thereby showing how difficulties can be used as refutations against a position which fails to resolve them.[49] The most pressing of these difficulties is connected with the subject-matter of mixed mathematical sciences like astronomy, optics, and harmonics. Aristotle draws a parallel between the pure science of geometry and the mixed science of astronomy, inasmuch as both sciences seem to be about ideal objects separated from sensible things. This parallel is crucial for generating the internal conflict that he finds in the position of the Platonists. If, on the one hand, there is an ideal heaven which is an independent substance apart from the sensible heaven then, insofar as it is a mathematical object, it must be immobile. But, on the other hand, it belongs to the very nature of the heaven and its parts to have motion (κίνησις) in some way.[50]

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[49] It would also appear from a passage in *De Caelo* (I.10, 279b6–7) that Aristotle considers difficulties brought against one view to be usable as proofs for the contrary view.

[50] Aristotle seems to assume that circular motion belongs naturally to *aither* as the material of the heavenly bodies, whereas no motion can belong to them as ideal and separate objects studied by astronomy. In the terminology of modern commentators, this difficulty is called a two-level paradox; cf. Owen (1968), Vlastos (1973b).

This is the same difficulty as that discussed in *Metaphysics* III (997b14–20), where the rhetorical question echoes the question here: How should one believe these things? On the one hand, it is not reasonable (εὔλογον) to assume that the objects of astronomy are immobile, since the heaven and its parts have a certain kind of motion by nature. Yet, on the other hand, it is quite impossible (παντελῶς ἀδύνατον) for these to be moving because they are also objects of mathematics. According to Aristotle's analysis, the mathematical astronomer who accepts the Platonist view of his science finds himself in a double bind. Whereas Platonic conditions for objectivity in science force him to regard the object of his inquiry as separated from the sensible flux, the subject-matter of astronomy (as distinct from pure mathematics) dictates that he study the natural motions of the heavenly bodies.

Thus Aristotle's objection is directed against Platonists like Speusippus[51] who make the objectivity of a science depend on the substantial separation of its objects from sensible things. But, as I noted for the relevant difficulty in *Metaphysics* III, this view of objectivity is particularly vulnerable in the case of the mixed mathematical sciences which obviously deal in some way with sensible things. So Aristotle can extend the same argument from astronomy to optics and harmonics in order to generate absurd consequences from the Platonist position. His basic assumption here is that the Platonists cannot make an exception for particular sciences because, as we have already seen, the argument from the sciences and the argument from objectivity both spell out the general conditions for scientific knowledge. So, for instance, a science of living things (i.e. biology) will not be about sensible animals but rather about some kind of animals separated from sensibles, and this is absurd.

III.1. *Argument from general theory of proportion*

The subsequent objection nicely illustrates the complex role which difficulties play in his method; i.e. of helping to refute an opponent's position, while also being among the phenomena to be saved by any solution that emerges from the refutation. In this case, furthermore,

[51] *Met.* 1090a27–30 (which Tarán (1981) claims is specifically about Speusippus) speaks about those who separate mathematical objects from sensibles on the grounds that mathematical axioms are not true of sensible things but rather 'greet the soul' (σαίνει τὴν ψυχήν).

the objection against the Platonists seems to be based upon Eudoxus' general theory of proportion, which was developed within the Academy itself. Thus the following objection strikes close to home:

> Again, some mathematical propositions are universally expressed by mathematicians in such a way that the objects signified are distinct from these mathematical substances. Accordingly, there will be other substances which are separate, which lie between the Forms and the Intermediates, and which are neither specific numbers nor points nor specific magnitudes nor time. If this is impossible, it is clear that the others, too, cannot exist separate from the sensible substances.[52]

Although Aristotle does not specify the referent here, the whole passage indicates that it is some kind of universal (καθόλου) theory in mathematics that is not confined to any particular kind of quantity. Such propositions are to be found in the general theory of proportion in Book V of Euclid's *Elements*.[53]

By way of illustration, Ps.-Alexander (729.21 ff.) supplies an example from this theory and another from the general axioms of equality, while Syrianus (89.30 ff.) also cites the same two examples. Similarly, modern scholars like Ross (1924 ii, 413) regard Eudoxus' theory of proportion as the best example of such a universal mathematics. Aristotle's objection works better, however, if one takes him to be referring generally to the objects about which proofs are given. In Book V of Euclid's *Elements,* for instance, the lines used to represent the quantities in proportion cannot be taken representationally, since the proofs are completely general and apply to all kinds of magnitude. Thus the lines stand for all kinds of quantity just as variables (e.g. 'x' & 'y') function in algebraic formulae. In fact, modern commentators on Euclid often rewrite Book V in algebraic notation, though this tends to obscure the geometrical character of the original proofs.[54]

From this perspective one can now see the power of Aristotle's objection when it is directed against the Platonists, especially those who accepted the general theory of proportion. Since this theory is

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[52] *Met.* 1077a9–14: tr. Apostle (1966).

[53] Cf. Heath (1925) ii, 112 ff.

[54] Cf. Heath (1925) ii, 138, Knorr (1975), Lachterman (1989) ch. 2. Schussler (1982) rightly points out that the modern notion of a symbolic number cannot be attributed to Aristotle because the universal relations between quantities involved in the general theory of proportion are still relations between discrete and continuous quantities and not quantities in general.

not specifically about numbers or points or lines or any other kind of quantity which the Platonists considered to be separate substances, they are faced with the following difficulty. One implication (1077a10–11) of their objectivity argument, when applied to the general theory of proportion, is that there must be some other substance (τις ἄλλη οὐσία) which is separated from (κεχωρισμένη) and between (μεταξύ) Forms and Intermediates. Furthermore, (to compound the difficulty) such a substance cannot be either a number or a point or a magnitude or time. If this result is impossible (ἀδύνατον), as appears to be the case, then it is also impossible for these other mathematical objects to exist apart from sensible things. The whole objection assumes that the separation of mathematical objects is equivalent to treating them as independent substances.

Just as in the previous objection, the Platonic objectivity condition is the target of Aristotle's refutation here.[55] If the truth of every science depends on it having an ideal object that is separate and substantial, this must also be the case for the universal science of mathematics that Eudoxus discovered. Since it is completely general, this science cannot have the same objects as any of the particular mathematical sciences; i.e. numbers, lines, planes or solids. Therefore, according to the Platonic argument from objectivity, one must look beyond these substances for some other separated substance as an object of the new science. Aristotle adds an extra twist to this objection by insisting that such substantial objects must lie between (μεταξύ) Forms and Intermediates, though his reasons for this are not immediately obvious. Given that it is more general than any of the particular kinds of mathematical object, it is plausible to argue that the object of universal mathematics cannot be either a number or a point or a magnitude or time. What is not clear from his objection is why such an object cannot be a Form nor why it must lie between Forms and Intermediates.

But some hints may be found in *Posterior Analytics* I.5 where Aristotle refers to the well-known proposition that proportionals alternate (τὸ ἀνάλογον ὅτι ἐναλλάξ). There he says (74a17–21) that at one time

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[55] Syrianus (*in Metaph.* 90.4 ff.) concedes that the general axioms require some corresponding objects but he does not think it absurd for a Platonist to accept these as *logoi* in the soul that are more simple and universal than particular kinds of mathematical object, since these 'notions' (ἔννοιαι) are true of such things. Although Syrianus seems to have conceded the possibility of universals supplanting Platonic Forms, it is more likely that he is giving a special Neoplatonic meaning to *logoi* in the soul.

the proposition used to be demonstrated separately for quantities insofar as they were numbers or lines or solids or times, even though it was possible to give a single proof for all of them. What is striking in both the *Analytics* and *Metaphysics* passages is the coincidence between the sorts of mathematical objects listed, even down to the inclusion of time. This becomes even more significant when Aristotle explains (74a21–23) that these quantities, which differ in species (εἴδει) from one another, used to be taken separately because there was not any single way of naming them all. The accuracy of this historical report is confirmed by the difference between Books V and VII of Euclid's *Elements*. In addition, Aristotle contrasts the historical situation with the present state of affairs in which the proposition is proved universally (καθόλου) for all of these mathematical entities.

But with regard to the question of why the object of such a general mathematics cannot be a Platonic Form, the important part of Aristotle's explanation is that there was no common name available to denote the property shared by numbers, lengths, times, and solids. For Plato it was at least a necessary if not a sufficient condition for the positing of a Form that there be some corresponding common name; cf. *Rep.* 596A. Aristotle also reports (*EN* 1096a17–19) that the Platonists refused to posit Forms for any series in which there is a prior and posterior. While ordinal numbers exemplify such a series, another example is point, line, plane, and solid, which was a paradigmatic schema of priority and posteriority for the mathematical sciences themselves. Thus, in his objection, Aristotle can infer that the Platonists would not posit a Form as an object of knowledge for the general theory of proportion. Yet a consistent application of the principles of objectivity requires them to posit some object for this universal science.

We can now appreciate the force of Aristotle's objection, which exposes another absurd result that follows from consistently pursuing one of the Platonic conditions for the objectivity of a science. When this condition is applied to the general theory of proportion, the objection is given extra power by the historical fact that this theory was one of the greatest mathematical achievements of the Academy. If the Platonists accept the truth of these general theorems which range beyond any of the particular kinds of mathematical entities that are posited as substances, they should in all consistency posit another kind of separated substance between Forms and Intermediates. But clearly they did not posit such a kind of substance, even though they accepted the truth of the general theory of proportions.

III.2. *Objections from common sense*

Unlike Cherniss, I think that Aristotle's objections are based on shared assumptions; so that they may be legitimate dialectical objections despite seeming unfair to his opponents. Thus I hold that we must search for such assumptions before passing judgment on Aristotle's fairness. This is important for the next objection which begins with a general appeal to truth and to the common beliefs about substance:

> In general, if one posits the mathematical objects as natures which are separate, conclusions contrary both to truth and to the accepted beliefs will follow. For these natures, because they are posited as such, must be prior to the sensible magnitudes, but with respect to truth they must be posterior; for the incomplete magnitude is prior in generation but posterior in substance to the complete magnitude. For example, this is how the lifeless is related to the living.[56]

The prominence given to the word ὅλως in this passage seems to indicate that Aristotle is moving onto a different level of argumentation where he intends to give a more general assessment of the Platonists' position on the ontological status of mathematical objects.[57] In other words, it is not purely a reductio-style argument designed to refute the position of opponents, even though that is its ultimate intention.

This new development is signalled by the introduction of an appeal to truth and ordinary beliefs as criteria for the assessment of that position. Annas[58] has claimed that this objection begs the question in as much as it presupposes Aristotle's own philosophical ideas about substance. While this seems true on the face of it, it is important to note that he explicitly appeals to truth and ordinary beliefs about substance. This suggests that Aristotle is drawing upon some consensus about substance, rather than inserting his own peculiar views.

Once again, I think we must view his objection within its proper dialectical context in XIII.2, where the guiding question is whether or not mathematical objects are substances besides sensible substances.

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[56] *Met.* 1077a14–20: tr. Apostle (1966).

[57] In fact, the use of ὅλως may be a signal that Aristotle is about to give a more general dialectical argument based on shared opinions, as distinct from those which are more peculiar to the Platonists.

[58] Cf. Annas (1976) 139 & 143. Annas (1987) 143 has since changed her mind about this because, even though she still thinks that Aristotelian ideas about substance are being presupposed, the Platonists can also be expected to have views about substance and so the argument must move on to this issue.

The Platonic position being refuted is clearly formulated near the beginning of the present passage as follows: "... if one posits mathematical objects to exist in this way as some separated natures."[59] According to Aristotle's objection, such a *protasis* yields consequences which are contrary to truth (τοῦ ἀληθοῦς) and to what is usually assumed (τοῦ εἰωθότος ὑπολαμβάνεσθαι). From the linguistic evidence it would appear that the latter criterion appeals to some standard which is implicit in our usual way of talking; cf. *Top.* 103a8, a25.

Given his characteristic approach to philosophy, I think it is more likely that Aristotle measures the correctness of his views against shared criteria which are derived from what is usually said about substance. In *Metaphysics* VII, for instance, we find him using such objective criteria to eliminate some of the candidates for substance. For instance, according to the common opinion that substance is what remains (ὑπομένον), it would seem that matter is substance (1029a1 ff.). But, on the other hand (1029a27–28), it seems (δοκεῖ) that the special marks of substance are to be a separated thing (τὸ χωριστόν) and to be a 'this something' (τὸ τόδε τι). When these latter criteria are applied, however, it appears that the form and the composite are substances to a higher degree than matter.[60] The verb δοκεῖν suggests that Aristotle does not view these criteria (or even the results of their application) as belonging exclusively to his own metaphysical viewpoint. Thus, a little later (1029a33–34), he talks about the generally accepted substances (ὁμολογοῦνται οὐσίαι) among sensible things and makes this consensus the starting-point of his inquiry.[61]

Within this methodological framework, therefore, we can understand Aristotle's attempt to link truth and standard assumptions as twin criteria for judging the claim that mathematical objects are separated substances. He made a similar move in previous arguments when he appealed to shared criteria of priority and, in fact, his explanation in the present objection is also given in terms of priority. What he says is that, if mathematical objects are posited as some separated natures, they must be prior to sensible magnitudes whereas, in truth (κατὰ τὸ ἀληθές), they are posterior. Since there are many senses of priority, however, the dialectical context requires a Platonic

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[59] εἴ τις θήσει οὕτως εἶναι τὰ μαθηματικὰ ὡς κεχωρισμένας τινὰς φύσεις, *Met.* 1077a15–16.

[60] διὸ τὸ εἶδος καὶ τὸ ἐξ ἀμφοῖν οὐσία δόξειεν ἂν εἶναι μᾶλλον τῆς ὕλης, *Met.* 1029a29–30.

[61] Cf. also *Met.* 1042a24–25.

meaning; i.e. priority according to nature and substance (κατὰ φύσιν καὶ οὐσίαν). In the present objection, however, he talks about things that are incomplete (ἀτελές), by contrast with things that are complete, and such talk suggests the introduction of his own teleological perspective.

Aristotle gives us an important clue as to what he means by 'complete' here when he illustrates it by means of a contrast between the lifeless (ἄψυχον) and the living (ἐμψύχον). That which is 'ensouled' is taken to be more complete than and therefore prior in substance (τῇ οὐσίᾳ) to what is without soul, even though the latter may be prior in generation (γενέσει). A common Greek assumption about the priority of living over lifeless things seems to guide the biological orientation of Aristotle, which typifies his opposition to the mathematicians of the Academy. For instance, in response to the Platonist claim that mathematical magnitudes are prior in some way to physical magnitudes, he distinguishes between what is prior in substance and in generation. Thus, in the passage under discussion, he partially concedes this claim when he admits that 'incomplete magnitude' (ἀτελὲς μέγεθος) is prior in generation to the sensible magnitudes mentioned in the previous line. When he insists that such mathematical magnitudes are posterior in substance, however, he seems to be opposing the reduction of philosophy to mathematics which was typical of Speusippus and Xenocrates.

By describing these magnitudes as 'incomplete,' Aristotle is virtually assuming them to be less substantial than physical bodies. Yet the Platonists may have given an account of the 'generation' of mathematical magnitudes which implies that they are incomplete in some way. This possibility emerges clearly in a later objection:

> Again, the way generation proceeds may clarify the matter. For what is generated first is something with length, then with width, lastly with depth, and completion has been attained. Accordingly, if that which is posterior in generation is prior in substance, the body would be prior to the plane and the line, and as such it would be complete and a whole to a higher degree than they are, seeing that it can become animate. But how can a line or a plane be animate? Such an axiom is beyond our senses.[62]

The sentence which introduces the argument refers to 'generations' (γενέσεις) and claims that these make something clear (δηλοῦσιν). Ross

[62] *Met.* 1077a24–31: tr. Apostle (1966).

(1924 ii, 414) takes this to refer to the 'modes of generation of the objects of mathematics' and Aristotle's subsequent argument bears him out. Yet the use of the plural seems to imply that there are many different generations rather than one single process.[63] Thus the generation of length would be somehow distinct from the generation of width and this, similarly, would be different from the generation of depth, perhaps by virtue of having distinct first principles.

If we are to give any credence to Aristotle's reports,[64] there is some basis in the Platonist tradition for such a differentiation, since both the One and the Indefinite Dyad appear to take different forms in each of the dimensions. For his objection to stand up, however, the most important claim which Aristotle must establish is that the Platonists somehow acknowledge the generation of body to be the completion of the whole process. This claim is implicit in the way he describes the final stages of generation: '. . . and lastly depth, and the process is complete.'[65] The notion of a goal or end is explicitly contained in the noun τέλος, and it is echoed in the adverb τελευταῖον.

But the important question for the issue of fairness is whether he is imposing a teleological perspective upon what may have been a purely logical Platonic schema. Within the context of a general discussion of motion and change in *Laws* X, Plato at least uses teleological language. In answer to the question of how *genesis* takes place, he gives (894A1–3) a description of three definite stages through which the generation of a sensible thing proceeds. Obviously these stages correspond to the three dimensions of length, width, and depth, through which mathematical magnitudes are successively generated. The process is described as beginning from a principle (ἀρχή) and growing in stages until it reaches a third stage where the result becomes perceptible.[66]

The terminology used by Plato is comparable to that used in Aristotle's objection which, in turn, is paraphrased by Ps.-Alexander

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[63] In view of the ambiguity surrounding 'generation' in this whole passage, we cannot rule out the possibility that Aristotle is implicitly referring to the different processes involved in generating material bodies, as distinct from geometrical bodies.

[64] Cf. *Met.* 1085a7–23, 1090b21–24. Cherniss (1944) is rather dismissive of all such reports as being inaccurate historically and as resulting from Aristotle's own method of interpretation.

[65] . . . τελευταῖον δ' εἰς βάθος, καὶ τέλος ἔσχεν, *Met.* 1077a25–26.

[66] Gaiser (1963) 173 ff. finds this passage in *Laws* X to be especially suggestive for the 'generation' of the dimensions of being from the One, which he takes to be a central motif in Plato's so-called 'unwritten doctrine.'

(731.16 ff.) in terms of growth and motion in general. Thus, if the Platonists speak in these terms about the 'generation' of mathematical objects (even if such talk is metaphorical), they are implicitly committing themselves to the view that the last stage is the completion of a process. Such a view is reinforced by the common Greek assumption that there are only three dimensions of magnitude and, hence, that body is the most complete magnitude. We find this conclusion in *De Caelo* I, for instance, where Aristotle says (268a31–b5) that we cannot pass beyond body to a further kind of magnitude as we passed from length to surface and from surface to body. In the present argument, therefore, it seems to be the same common assumption which justifies his conclusion that the process of generation has reached its goal (τέλος).

If this is the case, however, Aristotle is not foisting his own teleological perspective upon the Platonists so much as bringing out what is implicit in their views, and making it explicit in the criterion that what is posterior in generation is prior in substance. But, in effect, this criterion is a statement of the relationship between two different orders of priority each of which is produced by the application of other criteria. What it says is that the order of priority with respect to 'generation' (which is governed by its own criterion) is the complete reverse of the order of priority with respect to substance. For instance, the line and the plane may be prior in generation to the solid but the solid body (τὸ σῶμα) is prior in substance to them. Aristotle goes on (1077a28–29) to explain that, insofar as body is substantially prior it is complete (τέλειον) and more of a whole (ὅλον μᾶλλον) because it can become ensouled.[67] It would appear that the capacity of a body for becoming animate is what makes it prior in substance to a line or a plane, since it seems impossible that a line or a plane could become animate. Yet the priority in substance of a body may have already been decided by some other criterion (such as non-reciprocal dependence) which is shared by the Platonists. Here the outstanding difficulty for interpreters stems from two sets of related ambiguities which surround the concepts of 'body' and 'generation.'

In Aristotle's use of the word γενέσις here, both Bonitz (1849) and

[67] Moukanos (1981) 20 thinks that Aristotle's use of the word ἔμψυχος here clearly indicates that he is thinking of a material rather than of a geometrical body. If that were the case, however, the objection would have little bite against opponents who talked about the 'generation' of geometrical bodies. But perhaps Moukanos failed to

Ross (1924) find a serious ambiguity; i.e. that natural *genesis* is being confused with the 'generation' of mathematical objects. A similar opinion has, I think, led Annas (1976, 144–46) to charge that he consistently confuses mathematical solids with physical bodies. This seems justified by the fact that Aristotle uses the word σῶμα for a mathematical solid and then treats it as if it were a physical body when he talks about it becoming animate.

But perhaps there is some basis for Aristotle's objection in the way Platonists talk about the 'generation' of mathematical magnitudes. For instance, in *Metaphysics* XIV.4, their use of temporal indexes such as πρότερον enables Aristotle to deny (1091a28–29) that it is simply for the sake of theoretical analysis (τοῦ θεωρῆσαι ἕνεκεν) that they give an account of the generation of numbers (τὴν γένεσιν τῶν ἀριθμῶν). According to Ps.-Alexander (819.37–820.7), this defence was given by Xenocrates who claimed that any talk about the generation of numbers was not intended literally but only for the sake of instruction and knowledge (διδασκαλίας χάριν καὶ τοῦ γνῶναι) about the eternal and unchanging nature of numbers.[68]

But, just as in *De Caelo* I.10, Aristotle stubbornly refuses to accept such a defence from the Platonists. He insists that any talk about 'generation' with reference to mathematical objects must be taken in a physical sense that has definite temporal implications. With respect to the objection in XIII.2, therefore, I think that such an insistence is motivated by his interpretation of the Platonic view about the mode of being of mathematical objects. If these are substances which undergo some process of generation, then he feels justified in applying the general rule that things which are posterior in generation are prior in substance and in nature.[69]

It also becomes clear from the subsequent objection that Aristotle is determined to impose on mathematical objects what he considers to be the unity-conditions for sensible substance:

> Again, a body is in some sense a substance, for it is somehow already complete. But how can lines be substances? Neither as form or shape,

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see that Aristotle's objection trades on ambiguities in the notion of 'body' and in associated modes of 'generation.'

[68] But then it is unclear how we should understand Xenocrates' own definition of soul as 'self-moving number' or his talk about the motion of a point producing a line, and so on up to the solid; cf. *DA* I.2, 404b27 & I.4, 409a4–5.

[69] τῇ οὐσίᾳ καὶ τῇ φύσει πρότερα, *Met.* 1050a4–6, 989a15–16, *Phys.* 261a13, *PA* 646a24–27.

> if such is the soul for example, nor as matter, like the body for example; for we observe that no body can consist of planes or lines or points. If these latter were material substances of some kind, we would have observed them capable of being put together to form a body.[70]

The objection begins with the admission that body (σῶμα) is a kind of substance (οὐσία τις), as judged by the criterion of completeness. This is based on the common Greek assumption that body is the most complete magnitude because it has all three dimensions. In order to explain Aristotle's claim that body has a certain kind of completeness, Ps.-Alexander[71] also refers to *De Caelo* I, while taking pains to point out that this is not the absolute completeness (ἁπλῶς τέλεια) which physical bodies possess.

But there is no evidence in *Metaphysics* XIII.2 that Aristotle means to advert to the distinction between physical and mathematical body. In fact, all the evidence points to an intentional conflation of these two senses of 'body' for the purpose of dialectical refutation, and in the above passage there are clear signs that he is treating body as a material substance. When he asks (1077a33–34) the rhetorical question about how lines might be substances, for instance, one of the options given is that they might be substances in the material mode (ὡς ὕλη) like body. Based on the evidence of sense perception, however, Aristotle rejects (1077a34–35) the possibility that such mathematical entities might be material substances, since nothing appears (φαίνεται) to be capable of being compounded from lines or planes or points. His argument takes the logical form of *modus tollens,* which is implicit in the following conditional: If these were some kind of material substance then some sensible things would appear capable of undergoing this (composition).[72] But no sensible thing seems to be thus compounded from planes, lines, or points, and so it follows that they are not any kind of material substance.

Furthermore, he concludes (1077a32–33), they are not substances in the manner of some form or shape (ὡς εἶδος καὶ μορφή τις) because soul is the paradigm for this kind of substance. This conclusion is

[70] *Met.* 1077a31–36: tr. Apostle (1966).

[71] Cf. *in Metaph.* 732.4 ff. Moukanos (1981) 19 thinks that Ps.-Alexander is mistaken in taking Aristotle to refer to geometrical body because he does not take account of the fact that σῶμα is described as ἔμψυχος within the immediate context. But I think that Moukanos fails to see how the dialectical argument requires σῶμα to refer to both geometrical and physical body.

[72] εἰ δ' ἦν οὐσία τις ὑλική, τοῦτ' ἂν ἐφαίνετο δυνάμενα πάσχειν, *Met.* 1077a35–36.

not argued here, presumably because the previous objection has dismissed the possibility that planes and lines could become ensouled, as something that is beyond our senses. In both passages one should notice that sense experience is used as a touchstone for whether or not something is substantial. Once more, this suggests that Platonic claims about the substantiality of mathematical objects are being weighed against exemplary cases of sensible substance, especially material bodies.[73]

It must be this sense of 'body' that Aristotle has in mind in XIII.2 when he asserts that it can be become ensouled and that it is a kind of substance. Indeed, it is precisely this capacity for becoming animate that differentiates body as somehow substantial, in contrast with planes and lines which have no such potential. According to this criterion, therefore, Aristotle can eliminate the possibility that such mathematical objects are material substances. Presumably he also means to deny them any claim to substantiality as compounds of matter and form, since they are not capable of receiving any substantial form like that of soul. One may object, however, that the form belonging to mathematical objects is such that mathematicians would never feel the need to posit some principle of motion in them, since the pure mathematical sciences do not study motion.

Such a possible objection pinpoints another curious feature of Aristotle's whole argument in this section of XIII.2, which is closely connected with his neglect of the distinction between mathematical and physical bodies. It seems rather strange that he should insist upon biological form, to the exclusion of every other sort of form, as the paradigm of substantial form. Given the conflation of mathematical and physical body, this insistence yields the odd result that mathematical bodies may qualify as material substances insofar as they can become ensouled. Of course, one might infer from the context that Aristotle does not really entertain the possibility that mathematical objects are substances in any sense. But he himself does not draw this conclusion about mathematical solids, even though he has the logical equipment to distinguish them from physical bodies. Perhaps the neglect of this distinction is part of his dialectical strategy in refuting the Platonists. If they in turn failed to make the distinction then his

[73] See *De Anima* II which distinguishes substance as matter and as form from substance in the sense of a composite of matter and form. Since an ensouled body is such a composite substance, we may take it that body as material substance and as subject is the relevant sense in *Metaphysics* XIII.2.

ploy has a certain legitimacy in a dialectical joust conducted according to the rules set out in the *Topics*. I think it is a grasp of such rules, rather than any modern concept of fair play, which should govern our understanding of Aristotle's dialectical procedure.

We should notice in passing another peculiarity of this set of objections concerning the unity and generation of mathematical objects; namely, Aristotle's consistent use of sense perception as a touchstone for adjudicating their claim to substantiality. This seems rather strange, especially in view of the fact that he represents the Platonists as holding that mathematical objects are independent substances which are separated from sensible things. It can hardly have escaped Aristotle's attention that they usually treated Forms and other objects of knowledge as being inaccessible to the senses. Thus, I think it must have been the claim that such objects are substances which led Aristotle to judge them by the criteria for sensible substance. But even this explanation is rather unsatisfactory because in *Metaphysics* XII, for instance, he himself acknowledges that there is a kind of separated substance which is not accessible to the senses. Why shouldn't the same criteria which admit the existence of this kind of substance be applicable to mathematical objects, especially since they are candidates for supersensible substance? However, his arguments for the existence of a Prime Mover as the ultimate source of motion in the universe do begin from the perception of motion in sensible substances. But let me postpone these general cosmological questions to my next chapter, since they are not given any prominence in the special treatise at XIII.1–3.

In the final objection of XIII.2, Aristotle identifies the nub of his dispute with the Platonists about mathematical objects:

> Let it be granted that they are prior in formula to the body. But it is not always the case that what is prior in formula is also prior in substance. For A is prior in substance to B if A surpasses B in existing separately, but A is prior in formula to B if the formula of A is a part of the formula of B; and the two priorities do not belong to the same thing together. For if attributes, as for example a motion of some kind or whiteness, do not exist apart from substances, whiteness is prior in formula to the white man but not prior in substance; for whiteness cannot exist separately but exists always in the composite. By 'the composite,' here, I mean the white man. So, it is evident that neither is the thing abstracted prior, nor is what results by addition posterior; for it is by addition of whiteness that we speak of a white man.[74]

[74] *Met.* 1077a36–b11: tr. Apostle (1966).

The initial μέν here shows that Aristotle is prepared to concede that mathematical objects are prior in definition (τῷ λόγῳ πρότερα) to sensible bodies, but he minimizes the concession by saying that not all things which are prior in definition are also prior in substance (τῇ οὐσίᾳ πρότερα). He supports this distinction by citing different criteria for the two types of priority. Some thing A is prior in substance to something else B if A surpasses B in existing separately (χωριζόμενα τῷ εἶναι ὑπερβάλλει), whereas A is prior in definition to B if the definition of A is part of the definition of B. Aristotle warns that the two types of priority do not always belong to the same thing (οὐχ ἅμα ὑπάρχει).[75]

In *Metaphysics* VII.1, however, he says (1028a31–33) that priority in substance and definition coincide in the same thing when it is a primary substance. There he explains that none of the entities belonging to the other categories is separable (χωριστόν) but only substance is independent and, hence, prior in this manner. Furthermore, he claims, substance is prior in definition (τῷ λόγῳ) because it is necessary for the definition of each thing to contain in itself the definition of its substance.[76] Since what is captured in a definition is the essence of a thing, this provides further confirmation for the coincidence of priorities with respect to definition and substance. It also shows that Aristotle's claim in XIII.2 that the two kinds of priority do not always belong to the same thing together applies to accidental attributes, which are not separate from substances though they may be defined independently.

This becomes obvious when Aristotle introduces his examples with the following conditional statement: "For if attributes do not exist apart from their substances . . ."[77] This ontological condition has the following implication for an attribute like whiteness. Even though 'white' may be prior to 'white man' with respect to formula (κατὰ τὸν λόγον), it cannot be prior with respect to substance (κατὰ τὴν οὐσίαν) because it cannot exist as a separated thing but it is always together with the composite thing.[78] This explanation contains a clear

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[75] I am grateful to Alejandro Vigo (1991) for his criticism of my book (Cleary 1988) on this point, and I now accept his argument that a coincidence of priorities is the normal situation for substance, whereas they can be distinct for accidental composites like 'white man.'

[76] ἀνάγκη γὰρ ἐν τῷ ἑκάστου λόγῳ τὸν τῆς οὐσίας ἐνυπάρχειν, *Met.* 1028a35.

[77] εἰ γὰρ μὴ ἔστι τὰ πάθη παρὰ τὰς οὐσίας, *Met.* 1077b4–5.

[78] οὐ γὰρ ἐνδέχεται εἶναι κεχωρισμένον ἀλλ' ἀεὶ ἅμα τῷ συνόλῳ ἐστίν, *Met.* 1077b7–8.

application of the criterion of priority in substance; i.e. the quality of whiteness is not prior in being to the white man because it does not surpass the latter in its capacity for independent existence. But the example is chosen to show that whiteness may still be prior in definition, in the sense that the formula of 'white' is part of the formula of 'white man,' which is an accidental composite.

A parallel for this sense of priority may be found in *Metaphysics* V.11 where the accident is said to be prior to the whole with respect to formula.[79] The example given there is exactly parallel in that 'the musical' is said to be prior to 'the musical man' because the whole formula cannot be without its part.[80] Just as in XIII.2, however, Aristotle adds a caveat against thinking that this entails priority in substance, since it is not possible for 'the musical' to exist without someone who is musical.[81] Again we find here a clear application of the criterion for priority in substance, according to which the man as subject surpasses 'the musical' in the power of existing separately. In XIII.2, Aristotle's reason for choosing an example like 'white man,' which Annas[82] found puzzling, is that whiteness is not the essence of man, and so it illustrates how priority in substance and definition are not always identical.

In view of Aristotle's tendency in *Metaphysics* VII to reserve the term 'definition' for an account of the essence, perhaps we should follow Apostle in talking about priority in formula. This would be more consistent with the special sense in which the term 'composite' (σύνολον) is used by Aristotle in XIII.2. It does not refer to the substantial compound of matter and form (e.g. body and soul) but rather to the combination of an accident like 'white' with a substantial subject like 'man.' Since this is not the subject to which color predicates belong *per se,* 'white' can be defined without reference to 'man.'[83] Such is the logical basis for Aristotle's claim that 'the white' is prior in formula to 'the white man.' For, when one wants to define the latter compound thing, one must first define whiteness and then add the definition of man, since an accidental composite lacks even the

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79 κατὰ τὸν λόγον δὲ τὸ συμβεβηκὸς τοῦ ὅλου πρότερον, *Met.* 1018b34–35.

80 οὐ γὰρ ἔσται ὁ λόγος ὅλος ἄνευ τοῦ μέρους, *Met.* 1018b35–36.

81 καίτοι οὐκ ἐνδέχεται μουσικὸν εἶναι μὴ ὄντος μουσικοῦ τινός, *Met.* 1018b36–37.

82 Cf. Annas (1976) 147. It should be noted that she later (1987) treats this example as an illustration of the method of subtraction used on an accidental composite.

83 Cf. *Met.* VII.5, 1030b24–26 & VII.6, 1031b22–28.

horizontal unity of composites like 'snub nose.'[84] This fits with the simple criterion for priority in formula; i.e. that the formula of 'white' is part of the formula of 'white man.'

Despite the clear logical basis for this argument, it is still quite legitimate to ask how it constitutes an objection to the Platonist claims about the ontological status of mathematical objects. Given the whole topic of the treatise, it is rather curious that he chooses a quality like whiteness rather than some quantity, in order to make his point about the non-coincidence of two kinds of priority. According to his own categorial framework, however, both quantities and qualities are accidents of primary substance and so can be defined separately from it. Thus Aristotle's example suggests that the Platonists have been misled by this logical possibility. The fact that whiteness can be defined independently of sensible substances does not mean that there is some Whiteness Itself apart from sensible things, as the Platonists thought; cf. *Phys.* 193b35 ff. Although mathematical quantities are more separable from sensible things than qualities, one cannot infer that they are independent substances from the fact that their definitions do not presuppose any sensible subjects to which they belong *per se*.

This is the general thrust of Aristotle's rather strange conclusion in the present passage when he says (1077b9–11) that 'the result of subtraction' (τὸ ἐξ ἀφαιρέσεως) is not prior nor is 'the result of addition' (τὸ ἐκ προσθέσεως) posterior. The terminology of abstraction is introduced here quite suddenly, and the context provides little guidance as to how it should be interpreted, except for the explicit contrast with addition. Fortunately, Aristotle does give us a clue as to what he means by addition when he says that it is as a result of adding to whiteness that the white man is spoken of.[85] From the previous passage we may assume that he is here referring to the addition of a subject (i.e. man) that is not the primary subject to which the quality of whiteness belongs *per se*. Conversely, abstraction would be the process of taking away that subject and defining white separately. This fits with Aristotle's denial of priority to 'the result of subtraction,' since he previously said that 'the white' is not prior in substance to 'the white man' even though it may be prior in formula.

[84] Cf. *Met.* VII.4, 1029b25 ff. & VII.5, 1030b15 ff. Here I borrow Gill's (1989) distinction between the vertical unity that belongs to a primary substance and the horizontal unity that belongs to any composite, even if it should turn out to be a particular substance.

[85] ἐκ προσθέσεως γὰρ τῷ λευκῷ ὁ λευκὸς ἄνθρωπος λέγεται, *Met.* 1077b11.

In fact, it is quite clear from the whole passage that priority in substance is being denied to the so-called 'result of subtraction.' The ancient Greek commentators[86] assume that Aristotle is here referring specifically to mathematical objects, though they give no adequate explanation of why mathematical objects are referred to as 'the results of abstraction' or of what implications this terminology may have for their ontological status. This is a lacuna even in modern Aristotelian scholarship, which needs to be filled by explaining how such terminology applies to mathematical objects, given the peculiar fact that it is not used at all by Aristotle in XIII.3 for his positive account of the mode of being of mathematical objects.[87]

Finally, in summing up his attempt to refute the Platonists, Aristotle makes it quite clear that his previous concession to them amounts to no more than admitting that mathematical objects may be logically prior to sensible things:

> That the mathematical objects are not substances to a higher degree than bodies, nor prior in existence to sensible things but prior only in formula, nor yet capable of existing separately anywhere, has been sufficiently stated. But since it was shown that they cannot exist in sensible things, it is evident that, either they do not exist at all, or they exist in some sense and because of this they exist not without qualification; for "existence" has many senses.[88]

In the first part of this passage, one can count at least three theses directed against the multifaceted position of the Platonists: (a) that mathematical objects are not more substantial than bodies; (b) that they are not prior in being to sensible things but (c) that they are prior only in formula; and (d) that they are incapable of existing separately anywhere. Although Aristotle lists these conclusions separately, our analysis has shown that they are all interconnected.

The first conclusion is specifically directed against the claim that mathematical objects are substances to a higher degree than bodies. As we have seen, this sort of claim reflects the Pythagorean schema of point, line, plane, and solid, which was taken to reflect the order of reality. Hence the second conclusion may be taken as rejecting

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[86] Cf. Ps.-Alexander *in Metaph.* 733.23–24 & Syrianus, *in Metaph.* 93.22 ff.

[87] For instance, Moukanos (1981) 24 ff. simply claims that the conclusion of XIII.2–3 is that mathematics is about abstract objects, which exist through the separating reflection of mathematicians. But he fails to explain why the terminology of abstraction is conspicuously absent from XIII.3.

[88] *Met.* 1077b12–17: tr. Apostle (1966).

such a mathematical ontology, if being is given its primary meaning of substance. In the light of previous distinctions, this interpretation is very plausible since the third conclusion (c) states that mathematical objects are prior only in definition. Thus it is actually a continuation of the previous conclusion (b) and we are safe in assuming that the whole thing states the same distinction as that made earlier between priority in formula and priority in substance. As I have argued, this distinction is Aristotle's way of explaining the logical independence from sensible things of mathematical quantities, while denying the ontological implications which the Platonists drew from this separation. As with 'white man,' it is possible to define mathematical attributes independently of sensible things because the latter are not the subjects to which these attributes belong *per se.* Yet the Platonic condition for objectivity in science is satisfied by the logical fact that mathematical definitions are independent of sensible things.

Thus Aristotle can retain this objectivity condition without accepting the ontological condition that mathematical objects must be separated (κεχωρισμένα) from sensible substances. Now this is precisely what is denied in the fourth (d) conclusion which says that mathematical objects cannot exist anywhere (πoυ) as separated entities.[89] Aristotle's use of an adverb of place may betray a spatial interpretation of this separation, despite Plato's warning that Forms (and presumably mathematical objects also) are not in any place. Given the terms of reference within which he conducts this inquiry, however, if mathematical objects are to be independent substances they must be either separate (spatially) from sensible substances or *in* them. These are the two possibilities which he has now refuted to his own satisfaction and so he concludes, quite generally, that they do not exist without qualification (ἁπλῶς).

Yet two other possibilities still remain: (iii) either they do not exist at all (ὅλως οὐκ ἔστιν) or (iv) they exist in some way (τρόπον τινὰ ἔστι). These correspond exactly to the last two possibilities set out at the end of XIII.1, but the third possibility is not taken seriously because it would undercut mathematics completely. This leaves only the fourth possibility whose elucidation is the specific task for XIII.3, where he

[89] If we read πoυ as an indefinite adverb of manner, the conclusion would still make sense as the claim that mathematical objects cannot exist in any way as separated entities, since both options for this mode of being have been decisively rejected in XIII.2.

develops his own views on the mode of being of mathematical objects. All of these possibilities depend on Aristotle's claim that being is said in many ways, which by itself is an anti-Platonic thesis.

IV. *Untying the knots*

Since Aristotle thinks that he has refuted all the other possible views concerning the ontological status of mathematical objects, the remaining task is to give an alternative account which will avoid the difficulties raised. If his account manages to do this, while also saving the most authoritative phenomena, then it will be a successful resolution of the problem according to his own methodological criteria. Among these phenomena is Eudoxus' general theory of proportion which Aristotle already used against the Platonists and now makes the starting-point for his own proposed solution:

> Now, just as certain universal propositions in mathematics, which are about things not existing apart from magnitudes and numbers, are indeed about numbers and magnitudes but not qua such as having a magnitude or being divisible, clearly, so there may be propositions and demonstrations about sensible magnitudes, not qua sensible but qua being of such-and-such a kind.[90]

Here Aristotle seems to be appealing to the fact that mathematicians use general axioms and propositions about different species of quantity (though he does not use the term πόσον), without positing other objects besides magnitudes and numbers. Structurally, the argument draws a parallel between the fact that there are such general propositions and the possibility of statements and proofs similarly being made about sensible magnitudes. The first part of the parallel assumes as established (from the refutation of the Platonists in XIII.2) that the propositions of general mathematics are not about separated things (περὶ κεχωρισμένων) apart from magnitudes and numbers. Yet, while any proposition from the general theory of proportion is about magnitudes and numbers, it is not about them in so far as (ᾗ) these things have continuity or are discrete. Although geometry is about continuous quantity and arithmetic is about discrete quantity, such general propositions are not confined to either type of quantity.

[90] *Met.* 1077b17–22: tr. Apostle (1966).

Having established this starting-point from the general theory of proportion, Aristotle draws the parallel which is crucial for his own argument. He claims (1077b20–22) that, in a similar way, there can be propositions and proofs about sensible magnitudes, not insofar as they are sensible but insofar as they are such-and-such (ἀλλ' ᾗ τοιαδί). What he seems to mean is that one can select some definite quality (τοιαδί) of sensible magnitudes and construct demonstrations with respect to it as subject, while excluding the sensible aspect from consideration. Perhaps the parallel goes as follows: just as one can ignore the distinction between continuous or discrete quantity; so also sensible magnitudes can be considered not as sensible but only as magnitudes.[91]

This amounts to the anti-Platonist claim that a mathematical science like geometry is not about some separated magnitudes but is really about sensible magnitudes, only not qua (ᾗ) sensible. Aristotle thereby denies that ontological separation is a conditions for scientific objectivity, and suggests instead that a mathematical science like geometry attains its objects by ignoring the sensible aspects of the things which it studies. Here one might be tempted by Jacob Klein's (1968, 100–13) psychological interpretation of this process as a mental disregarding of sensible attributes by means of which the geometer perceives the unchanging aspect of sensible magnitudes that will satisfy the demands of a science. No doubt the mathematician must play some role in reaching this scientific perspective but I think that the most important point concerns the logical situation. Given the Platonic assumption that sensible things cannot be objects of science, it makes sense to say that Aristotle's primary consideration will be to establish that true knowledge of sensible magnitudes is logically possible through a disregarding of their sensible aspect. In this connection, let us examine some remarks which he makes in *Posterior Analytics* I.5 about the general theory of proportion.

The context for these remarks is provided by a discussion (74a4 ff.) of the different ways in which one might be mistaken in thinking

[91] Syrianus (*in Metaph.* 95.13–17) expresses some surprise at what he sees as Aristotle's attempt to find a parallel in ontological status between universals and mathematical objects, since the former are logical entities belonging in the soul, whereas the latter are held to be in sensibles and to be also mental abstractions (ἐπινοίαις ἀφαιρούσαις) from sensibles. But such remarks tend to suggest that a general theory of abstraction was the product of commentators like Alexander who proposed it as official Aristotelian doctrine, which Syrianus is here opposing.

that one has proved an attribute to belong primarily (πρῶτον) and universally (καθόλου) to a certain subject. In a prior discussion (I.4) of the conditions for universality and necessity in demonstration, Aristotle had established (73b25 ff.) that an attribute must be shown to belong to a subject *per se* (καθ' αὑτὸ) and qua itself (ᾗ αὐτό). In most cases these two conditions seem to have identical force; e.g. 'point' and 'straight' belong *per se* to 'line' and they also belong to it qua line (ᾗ γραμμή). The attribute of having the sum of its interior angles equal to two right angles belongs to the triangle qua triangle (ᾗ τρίγωνον) and hence satisfies the conditions for necessity and universality.

The crucial test for such universality is whether the attribute in question can be shown to belong to any chance instance (ἐπὶ τοῦ τυχόντος) of the relevant subject and to belong to it primarily. This test can be used to weed out errors which would undermine the generality of demonstrations. For instance, one might be mistaken in thinking that 'figure' rather than 'triangle' is the primary subject to which some attribute belongs. Yet there are other possible mistakes about the primary subject which cannot be directly eliminated through this test, since it presupposes that we already have some access to the subject. For instance, if we thought that all triangles were isosceles then we might give a perfectly valid proof that the internal angles equal two right angles but it would not be generally sound. The first of the above tests would not enable us to detect the error because any chance isosceles triangle will have its internal angles equal to two right angles. Still, as Aristotle points out (73b39), this is not the primary subject of this particular attribute because the triangle is prior (πρότερον). So he relies (74a32 ff.) on the second test for determining the primary subject of any given attribute and, in fact, this is central to his method of subtraction.[92]

Within this logical context, the theory of proportion is used to illustrate the second way in which one might be mistaken about the primary subject to which an attribute belongs. The first way of error is associated with cases where we cannot grasp the higher term (ἀνώτερον), such as 'triangle,' apart from the particular species of triangle; while the second way is related to cases where there is an unnamed term (ἀνώνυμον) above different species of things. Later he

[92] For more detailed analysis of the function of 'subtraction' here see Cleary (1985) 21–25.

illustrates the second kind of error by referring to the proposition that proportionals alternate, which is part of the general theory of proportion.[93] Before the discovery of the general theory of proportion, Aristotle says, the proposition in question was once proved separately for numbers, lines, solids, and times; even though a single demonstration could have been given for all of them.[94]

According to Aristotle's analysis, therefore, the reason for this error was the lack of a single (generic) name to cover all of these things, which differ from each other in species (εἴδει) and which were therefore taken separately. But the mistake has now been rectified:

> Nowadays, however, the theorem is proved universally; for alternation belongs to these not qua lines or qua numbers or etc., but qua being this (thing), and it is to this (thing) that alternation is now assumed to belong universally.[95]

Unfortunately, Aristotle does not supply any name for this aspect which is being treated as the generic subject for attributes like alternation that belong *per se* to it according to the general theory of proportion. In the light of the *Categories* (4b22 ff.), one might follow the suggestion of Heath[96] that quantity (ποσόν) seems to be an appropriate term for that generic aspect shared by numbers, lines, surfaces, bodies, and times.[97] Euclid used the term 'magnitudes' (μεγέθη) for the subject matter of Book V but he may have coined this himself. What Aristotle's historical remark suggests is that there was no

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[93] Cf. Euclid's *Elements*, Book V, Prop. 16.

[94] ὥσπερ ἐδείκνυτό ποτε χωρίς, ἐνδεχόμενόν γε κατὰ πάντων μιᾷ ἀποδείξει δειχθῆναι, *APst.* 74a19–21.

[95] *APst.* 74a23–25: tr. Apostle (1981) with his own insertions in parentheses. Apostle's translation eliminates an ambiguity in the last clause of this passage which goes as follows: ὃ καθόλου ὑποτίθενται ὑπάρχειν. While the relative pronoun may be referring back to the 'this' (τοδί) of the previous phrase, it does not seem to function grammatically as a subject to which the attribute of alternation belongs. If it did then we should expect to find the dative of the pronoun (ᾧ), rather than the nominative, accompanying ὑπάρχειν. Thus I think that the qualifying phrase should be taken simply as referring to some particular aspect (τοδί) which mathematicians assume to belong universally to numbers, lines, times, etc.

[96] Cf. Heath (1949) 43–44. For instance, Valditara (1988) has no difficulty in accepting that abstract quantity was the subject-matter of the universal mathematics developed within the Academy.

[97] Mendell (1985) 232, however, argues from the absence of the word πόσον to the conclusion that Aristotle cannot have quantity in mind as the subject-genus for the general theory of proportion. It is indeed remarkable that Aristotle studiously avoids using his own categorial term, given that the Eudoxean theory covers all the species of quantity, while uniting them under a single general perspective. Perhaps the reason is that a univocal subject-genus cannot be posited to cover the species of

such generic term available to Eudoxus when he discovered the general theory of proportion.

Such a historical background lends further significance to the word τοδί here, which seems to draw attention to a certain universal aspect of numbers, lines, planes (etc.) about which the proposition that proportionals alternate is being proved.[98] This fits with Aristotle's claim that the proposition is not being demonstrated primarily and universally for numbers qua numbers or for lines qua lines, though it still applies to them insofar as they are quantities or magnitudes. In other words, this general quantitative aspect indicated by τοδί is the primary subject to which the relevant attribute belongs *per se* and qua itself. Thus, it is due to the fact that numbers and lines are quantities that the general theory of proportion applies to them and not in virtue of the fact that they are simply numbers or lines.[99]

We can now see why Aristotle in *Metaphysics* XIII.2 & 3 could make double use of this theory both as a refutation of Platonism in mathematics and as a model for his own alternative account of the objects of mathematics. It is obvious, on the one hand, that the propositions of a general theory of proportion are not about things separated from specific magnitudes and numbers. Yet, on the other hand, it is also clear that the theory is not about any specific kinds of quantity as such, since it considers them only insofar as they share the general characteristic of being quantities. Here, drawing upon the general theory of proportion, Aristotle shows that by ignoring

quantity which constitute a series whose members are related as prior and posterior. This would be consistent with Plato's refusal to posit a universal Form to cover such a series, and with Aristotle's report (*APst.* I.5) that there was no general name for quantity, and that proportions were proved separately for each kind of quantity before Eudoxus.

[98] In view of the absence of a concrete term, however, one might also treat τοδί as Aristotle's attempt to indicate some intelligible aspect of things which is only accessible to the kind of thinking that is characteristic of universal mathematics.

[99] In *Posterior Analytics* II.17 (99a8–11), Aristotle uses the theorem that proportionals alternate to illustrate how the same effect may be produced by different causes, which are yet the same under a more generic aspect (ὡς ἐν γένει). Thus, for instance, the cause of the alternation of proportionals is different for lines and numbers, when these are considered as lines and numbers; yet the cause is also the same, when they are considered as acquiring such-and-such an increment (ᾗ ἔχον αὔξησιν τοιανδί). The deliberately vague description of this generic cause is typical of Aristotle's other references to the quantitative aspect of things that is relevant to the general theory of proportion. Such vagueness suggests that the Eudoxean theory does not have a strictly univocal subject-genus, according to Aristotle, nor one that is purely equivocal but rather one with some kind of analogical unity that lacks an exact linguistic correlate.

differentiae such as continuity and discreteness it is possible to have a science of quantities without positing some separate entity called 'quantity' apart from numbers, lines, planes, solids and times.

Similarly, he argues (1077b22), it is possible for geometry to make demonstrations about sensible magnitudes, not qua sensibles but qua such and such (ᾗ τοιαδί). Once again we find the deictic form of a pronominal adjective being used to indicate some aspect of sensible magnitudes which is being singled out as the primary subject for mathematical attributes. As usual, it is accompanied by the 'qua' (ᾗ) locution, which itself serves as an index to the aspect that is isolated for scientific inquiry. Thus, for instance, it is logically possible to consider sensible magnitudes qua solids or qua planes or qua lines. In this way, as Aristotle puts it, one is not considering them qua sensible, even though the science is (in some sense) about sensible magnitudes. But one might wonder how mathematics studies sensible magnitudes if it ignores their sensible aspect. In effect, this is to ask about how Aristotle's position differs from that of the Platonists who say that mathematics cannot be about sensible things. These are questions which must be answered in a logically convincing manner if Aristotle is to provide an alternative account of the foundations of mathematics.

IV.1. *Aristotle's theory of subtraction*

My claim is that the key to his answer lies in the 'qua' locution and in a related method of subtraction, both of which are to be found in the *Posterior Analytics* (I.5) where Aristotle explains the logical basis for a general theory of proportion. Immediately after the passage quoted above, he goes on to illustrate a third mistake that is possible when one demonstrates that some attribute belongs to a subject. For instance, one might prove for each kind of triangle (i.e. equilateral, scalene, & isosceles) that its internal angles are equal to two right angles. Still, as he points out, one would not know that it belongs to triangle qua triangle (ᾗ τρίγωνον), even if one had enumerated every species of triangle, because one had failed to identify the *primary* subject of predication, which is here indexed by the 'qua' locution. As we have seen, this is the strictest condition for universal scientific knowledge, so it is natural for Aristotle to address (74a36–37) the question of how we ascertain when we have such knowledge. His rather obscure answer seems to involve a method of subtraction which isolates

the primary subject to which an attribute belongs. For instance, the attribute of having internal angles equal to two right angles belongs to the bronze isosceles triangle. Yet, when 'bronze' and 'isosceles' are subtracted (ἀφαιρεθέντος), this attribute will still belong to the triangle and such a result shows that 'triangle' is the primary subject to which it belongs. Here subtraction is used on a sensible compound to identify the primary subject of certain geometrical attributes.

Thus it would appear that subtraction (or abstraction) is a logical procedure which allows one to isolate the primary subject of a given attribute in the following manner. One focuses upon a particular attribute (e.g. having internal angles equal to two right angles) and asks: to which aspect of a sensible triangle does this attribute belong primarily? Unless one can answer this question it is not possible to have scientific knowledge in the strict sense with respect to sensible triangles or any other sensible things, given their initial appearance as a confused jumble of aspects; cf. *Phys.* 184a21–22. Finding an answer through subtraction seems to presuppose a certain order in which aspects are taken away; e.g. the bronze aspect of the sensible triangle is subtracted before the isosceles aspect. After each step in the procedure, one can ask whether the attribute in question has been eliminated along with the particular aspect that has been conceptually subtracted. Attributes like 'heavy' and 'light,' for instance, disappear along with the bronze aspect which is therefore their primary subject. Similarly, the attribute of having the sum of its internal angles equal to two right angles is eliminated when one subtracts the aspect of triangularity from this sensible figure.

Of course, as Aristotle himself realizes, this attribute would also disappear if one removed the aspect called 'figure,' yet he insists (*APst.* 74b2–3) that this would not make figure the primary subject to which the attribute belongs *per se.* The logical basis for this has already been given when he says (73b35–36) that such an attribute cannot be proved of any chance figure. We recall that this was one of his tests for establishing whether an attribute belongs universally to a given subject as primary. So the method of subtraction is a comprehensive logical process for identifying the primary subject of any given attribute. Furthermore, it would appear that the order in which aspects are removed makes a significant difference. For instance, in the case of the triangle, it seems that the order of subtraction inverts a successive division of descending genera into their subordinate species; i.e. figure, triangle, isosceles. Since such divisions are typical of

the practice of dialectic within the Academy, one might claim that the method of subtraction presupposes the results of a discipline which divides reality at its natural joints. Such a claim is consistent with the use of 'abstraction' terminology[100] in the *Topics,* which itself may be taken as a handbook for Academic dialectic.

Having established this historical and logical perspective, let us see if the argumentation of *Metaphysics* XIII.3 becomes more intelligible in that light. The argument based on the theory of proportion draws the following logical parallel: just as it is possible to have a universal science about numbers and magnitudes in so far as they have some general aspect, without the ontological separation of any universal entity; so also one can have a science of sensible magnitudes in so far as they are such and such (ᾗ τοιαδί). Perhaps Aristotle is being deliberately vague here so as to make the point that the 'qua' locution can pick out any aspect of sensible magnitudes and treat it as the subject-matter of a particular science. It also establishes the possibility of demonstrative knowledge of that unseparated aspect because, as we have seen from the *Posterior Analytics,* the 'qua' locution indexes the primary subject of whatever attributes are proved to belong to something qua such-and-such.

On this logical basis Aristotle draws a second suggestive parallel:

> For just as there are many propositions concerning sensible things but only qua moving, without reference to the whatness of each of these and the attributes that follow from it—and it is not necessary because of this that there should exist either a moving of a sort which is separate from the sensible thing or is some definite nature in the sensible thing—so also there will be propositions and sciences about things in motion, not qua in motion but only qua bodies, or only qua planes, or qua lengths, or qua divisible, or qua indivisible with position, or just qua indivisible.[101]

As in the previous argument, the general structure of this argument is that of an explicit parallel which is drawn between an actual and a possible state of affairs. Here Aristotle starts from the fact that there are many statements about things only in so far as they are changing (ᾗ κινούμενα μόνον), quite apart from the particular essence of such things or their essential attributes.

It is difficult to decide whether Aristotle is appealing to the science

[100] Cf. *Top.* III.3, 118b10 ff. & III.5, 119a12 ff.
[101] *Met.* 1077b22–30: tr. Apostle (1966).

of physics or whether he is merely using the commonsense intuition that it is possible to make assertions about sensible things qua moving.[102] In either case his point remains the same; i.e. that the subject-matter (as indicated by the 'qua' locution) is distinct from the essence or the essential attributes of sensible things. Aristotle insists, however, that this logical separation does not imply either that there is some changing thing separated from sensibles or even that there is some distinct nature in them.[103] Once again, he is excluding the two possible modes of being for mathematical objects which he has already rejected in XIII.2. But notice also the linguistic contrast between logical distinctness (χωρίς) and ontological separation (κεχωρισμένον). Such a contrast is the basis for Aristotle's opposition to the Platonists who confused the two types of separation when setting out the objectivity conditions for any science. In order for a science of moving things to be possible, according to the Platonists, there must be apart from sensible and changing things some Form of Motion which is the permanent and eternal object of that science. In the course of his refutation, Aristotle has shown that such an objectivity condition leads to absurd results and now he proposes to replace it with a more satisfactory one.[104]

It is clear that what he is proposing as a basis for the truth and objectivity of any science is the possibility of logically separating its subject-matter from the confused appearances of sensible things. For instance, he emphasizes that we are able to make true statements about sensible things qua moving, while leaving out of account both the essence of these things and all other related attributes. Obviously, such a 'leaving out' is logical because the essence of anything is ontologically inseparable from it and it could not be ignored, for instance, if we were considering something under its species-form

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[102] Ps.-Alexander (734.33 ff.) suggests that the reference is to the science of physics, while Syrianus (95.19–22) thinks that Aristotle has in mind the application of some universal axioms to motion; e.g. things that are moved at equal speeds traverse the same distance in an equal time.

[103] ἢ κεχωρισμένον τι εἶναι κινούμενον τῶν αἰσθητῶν ἢ ἐν τούτοις τινὰ φύσιν εἶναι ἀφωρισμένην, *Met.* 1077b25–26.

[104] Wieland (1962) 197–200 argues that Aristotle's discovery of the 'qua'-structure enabled him to overcome the danger of hypostatizing the predicate and permitted him to view it only as an aspect of some subject. So the Aristotelian theory of mathematics is grounded in the 'qua,' since it makes possible the distinction between things and the qualities of things which the mathematician studies. According to Wieland (1962) 213, this 'qua'-structure involves a radical and decisive critique of Plato for confusing things and their qualitative aspects.

description. It is important to notice, however, that Aristotle mentions the possibility of leaving the essence out of account through this logical technique of subtraction. If this were not possible then there might be only one science of sensible things; i.e. a science of natural kinds. But he clearly rules out this possibility in the final section of the above passage when he draws the second part of his parallel: just as there are propositions about sensible things qua moving, so also there can be propositions and sciences about moving things, not qua moving but qua bodies only (ᾗ σώματα μόνον). In other words, just as one can select some aspect of sensible things as a primary subject for certain attributes related to motion, so also one can select the bodily aspect of moving things as a subject to which certain attributes belong primarily and universally.

Despite the usual ambiguity of the term σώματα, it seems most likely that Aristotle has in mind the solids (στερέα) whose *per se* attributes are studied by the science of stereometry. The selection of solids as the primary subject of such attributes is indicated by the 'qua' locution and is achieved through subtraction. Indeed, the passage goes on to list a series of such subtractions which itself seems to have an inherent order.[105] First, one considers moving or changing things, not qua moving but only qua solids. This step involves the logical subtraction of the sensible and changing aspects of things, together with the *per se* attributes that belong primarily to this aspect; e.g. sensible contraries like hot/cold, light/heavy, wet/dry. The analogous step in *Posterior Analytics* I.4 (74a33–b4) was the subtraction of 'bronze' from the complex subject 'bronze isosceles triangle,' thereby eliminating certain sensible attributes. Such a logical step makes possible the isolation of the solid as a primary subject for the attributes which stereometry will demonstrate as belonging to it *per se*.

The method of subtraction can be used again in a logical way to 'strip off' (ἀφαιρεῖν) the third dimension and thereby eliminate its *per se* attributes; cf. *Met.* 1029a10 ff. & 1061a28 ff. This seems to be what Aristotle has in mind at XIII.3 when he says that there can be a science of sensible things qua planes (ᾗ ἐπίπεδα); i.e. plane geometry. Similarly, the second dimension can be logically removed so as to make possible the study of sensible things qua lengths (ᾗ μήκη). It

[105] Thus abstraction presupposes that logical reality is ordered and that one can systematically search for primary subjects and their related attributes, even though all these aspects are jumbled together in sensible things.

is not necessary for Aristotle's argument that there be a science of line geometry apart from plane geometry nor does he suggest that there is such a distinct science. The point is that the method of subtraction allows one to identify certain attributes as belonging universally to the line as a primary subject; e.g. straight and curved belong to bodies in so far as they contain lines. Therefore, strictly speaking, it is only qua line that a sensible thing can be said to be either straight or curved. Although he does not mention Protagoras within this context, one can see how he might defuse the well-known objection that mathematical definitions (e.g. for the tangent of a circle and a line) are not true of concrete sensible things.[106]

Plato and Aristotle both recognize that such an objection is beside the point, but their grounds for dismissing it are quite different. Furthermore, the objection is more serious from Aristotle's point of view because he wants to claim that mathematical propositions are not about separate entities but are true of sensible things in a certain way. I think it is clear that he can perform this logical tightrope act only by appealing to the method of subtraction and the related 'qua' locution. By subtracting all of the dimensions, for instance, it is possible to treat sensible things as indivisibles with position (ᾗ ἀδιαίρετα ἔχοντα θέσιν), just as the Greek astronomers treated the stars as points for the purposes of their science, even though none of them believed that they really were points. Are astronomers therefore guilty of making false assumptions? Aristotle would deny the accusation implicit in this question since he can logically justify their procedure in terms of subtraction.

Furthermore, the arithmetician can count sensible bodies while treating them as indivisibles only (ᾗ ἀδιαίρετα μόνον). The additional step here, over and above treating bodies as indivisibles with position, is the removal of the aspect of relative position so that bodies are treated simply as interchangable units to be counted by the arithmetician. Later (1078a23–25) he gives a different justification in terms

[106] In a textually problematic passage in *De Anima* (403a10–16), Aristotle says that the question about the separation of soul from body will depend on whether or not it has any attributes peculiar (ἴδιον) to it apart from body. If not then it cannot be separate (χωριστή) but will be like the straight qua straight, which has many implications like touching a (bronze?) sphere at a point. The passage goes on to suggest that a separate 'straightness' could not make such a contact because contact always involves a body. In view of the familiar ambiguity about 'body,' it is difficult to determine whether Aristotle means that the very notion of 'contact' implies sensible body or merely mathematical body.

of counting sensible things as instances of natural kinds, but the notion of treating a body as a unit without position fits better into the kind of schema which is typically presupposed by the method of subtraction: solid, plane, line, point, and unit. While this schema may have been Pythagorean in origin, it seems to have been a conventional way of treating mathematical entities; cf. *Met.* 1016b24–31. Thus, in order to show that such entities are not ontologically separated, Aristotle must establish that it is logically possible to treat sensible things under these descriptions; e.g. as units or as points.

The whole of the above passage, therefore, represents his attempt to show that different branches of mathematics can be about sensible things qua such-and-such (ᾗ τοιαδί). As the deictic word from the previous passage suggests, the 'qua' locution picks out the different aspects of sensible things which are the primary subjects of *per se* attributes that each science demonstrates as belonging universally to them. These subject-aspects are identified through the method of subtraction which logically removes different aspects in an orderly way until one reaches the primary subject of the attribute in question. It should be noted that the order of subtraction in this passage exactly reverses the order of priority within the standard Academic schema of unit, point, line, plane, and solid. This is no coincidence because some such schema always guides the process of subtraction. Keeping this in mind, we can link 'abstraction' with Aristotle's statements about priority, simplicity and accuracy.

IV.2. *Arguments from truth*

While Aristotle makes every attempt to show that the mathematical sciences can be about sensible things, he also seems to talk about mathematical objects as distinct entities, even if only in a logical manner. Since he has rejected the mode of being attributed to them by the Platonists, yet still accepts their assumption that a true science must have real objects, the major task of XIII.3 is to delineate an alternative mode of being for mathematical objects. He seems to have some such task in mind when he draws an initial and tentative conclusion from the first two arguments of the chapter:

> Thus, since it is true to say, without specifying, that not only what is separate exists but also what is not separate (for example, that the moving exists), it is also true to say, without specifying, that the math-

> ematical objects exist, and to be indeed such as mathematicians say they are.[107]

On the face of it this passage seems to contradict his previous conclusion at the end of XIII.2; i.e. that mathematical objects do not exist without qualification (οὐχ ἁπλῶς ἔστιν, 1077b16). More careful inspection, however, reveals that the adverb ἁπλῶς is paired with verbs of speaking (λέγειν and εἰπεῖν) here rather than with the verb εἶναι as it was in the previous passage.[108] This suggests that the above passage is mainly concerned with what it is true to say about the being of mathematical objects.

I take his point to be that, since it is true to say (speaking without qualification) that non-separated things like motions exist just as much as separated things, it is also true to say (again speaking simply) that mathematical objects exist and are such as they are said to be. The general distinction between separated and non-separated beings seems to correspond with the distinction between independent substances and dependent attributes. This is confirmed by his use of moved things (κινούμενα) as an illustration of non-separated entities.[109] In the light of XIII.2, one might infer that mathematical objects have a dependent mode of being akin to that of attributes. Since attributes can be truly said to exist (when one speaks simply or without qualification), the same is true of mathematical objects which 'are such as they are said to be' (1077b33–34). The veridical nuance here indicates that Aristotle is concerned with the truth and objectivity of mathematics. The science itself would be false and ungrounded if its objects did not exist or if they did not correspond exactly to its definitions and proofs. This is the reason for his insistence that mathematical objects exist in some sense, even if they do not exist as independent substances.[110]

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107 *Met.* 1077b31–34: tr. Apostle (1966).

108 Hussey (1992) 109 suggests that we read this as 'straightforwardly true to say,' which involves taking ἁπλῶς with ἀληθες rather than with λέγειν. This reading shifts the emphasis from what we say to the truth conditions governing claims about mathematical objects.

109 Cf. also *Phys.* VIII.1, 251a10–11. In Aristotle's categories motion is quite inferior to substance and even dependent on quantity; cf. *Met.* IV.13, 1020a28–32.

110 Ps.-Alexander (735.24 ff.) thinks that Aristotle is responding to a possible objection against his own view that mathematical objects have the mode of being of abstractions (ἐξ ἀφαιρέσεως τὸ εἶναι ἔχει). Syrianus (96.17 ff.) also seems to accept this as an accurate account of Aristotle's view, even though the terminology of abstraction is not used in XIII.3 for the mode of being for mathematical objects.

So far he has only indicated that they are dependent like attributes rather than independent like substances. As I see it, this is merely the first step toward establishing alternative ontological foundations for the mathematical sciences. Thus, in the subsequent passage, we find him drawing parallels with other areas of knowledge about sensible things:

> And just as it is true to say, without specifying, that each of the other sciences is concerned with certain things—not with what is accidental to those things (for example, not with the white, if the healthy is white and if the science is concerned with the healthy) but only with those things qua such, that is, with the healthy qua healthy if it is the science of the healthy, and with a man qua man if it is the science of man—so it is with geometry. If the objects of the mathematical sciences also happen to be sensible but are not investigated qua sensible, this does not mean that those sciences will not be concerned with sensible things, or that they will be concerned with other things separated from the sensibles.[111]

Once again, it is worth noting that the structure of this argument follows the pattern already established in earlier arguments. Aristotle draws a parallel between what he considers to be an accepted state of affairs and a possible state of affairs that he wants to establish as true.

In this case the parallel goes as follows: Just as it is true to say, speaking without qualification (ἁπλῶς εἰπεῖν), that the other sciences are about this (τούτου), so it should also be true to say the same for the science of geometry. From the broader context it seems that the word τούτου here performs the same pointing function as the words ᾗ τοιαδί in a previous argument (*Met.* 1077b22). That is to say, it indicates a certain aspect of sensible things which is isolated as the subject-matter of some science. For instance, medicine studies man qua healthy but not qua pale, even if healthy men happen to be pale. This example is intended to illustrate Aristotle's claim that the other sciences are not about the accidental (τοῦ συμβεβηκότος) but about that which is the subject of each science (ἐκείνου οὗ ἐστὶν ἑκάστη). If the science is medicine, for instance, it studies men in so far as they are healthy (or sick) but if the science is of man as such, it studies them in so far as they are men.

At the end of the passage quoted above, Aristotle touches briefly

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[111] *Met.* 1077b34–1078a5: tr. Apostle (1966).

on the implications of the apparent separation of its subject-matter which makes each science possible. Once more, he insists that this does not imply the existence of some other separated things (κεχωρισμένα) apart from sensible things. On the other hand, however, he does not seem to hold that the mathematical sciences are about sensible things as such because this would leave him open to well-known Protagorean objections. Yet, to avoid lapsing into Platonism, he must argue that the sciences are about things whose sensible aspect can be treated as incidental for the purpose of scientific inquiry. In other words, he must show that each science establishes a special perspective on sensible things from which many aspects of these things can legitimately be treated as accidental.

This is what I take to be the significance of his talk about accidents here, since he says elsewhere that there can be no science of accidents. What he has in mind are not accidents as such but rather aspects of sensible things that are seen as incidental from a certain point of view. The attribute of being healthy, for instance, is completely accidental from the perspective of the geometer who studies a sensible body in so far as it is a solid. Yet the same attribute is considered essential by the physician who views it as a living body. Of course, the physician treats as incidental for his inquiry other aspects of body unless they are directly relevant to its health. Even if a healthy body happens to be white, for instance, the science of medicine will not be concerned with it qua white but rather qua healthy; e.g. if a white pallor is a sign of health or sickness.

Similarly, Aristotle argues, even if the mathematical sciences happen to be about sensible things, they will not consider them qua sensible (ᾗ αἰσθητά). Here the 'qua' locution indicates the aspect of concrete things that is being ignored when some other aspect is singled out for scientific inquiry. The geometer ignores the sensible aspect of things because he is not concerned with sensible attributes like heavy / light or hot / cold which belong *per se* to this aspect as subject. But yet, when he considers these things as bodies or as planes, there is a definite way in which the mathematician is dealing with sensible things under a certain description. In order to steer a course between Plato and Protagoras, Aristotle must specify its logical basis more precisely and show that its ontological implications are compatible with his own metaphysical assumptions. That is the task which he has undertaken in XIII.3 and, in the present passage, he lays some logical foundations for such a solution.

Aristotle's basic claim is that it is true to say that certain mathematical attributes belong *per se* to sensible things under a certain description. In order to buttress that claim, he draws a comparison with an existing logical situation that he considers unproblematic:

> If A belongs to a thing, many other essential attributes of A will also belong to that thing qua A. For example, proper attributes belong to the female animal qua female, or to the male animal qua male, although no female or male exists separate from animals. So, proper attributes belong also to sensible things qua having lengths or qua having surfaces.[112]

Instead of talking about 'essential attributes, I would prefer to translate the words συμβέβηκε καθ' αὑτά as 'belonging *per se*' because I think that Aristotle's use of this terminology indicates that he has in mind *per se* attributes (τὰ καθ' αὑτὰ συμβεβηκότα).[113] In a certain way, of course, such attributes are also essential but we must beware of thinking that they are part of the essence of the composite sensible thing to which they belong. If that were the case then of each kind of sensible thing there could be only one science, which would study its essential attributes. But this would block Aristotle's whole effort to establish that all the sciences (except perhaps theology) are about sensible things qua such-and-such. His argument for this position is supported by a carefully chosen example of attributes which belong *per se* to an animal qua male or qua female. Contrary to the Platonic conditions for objectivity, it is not possible to separate male or female from animal as ontological subject, even though one can prove different attributes to belong to animal under each description.

A logical peculiarity of the examples which illustrate such *per se* attributes is that it is only as exclusive disjuncts that they belong necessarily to their subjects. Every animal must be either male or female, for instance, just as it is necessary that every line be either straight or curved. Even though Aristotle refers to them here as 'peculiar attributes' (ἴδια πάθη), we should note that they are not properties, strictly speaking, since they do not satisfy the condition of counterpredicability. Indeed they might even be classed as accidents, given that one of the pair may or may not belong to the same subject; cf. *Top.* 102b4–14. But they are different from purely contin-

[112] *Met.* 1078a5–9: tr. Apostle (1966).

[113] According to Aristotle, the isolation of such attributes and their primary subjects is crucial for the possibility of demonstrative sciences; cf. *APst.* I.4.

gent accidents inasmuch as they belong necessarily to their primary subjects by disjunction.[114] According to the *Posterior Analytics* (74b5–13), this kind of necessity is adequate for the purposes of demonstration. In fact he rarely gives examples of demonstrations based on counterpredications, possibly because such proofs appear to be circular and uninteresting; cf. *APst.* 73a7–20, 78a7–14. It is only in a science like mathematics, which works largely from definitions, that such reciprocal predication is common. In general, however, Aristotle considers *per se* attributes (τὰ καθ' αὑτὰ συμβεβηκότα) to be the building-blocks of a demonstrative science and so he gives them their due prominence; cf. *APst.* 75a28–37, 75a41–b2. For him it is important to identify the underlying subject-genus (τὸ γένος τὸ ὑποκείμενον) to which these attributes belong *per se,* because its existence is one of the unproven hypotheses of the science. In some cases its existence is obvious to the senses and, even where this is not the case, he thinks that it is possible to identify the subject-genus in some other way such as through the 'qua' locution.[115]

This logical situation is relevant to XIII.3, where Aristotle is trying to establish that the mathematical sciences are about sensible things under some description, and not about separated entities. In the course of that effort, he makes the general claim that many things belong *per se* to things insofar as each of them has such-and-such a character. Then he draws a comparison between considering an animal qua male or qua female and studying sensible things qua lengths or qua planes. It is not entirely clear whether the parallel is intended to be exact or whether it is merely a loose analogy of the following sort: just as certain *per se* attributes belong to animal qua male, so

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[114] In the secondary literature there has been some debate about how to fit *per se* attributes into the list of "predicables" as definition, genus, property, and accident; cf. Barnes (1970) 136–55 & Wedin (1973) 30–35.

[115] In his logical analysis of what he calls Aristotle's theory of reduplication, Bäck (1979) points out that such a qua proposition is actually a condensed demonstrative syllogism in which the qua term functions as a middle term and as a cause; e.g. an isosceles triangle has this property because it is a triangle. In a forthcoming book, he argues that the qua phrase is attached to the predicate and does not change the reference of the subject term, which he takes to be a particular existent like this bronze triangle. He objects (in correspondence) that my approach of making qua propositions fix our attention on the primary subjects has the consequence of changing the reference of the subject term to some kind of Platonic entities about which it would be difficult to verify any knowledge claims. But I hold that the distinction between natural and logical priority in Aristotle separates the *de dicto* question of the primary logical subject from the *de re* question about the fundamental subject as a substance.

also there will be other *per se* attributes that belong to sensible things qua lines. If the comparison is to be taken loosely then mathematical attributes are not essentially related to sensible things.[116]

IV.3. *Arguments from accuracy*

If, on the other hand, Aristotle holds the mathematical sciences to be about sensible things then he faces an obvious objection (as made by Protagoras) about their truth and accuracy. However, he rejects the Platonic assumption that the truth (ἀληθεία) and accuracy (ἀκριβεία) of mathematics depends on the ontological separation of its objects from sensible and changing things. In the *Philebus* (57D), for instance, this assumption grounds the distinction between practical and philosophical mathematics, which differ in accuracy and truth according to the purity of their objects which, in turn, depends upon the separation of these objects from the impure and mixed state of sensible things. Aristotle is enough of a Platonist to accept the connection between purity and accuracy, even though he rejects the ontological implications drawn from this link.

In fact, as we have seen from *Metaphysics* XIII.2, he denies that the truth of mathematics depends on intelligible substances existing apart from sensible things, yet his own account of the mode of being of mathematical objects must be able to explain the accuracy of these paradigm sciences. This is the point of the following passage from XIII.3:

> And to the extent that we are investigating what is prior in formula and is simpler, to that extent the result will be more accurate, and by "accurate" I mean simple. Thus, the science which leaves out magnitude is more accurate than the one which includes it, and the science which leaves out motion as well as magnitude is the most accurate of the three. And if a science is concerned with motion, it is most accurate if it is concerned with the primary motions; for these are the simplest, and of the primary motions the even motion is yet the simplest.[117]

What seems to be recalled here is his concession to the Platonists at the end of XIII.2; i.e. that mathematical objects may be prior in formula (τῷ λόγῳ πρότερα) to sensible things, even though they are

[116] One might think that Aristotle will hardly want such a relation if he wishes to explain how mathematics can 'abstract' from sensible things but sometimes he hints at a close connection between body and geometrical concepts like tangency; cf. *DA* I.1, 403a12–16.

[117] *Met.* 1078a9–13: tr. Apostle (1966).

not prior in substance (τῇ οὐσίᾳ). By means of this distinction between different senses of priority, Aristotle tries to explain the undeniable accuracy of the mathematical sciences, so as to undermine the Platonic case for the ontological priority of its objects.

Using the notion of simplicity as his middle term, Aristotle establishes a direct correlation between the accuracy of a science and the logical priority of its objects. Thus, by explicitly equating what is accurate (τὸ ἀκριβές) with what is simple (τὸ ἁπλοῦν), he is able to claim (1078a9–10) that in as much as (ὅσῳ) we are dealing with what is prior in definition and more simple to this extent (τοσούτῳ) our science will have greater accuracy. For instance, he says, the science which leaves magnitude out of account is more accurate than one which includes it, while the science which excludes motion is especially accurate. Yet, even among the sciences that deal with motion, there are degrees of accuracy and simplicity. For instance, those sciences which deal with primary motions will be more accurate because these are the simplest and, among these motions, uniform motion will be most simple.

This condensed argument presupposes a universe in which locomotion is prior to all other kinds of change (*Phys.* 243a6–10, 260a27–29), and in which the only simple motions are rectilinear and circular (*Cael.* 268b16–19). At *Physics* 265a16–17, for instance, he argues that circular locomotion is prior to rectilinear because the former is simpler and more complete. This argument also rests on the assumption that a finite universe cannot contain infinite linear motion. In the physical universe, therefore, circular motion is prior in nature (φύσει) and definition (λόγῳ) and time (χρόνῳ) to rectilinear motion because it can be complete and eternal; cf. *Phys.* 265a22–27. Thus the science of uniform circular motion (astronomy) is simpler and more accurate than a science of uniform rectilinear motion (physics).

Logically speaking, however, Aristotle accepts that a straight line is prior to and simpler than a circle because curvature admits of the more and the less. In Euclidean geometry, for instance, the straight line may be treated as being logically prior to the circle because the latter is defined in terms of it but not *vice versa.* Thus a science which dealt only with straight lines would be simpler and more accurate than a science that dealt with circles and other curves. In this way one can see that the claims which are made about greater accuracy and simplicity in comparable sciences depends a great deal on the schema of priority which one applies to their respective objects. While

the Platonists emphasized the accuracy of mathematics, Aristotle's response is that scientific precision may have logical though not ontological implications. Indeed, he suggests that such accuracy is the result of leaving out of account more complex aspects of sensible things, such as motion or even magnitude. For example, arithmetic may be taken as being more accurate than geometry because it ignores magnitude and treats things as indivisible units.

At *Posterior Analytics* I.27 arithmetic is judged to be prior to and more accurate than geometry because it depends on fewer posits (ἐξ ἐλαττόνων), whereas geometry starts from additional assumptions (ἐκ προσθέσεως); cf. 87a35–37. In order to illustrate what he means here, Aristotle appeals to the distinction between a unit and a point. Whereas the unit is described as a substance without position (οὐσία ἄθετος), the point is said to be a substance with position (οὐσία θετός).[118] When he explains that the latter is the result of addition (ἐκ προσθέσεως), he can only mean that relative position has been added. Even though the point itself does not have any magnitude, its definition presupposes continuous magnitude in at least one dimension because it is either actually or potentially a common boundary (κοινὸν ὅρον) for parts of a line; cf. *Cat.* 5a1–2. By contrast, the units that are part of number have no common boundary at which they join together. In his treatment of quantity in the *Categories* (4b20 ff.), Aristotle distinguishes between discrete and continuous quantities in terms of whether or not their parts have position (θέσις) in relation to one another. Thus, to describe the unit as a substance without position is to disconnect it from continuous magnitude and its associated paradoxes.

In this light it is intelligible for Aristotle to claim, in the above-quoted passage from *Metaphysics* XIII.3, that the science which deals with continuous magnitudes is less accurate than the science which does not. Clearly, he has in mind a comparison between arithmetic and geometry, since this would also fit with his talk about logical priority. In terms of the above example, one can say that the unit is logically prior to the point because the definition of the latter involves the former; e.g. point =df unit with position. But all talk of substance with respect to these entities must now be dropped on account of Aristotle's insistence that logical and ontological priority

[118] Tarán (1981) insists that among the Platonists in the Academy only Speusippus posited the point as a separate substance but, since he also seems to have been a staunch Pythagorean, there is some reason to believe that Aristotle may be here citing a traditional distinction.

do not coincide in the case of mathematical objects. When he explains the accuracy of these paradigm sciences in terms of simplicity and logical priority, he is undermining one of the major Platonic arguments for the existence of separate objects of knowledge. In fact, the link between simplicity and accuracy is consistent with the etymology of the word ἀκριβεία because there is less room for error in judging (κρινεῖν) when the matter is simple. But, according to Aristotle, the simplicity of the subject-genera of mathematics is merely the result of logical subtraction from more complex sensible things. Such subtraction is warranted by the logical fact that mathematical attributes do not belong *per se* to sensible things qua sensible. Just as in the case of the bronze isosceles triangle, the method of subtraction can be used to identify the primary subject to which such attributes do belong *per se*.

IV.4. *Questions about false assumptions*

Thus mathematical attributes can be defined independently of the sensible things to which they happen to belong and, in this sense, are logically prior to them. Although the parallel is not exact, Aristotle illustrates this kind of logical priority at the end of XIII.2 with the example of 'white' in relation to 'white man.' The point he makes there is that a given attribute may be logically separated from the subject to which it happens to belong without this involving any ontological separation. It is obvious that he intends to make the same point about mathematical attributes, and he also wants to defend mathematicians against the accusation of making false assumptions when they begin with precise definitions and hypotheses. In a subsequent passage in XIII.3, he considers this as a pressing issue for the 'mixed' sciences:

> The same statements apply to optics and harmonics; for they investigate sight and sound, respectively, not qua sight or qua sound, but qua lines and numbers, which, however, are proper attributes of sight and sound respectively. Mechanics, too, proceeds in the same way. So, if one lays down as separate certain attributes and inquires into these qua what they are, he will not by so doing inquire into what is false, just as he will not speak falsely when he draws a line on the ground and calls it a foot long when it is not a foot long; for this latter falsehood is not in the premises.[119]

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[119] *Met.* 1078a14–21: tr. Apostle (1966).

When Aristotle talks here about 'the same account' (ὁ αὐτὸς λόγος) being given for harmonics and optics, he seems to be referring back to the account of accuracy given in the previous passage. But, as we have seen, this account is essentially comparative; i.e. one science is judged to be more accurate than another to the extent that its characteristic objects are simpler and prior in definition. Thus the present passage may be comparing harmonics, optics, and mechanics with their respective superior sciences, which are more accurate because they show 'the reason why' (τὸ διοτί), whereas the subordinate sciences only show 'the fact' (τὸ ὅτι). Such an implicit comparison in terms of accuracy might be part of Aristotle's explanation that neither optics nor harmonics considers sensible things qua sight (ᾗ ὄψις) or qua voice (ᾗ φωνή) but rather qua lines (ᾗ γραμμαί) or qua numbers (ᾗ ἀριθμοί), since the latter are the subject-matters of plane geometry and arithmetic, respectively. But his explanatory remark about 'proper attributes' (οἰκεῖα πάθη) directs us to the discussion of *per se* attributes as the proper context within which to understand the present passage.

Despite the brevity of this explanatory remark, its Greek construction suggests that Aristotle is claiming that lines and numbers (as the referents of ταῦτα) are proper attributes of sight and sound (the referents of ἐκείνων). In his commentary, Ps.-Alexander (738.1 ff.) seems to assume that the mathematical entities are *per se* attributes of these sensible subjects. But if that were the case then lines, for instance, could not be defined except with reference to sight or at least with reference to the path of light rays.[120] While this might be plausible in modern physics, it seems incompatible with Aristotle's logical view that the objects of 'pure' geometry can be defined independently of their sensible subjects.

Thus, in the case of 'mixed' sciences like optics, it will be difficult for him to establish a satisfactory logical relationship between the attributes studied and the ostensible subject-matter of each science; e.g. sight & sound. If the mathematical attributes belong accidentally to these subjects then, according to Aristotelian principles, there cannot be any science of them. But the actual existence of the sciences of harmonics and of optics forces him to give some logical legitimation for these inquiries. However, as we saw from the review of diffi-

[120] In support of this suggestion, one might point to Euclid's definition of a straight line as that which lies evenly on itself—plainly an appeal to vision and the path of light rays.

culties, the existence of such 'mixed' sciences poses a greater obstacle to Platonism because they are not about some separated sights or sounds. Yet the difficulties of opponents will provide cold comfort for Aristotle unless he can give an alternative account that shows how these sciences can retain the precision of mathematics while being about sensible things in some real sense.

Hippocrates Apostle (1966, 414n15) makes the promising suggestion that proper attributes (as distinct from properties) may be in another genus (e.g. that of arithmetic or geometry) and yet belong to sight or sound necessarily or for the most part. This would satisfy the conditions for an Aristotelian science and still allow the 'mixed' sciences to be about sensible things in more than an accidental sense. One might show, for example, that the equality of the angles of incidence and reflection belongs to the path of a light ray. This example illustrates how a proposition in optics might be proved through plane geometry and it also gives us an indication of the role of the applied sciences in Aristotle's whole argument. These sciences provide him with important evidence for his claim that mathematical attributes do belong in some essential manner to sensible things, yet can be logically separated for the purposes of scientific study.[121] Such a separation, however, does not carry any ontological implications nor does it involve a falsification of reality because sensible things qua sensible are not the primary subjects to which mathematical attributes belong *per se*.

In the present passage, therefore, he can insist that one does not fall into error if one posits these attributes as separate from accidents and studies them as such.[122] Here the 'qua' locution is once again used to indicate that a certain attribute, say linearity, can be logically isolated as the subject of other *per se* attributes; e.g. straight or curved. The ontological overtones of the word κεχωρισμένα recall the Academic habit of treating mathematical entities, such as lines and planes, in a quasi-substantial manner which facilitated the clustering of *per se* attributes around them as logically separated subjects. But how does Aristotle avoid the Platonic tendency to hypostatize mathematical objects?

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[121] As an objection against Platonism, Aristotle brings forward the fact that mathematical attributes do belong to sensible things; cf. *Met.* 1090b1–4.

[122] εἴ τις θέμενος κεχωρισμένα τῶν συμβεβηκότων σκοπεῖ τι περὶ τούτων ᾗ τοιαῦτα, *Met.* 1078a17–18.

One means of finding out is to ask about the so-called 'accidents' (συμβεβηκότα) from which mathematical attributes are said to be separated without error. Aristotle talks about a sensible diagram that does not conform with the initial geometrical assumptions, but he seems more concerned with defending geometers against the Protagorean objection when he insists that the falsehood is not in the premisses of the science.[123] Presumably, he means that the truth of mathematics does not depend upon its propositions corresponding exactly with any sensible diagrams that are used as aids to reasoning.

This provides an answer to Protagoras without committing Aristotle to any form of Platonism in mathematics. In contrast to the *Categories,* Aristotle no longer treats numbers and lines and surfaces as quasi-substantial entities but rather talks about sensible things qua numbers, qua lines and qua surfaces. He insists on such language when he is talking about the mode of being of mathematical objects, although he implicitly accepts that mathematicians talk like Platonists. In order to validate their actual procedure, however, he appeals to the logical independence of quantity from quality. Although mathematicians posit subject-genera *as if* they were separate, they are no more in error than if they were to call some line in a sensible diagram one foot long when it is not. Just as nothing depends on the sensible diagram, so also in mathematics no propositions or conclusions depend on the actual separation of its subject-genera from sensible things.

Thus Aristotle gives an account of mathematical objects in terms of the 'qua' locution, while rejecting the ontological implications which the Platonists drew from the separating activity of mathematicians. For instance, the geometer defines a line as 'length without breadth' and the Platonists posit a corresponding ideal object, since it is clear that such a thing does not exist by itself in the sensible world. Aristotle's counterclaim is that the truth of this definition does not depend on the existence of such an independent entity because sensible things can be considered qua lines or as lengths without breadth. A modern version of the same claim might be to say that neither the separation nor the dependence of mathematical entities counts as positive information in the sciences themselves.[124] While the mathematician actually separates his objects of study from sensible things, this is a

[123] οὐ γὰρ ἐν ταῖς προτάσεσι τὸ ψεῦδος, *Met.* 1078a20–21.

[124] Cf. Lear (1982) & Mignucci (1987). Like Annas (1987) and Lennox (1987),

purely methodological separation that promotes ease of inquiry and an increase in exactitude.

Thus, in line with his goal in *Metaphysics* XIII.1–3, Aristotle spells out a mode of being for mathematical entities which is consistent with the actual practice of mathematicians:

> A thing can best be investigated if each attribute which is not separate from the thing is laid down as separate, and this is what the arithmetician and the geometrician do. Thus, a man qua a man is one and indivisible. The arithmetician lays down this: to be one is to be indivisible, and then he investigates the attributes which belong to a man qua indivisible. On the other hand, the geometrician investigates a man neither qua a man nor qua indivisible, but qua a solid. For it is clear that the attributes which would have belonged to him even if somehow he were not indivisible can still belong to him if he is indivisible. Because of this fact, geometers speak rightly, and what they discuss are beings, and these are beings; for 'being' may be used in two senses, as actuality and as matter.[125]

What Aristotle here proposes as a solution, i.e. that mathematical objects exist 'as matter' (ὑλικῶς), has itself prompted many different interpretations.[126] But, given his aporetic approach, the acid test is whether it resolves the difficulties associated with the aporiae that guided the whole inquiry.

The above passage begins with a methodological recommendation for the other sciences based on the procedure of mathematicians. I take the word οὕτω to refer back to that procedure, which is then redescribed in a conditional clause as follows: "if one posits as separate what (in reality) is not separated . . ."[127] This clause contrasts the logical and ontological implications of the positing activity of the arithmetician and the geometer. Although their subject-matter is treated as logically separate, Aristotle insists that it is not separated in reality. Therefore he recommends this procedure for each of the other sciences because it promotes greater accuracy without leading to error.

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however, I do not accept Lear's talk of 'helpful fictions' because it implies that mathematics for Aristotle is somehow based on a falsehood, whereas he held mathematical propositions to be actually true of sensible things considered under a certain description.

[125] *Met.* 1078a21–31: tr. Apostle (1966).

[126] These interpretations have been collected and classified by F.A.J. de Haas (unpublished mss.).

[127] εἴ τι τὸ μὴ κεχωρισμένον θείη χωρίσας, *Met.* 1078a21–22.

The most obscure part of this passage, however, is the description of how the arithmetician considers a man as one indivisible thing, while the geometer treats him as a solid. We are tempted to object that the mathematician does not deal with man at all, whether as unit or as solid, but that would be to miss the whole point of his argument. For Aristotle clearly does not mean to claim that mathematics is about mankind, though he does wish to establish that these sciences can be viewed as dealing with sensible things under highly specific aspects.[128]

Yet it is clear that Aristotle is concerned with the truth of mathematics which, according to his correspondence theory, depends on the existence of real entities. For instance, the statement about the arithmetician begins with an explicit comparison between what is posited by him and what is actually the case. On the one hand, Aristotle says, a man qua man is one and indivisible (ἓν μὲν . . . καὶ ἀδιαίρετον) while, on the other hand, the arithmetician posits the unit as indivisible (ὁ δ' ἔθετο ἓν ἀδιαίρετον) and then considers whether any attributes belong to the man qua indivisible. The point implicit in the Greek construction seems to be that the arithmetician has not assumed any falsehood, in spite of the fact that he hypothesizes the unit without reference to the sensible world. Aristotle's language suggests that the arithmetician simply posits an indivisible unit without reflecting on his ontological assumptions, and this conforms quite well with what Aristotle says elsewhere[129] about the practice of mathematicians. Indeed he doesn't think that it is their business to investigate foundational questions.[130] But, as a philosopher, Aristotle must anchor the mathematical sciences in the sensible world, especially since he has undermined the foundations which the Platonists gave them.

Thus, in the present passage, he tries to establish that these sciences are true of sensible things under a certain description. For instance, one can count men without falling into error because a man qua man conforms to the definition of a unit which is posited

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[128] Here Ps.-Alexander (*in Metaph.* 738.24–29) may be correct in thinking that Aristotle is trying to counter a possible objection of the following sort: if mathematical sciences are not about separated numbers or figures, then mathematicians are talking about non-beings (περὶ μὴ ὄντων) because they are concerned with 'immaterial' (ἄυλα) rather than 'enmattered' (ἔνυλα) things. Syrianus (*in Metaph.* 99.17–36) offers a similar reading but he does not think that Aristotle answers the objection satisfactorily

[129] Cf. *APst.* 76a31–36, 76b3–11, 92b15–16, 93b21–28.

[130] Cf. *Met.* 1025b3–18, 1059b14–21, *Phys.* 184b25–185a5.

by the arithmetician. By contrast, if one tried to count the same things qua colored, the possibility of error and confusion is greater. In modern terms, the difference between 'man' and 'color' is that the former is a sortal term which divides its reference cleanly, whereas the latter is a mass term that does not.[131] There is a corresponding distinction in Aristotle when he recognises that only certain concepts provide us with a measure for counting a collection of things; cf. *Met.* 1014a26–31, 1088a4–11. In the present passage, for instance, it is clear that he has distinguished very carefully the aspect under which an arithmetician might consider a sensible thing such as a man. Even though a man is one and indivisible in so far as he is a man (i.e. under the species description), the arithmetician is not interested in him as such; otherwise he would be engaged in some kind of biology. Indeed, the mathematician only deals with a man in so far as he is an indivisible unit and in so far as some numerical attributes belong to him under that description.

Thus, contrary to the claims of the Platonists that arithmetic studies some pure units or numbers apart from sensible things, Aristotle argues that the science is about sensibles under a certain qua-description which picks them out as discrete quantities.[132] Similarly, he claims, geometry considers the same things under a different and incompatible description; e.g. if the geometer studies a man qua solid (ᾗ στερεόν), he is treating him as a continuous quantity that is indefinitely divisible, which directly contradicts the description under which he is counted by the arithmetician. While this approach is consistent with Aristotle's sharp distinction between the two sciences, he appears to be flouting the principle of non-contradiction. How can it be the case, one might ask, that a man is simultaneously indivisible and infinitely divisible?

In the above-quoted passage from XIII.3, Aristotle shows an awareness of this possible objection, but his response is less than clarity itself. For example, he explains that the geometer can study the man qua solid because attributes that would belong to the man, if he were somehow not indivisible could obviously belong to him without

[131] Cf. Strawson (1959) 167 ff.

[132] It is a moot question whether the 'qua' locution refers to sensible things as ontological subject or whether there exist some corresponding 'qua' entities which are its real referents. While the latter seems to tally with some of Aristotle's statements, it would commit him to a kind of Platonism of quasi-substantial entities that he took pains to reject.

this.[133] The Greek here is difficult, perhaps because Aristotle is trying to express a complex set of possibilities through a counterfactual conditional as follows: if it should happen that a man were not indivisible (though he is one and indivisible as a man) then the attributes that would belong to him clearly could belong to him without indivisibility. In reading the text, it makes more sense if we take the words ἄνευ τούτων as referring back to the singular subject ἀδιαίρετος and its cluster of *per se* attributes.[134]

The philosophical point at issue is that if τούτων refers back to both humanity and indivisibility, as Ross[135] thinks, then Aristotle would appear to be putting the study of man qua man on a par with the study of man qua indivisible. In other words, there is no privileged or comprehensive science of man but rather many particular aspects which are treated by different sciences. This issue has a direct bearing on the question raised by Julia Annas[136] about whether or not Aristotle is committed to making the distinction between essential and accidental properties relative to the description under which a thing is considered.

Such a modern-sounding relativization of the distinction might be more plausibly attributed to him if it could be shown that he regarded all sciences as studying merely particular aspects of things. But that possibility seems to be ruled out when he introduces the science of first philosophy as having a privileged view of what properties are essential to a thing, as distinct from those which are accidental.[137] Furthermore, a science which studies man qua man considers the truly essential attributes that belong to his substantial form. Since quantity is not part of the essence of man, the arithmetician and the geometer will not study such essential attributes but only those which belong to man qua indivisible or qua solid. In order to satisfy the strict conditions for a demonstrative science, of course, the attributes studied by mathematics must belong *per se* to their primary subjects.

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[133] ἃ γὰρ κἂν εἰ μή που ἦν ἀδιαίρετος ὑπῆρχεν αὐτῷ, δῆλον ὅτι καὶ ἄνευ τούτων ἐνδέχεται αὐτῷ ὑπάρχειν, *Met.* 1078a27–8.

[134] This reading is supported by Ps.-Alexander (739.13 ff.) who writes τούτου instead of τούτων in his transcription of a received text.

[135] Cf. Ross (1924) ii, 417.

[136] Cf. Annas (1976) 149–50. More recently (1987) she has given up the objection because she no longer sees relativization to subject-matter as a disadvantage for the qua-idea, which she has accepted from Lear (1982) as being central to Aristotle's position in XIII.3.

[137] Cf. *Met.* IV.4, 1007a20 ff.

That is why the distinction between essential and accidental attributes is relativized to the description under which something is taken. From the point of view of the geometer, for instance, an essential attribute of man as a solid body is indefinite divisibility; whereas indivisibility is an essential attribute of man considered from the perspective of the arithmetician. So for Aristotle there is no doubt about the truth of arithmetic because a man qua man is an indivisible form and hence it is quite plausible to treat him as an instance of the unit as it is defined by the arithmetician.[138] There is a difficulty about the truth of geometry, however, if one insists (as does Aristotle) that it is about sensible things like a man. In contrast to the procedure of the arithmetician, the geometer must ignore the reality of a man as a unified whole and treat him merely as a divisible solid or as a surface or as a line. Yet even with such exclusions, it is doubtful whether man as a solid corresponds exactly with the definition of a solid posited independently by the geometer. The precision of geometry comes from the sort of idealization that fits better with a Platonic account of the foundations of that science.

But Platonism is being resisted here by Aristotle as he struggles to find a plausible way of connecting the science of geometry with sensible things. This is why he uses a counterfactual conditional to talk about what could belong to a man if he were not indivisible and so it is only through the 'qua' locution that he can establish the logical possibility of talking about a man insofar as he is a solid (ᾗ στερεόν). When this aspect has been isolated as a primary subject, it is possible to claim without contradiction that a man has certain *per se* attributes which are directly opposed to those which belong to a man qua unit. In addition to the logical situation, however, the mode of being of this aspect must be clarified before one can be assured of the truth of geometry as a science concerned with sensible things.

This appears to be what Aristotle has in mind when he insists that geometers speak correctly (ὀρθῶς) and that they are speaking about 'beings' (ὄντα) which really do exist. In support of this claim, he appeals to two general senses in which 'being' is used; namely, being in the sense of actuality (ἐντελεχείᾳ) and being in a material sense (ὑλικῶς). In view of his usual distinction, it is natural to suppose that

[138] Of course, this will not seem plausible to our modern intuitions that have been informed by Gottlob Frege's analysis of number according to which it is not a first-order property of things but rather a second-order predicate; cf. Frege (1950).

ὑλικῶς must stand for potential being, but we must consider Aristotle's purpose in choosing this word rather than δύναμις. However, we should examine how the conclusion is to be understood within the context of the whole argument in XIII.1–3, especially in view of the linguistic hint that mathematical objects may have a mode of being analogous to that of matter.

The simplest solution is to agree with Annas (1987) and Barnes (1985) that by ὑλικῶς Aristotle means to indicate that mathematical objects have the dependent mode of being of non-separated things, and that this is analogous to the mode of being of matter. This would fit with the anti-Platonic character of the whole treatise, which has insisted upon mathematical objects not being separated things. It is also consistent with the sudden introduction of matter, which is left unexplained as though it should be obvious that this is also a non-separate aspect of sensible things. If we accept that a simple parallel with matter is intended, then the whole chapter may lose the appearance of being puzzlingly incomplete, as it seems to Hussey (1992). In fact, the treatise is complete because the solution is that mathematical objects are non-separate things just like matter. This solution was prefigured earlier (1077b31–34) when Aristotle declares that non-separate things like motion can be simply said to exist, just like mathematical objects. Such a quick and easy solution may be seen as satisfactory, if we take his problem to be that of finding some other mode of being for mathematical objects besides that of substances.[139]

Given Aristotle's own method of inquiry, we must also briefly consider whether this solution saves most of the phenomena associated with mathematical objects. For instance, in *Metaphysics* III.5 he claims that points, lines and planes cannot be substances because their 'generation' does not involve a temporal process, since they instantaneously come to be or cease to exist as bodies are either divided or joined together. Furthermore, it does not seem that such changes take place in a material substratum like that which underlies substantial change, since the division of a line appears to yield two points (as end-points of two subsections) where previously there was an indivisible point. In fact, the situation is more like that of the

[139] Previously, I found it difficult to accept that this could be an adequate solution, since I took ὑλικῶς to indicate a potential existence that must be somehow actualized by the mind. I was finally convinced by F.A.J. de Haas (in personal correspondence) that my own logical interpretation of subtraction made such an ontological reading unnecessary.

present moment in time, which is always becoming different without undergoing any process of change. Aristotle confirms (1002b5 ff.) this parallel by saying that the 'now,' just like the point, the line and the plane, is a limit or division of something.

We recall that all of these claims were made in an aporetic context, where we would not expect to find an elucidation of his own views. But now we must ask whether they help to clarify Aristotle's obscure claim that mathematical objects exist 'materially' rather than actually like independent substances. The simple truth is that they do not help a great deal without looking at some parallel passages where Aristotle talks about entities that have a similar mode of generation, which is quite different from that of sensible substances. For instance, in *Metaphysics* VI.3, he characterizes accidental causes in terms of their coming to be and cessation without any temporal process of generation and corruption. Furthermore, in VIII.5 (1044b21–24), he cites points as an example of things which are and are not without generation and corruption. In general, he says that particular forms are like this, since it is not 'the white' which is generated but rather 'the white log.' His general point seems to be that a particular form which is not yet conjoined with a material substratum is not subject to generation and corruption, since matter is the ground for temporal change.[140] But this only deepens the mystery as to why Aristotle should say that mathematical objects have a 'material' mode of existence.

Another possible approach to the puzzle emerges out of a passage in *Metaphysics* VII.10 (1036a5–8) which implies that particular mathematical objects have this peculiar mode of instantaneous generation when they are being brought under some universal by the mind. What Aristotle says (1036a2 ff.) is that when these intelligible particulars are not being thought, it is unclear whether or not they exist, since they are always spoken about and recognized through a universal account. Within the same context he adds that matter in itself is unknowable, so presumably he means this to apply also to intelligible matter. What this passage shows very clearly is that the parallel between mathematical objects and matter as both having a material mode of being cannot be pushed too far without undermining the

[140] In *Physics* VI.10 (240b8 ff.), Aristotle explains that it is not possible for something partless to be moved, except accidentally. Subsequently, the partless is defined as what is indivisible in quantity, like a point or a moment in time.

science of mathematics. There is a similar danger attached to the parallel with accidental entities, since Aristotle says elsewhere (1026b25–27) that accidental causes are indefinite and unknowable.

Finally, we must briefly consider the relationship between the mode of being of universals and that of mathematical objects, which include many universals like 'triangle,' corresponding to definitions and functioning as subjects for the attributes that are proved to belong to them. But it is not quite clear what precise mode of being Aristotle ascribed to universals, except that he denied them independent existence (ante rem) like Platonic Forms. That leaves open several possibilities; i.e. that they could be common predicates in sensible things (in re), or general concepts abstracted from sensible things (post rem) or even mere names for classes of particular things. As the history of Aristotelian commentary shows, it remains unclear whether he committed himself to one or other of these possibilities, or even whether he regarded them as being mutually exclusive. Indeed, as George Brakas (1988) has suggested, it is possible that the concept of the universal changed between the *De Interpretatione* and the *Metaphysics,* where he claims to find a less realist view. Since that is not relevant to my concerns here, I will confine my remarks to what the *Metaphysics* says about the mode of being of universals that might throw some light on the mode of being of mathematical objects.

In XII.5 Aristotle suggests that the universal as a principle or cause is merely potential, while the particular is actual; for instance, man generates man but Peleus is the actual cause of Achilles. Within the same context, he describes matter as potential by contrast with form which is actual. Elsewhere in the *Metaphysics* (VIII.6, 1045a33–35), he identifies the genus as a potentiality like matter, while the differentia or form is a corresponding actuality. Since both passages suggest that the universal is an entity with a potential rather than an actual mode of being, one wonders whether the objectivity of the sciences is thereby undermined, since universals were introduced by Aristotle to replace Forms as objects of knowledge.

An answer may be found in XIII.10 where he introduces an exact correlation between two types of knowledge and their corresponding objects. On the one hand, universal knowledge and its universal object are indefinite like matter whereas, on the other hand, actual knowledge and its particular object are definite. Aristotle is forced to use this peculiar distinction in order to resolve the aporia about whether or not there can be knowledge of substance, since it is particular

while knowledge is universal. He was drawn into this dilemma by his insistence that universals are not independent substances like Platonic Forms but are rather common predicates of particular substances. This opens up a gap between knowledge and substance that Aristotle tries to fill by introducing a particular form which is the substance of a particular composite, and which is also the direct object of actual knowing.[141] In his characteristic manner, Aristotle wants to have it both ways by giving knowledge direct access to substance, while retaining the universal as the general (though potential) object of knowledge.

My concern with this special aporia here is merely to consider whether or not he uses a similar conjuring trick in the case of mathematical objects, even though the problem about knowledge of substance does not arise. It appears that he thinks of mathematical objects as being both universal and particular, just like sensible things, except that they are individuated by intelligible matter and realized primarily in thinking. But, since particular mathematical objects are not independent substances, he says that they have a material mode of being even though they will be the objects of actual mathematical knowledge. Correspondingly, the mathematical universals under which the particulars fall will be the objects of potential knowledge, which is realized every time the mind grasps a particular object either through perception or imagination. I want to postpone further consideration of these epistemological questions to my last chapter, but with regard to mathematical objects it remains unclear whether their ὑλικῶς mode of being should be explained in terms of the potency of universals or merely in terms of the dependency of accidental attributes. What is clear, however, is that he is most concerned to deny that they have the actual mode of being that belongs primarily to sensible and supersensible substances.

V. *Mathematics and the Good*

Plato's *Timaeus* may provide the appropriate background for bringing the concluding passage in *Metaphysics* XIII.3 into some kind of

[141] For a detailed analysis of this aporia, see Code (1982 & 1984) & Cleary (1987b). On the issue of particular forms in Aristotle, see M. Frede (1978 & 1987) & Whiting (1986).

intelligible perspective, since it does not seem to have any intrinsic connection with the previous discussion of the mode of being of mathematical objects. The passage goes as follows:

> Now since the good is distinct from the beautiful (for the good is always in action but the beautiful may also be in what is immovable), those who assert that the mathematical sciences say nothing about the beautiful or the good speak falsely. For they do speak about and show these, and in the highest degree. The fact that they do not use the names, while they do exhibit constructions and theorems about them does not mean that they say nothing about them. Now the most important kinds of the beautiful are order, symmetry, and definiteness, and the mathematical sciences exhibit properties of these in the highest degree. And since these (that is, order and definiteness) appear to be causes of many things, it is clear that the mathematical sciences must be dealing in some way with such a cause, that is, the cause in the sense of beauty. However, we shall speak about these matters at greater length elsewhere.[142]

Most commentators agree that this last promise is not fulfilled in Aristotle's extant works. But the main purpose of the passage itself seems to be the refutation of those who claim that mathematics fails to deal with the beautiful and the good. Ancient commentators like Ps.-Alexander (739.21 ff.) have taken Aristotle as referring to people like Aristippus, the sophist.

As corroborating evidence there is a parallel passage in *Metaphysics* III.2 (996a32–b1) where Aristotle notes the ridicule which sophists like Aristippus heap on mathematics. For instance, they claim that mathematics is useless for practical living because, in contrast even to the lowest arts, it says nothing about goods or evils (περὶ ἀγαθῶν καὶ κακῶν). However, as Ross[143] points out, there is no mention of beauty (τὸ καλόν) in this passage but only of the good (τὸ ἀγαθόν) and its supposed absence from mathematics. Yet it might be the case that the latter is being used as a general term to cover both meanings, so that Aristotle can now answer these sophistic critics by distinguishing the beautiful from the good. While this provides a fairly plausible interpretation for the present passage, it is odd that such a distinction between the beautiful and the good is without parallel anywhere else in Aristotle's works, even though τὸ καλόν sometimes seems to have a wider meaning. It would also be unusual to find

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[142] *Met.* 1078a31–b6: tr. Apostle (1966).
[143] Cf. Ross (1924) ii, 418.

him restricting τὸ ἀγαθόν to the realm of action, given that he considers it to be a πρὸς ἓν equivocal that applies to every category, including divine substance; cf. *EN* 1096a23. If we are to make some sense of the distinction here, therefore, I think that we must accept Ross's suggestion that for the sake of argument the meaning of τὸ ἀγαθόν is being restricted by Aristotle to what is morally good. This fits with the narrow meaning which the sophists gave to the term, according to the reports in III.2. So, by way of answer to such people, Aristotle is justified in introducing his rather artificial distinction between the beautiful and the good. But the broader issue that is more relevant to our discussion is what kind of causes are to be found in mathematics.

In the above-quoted passage, the issue arises as part of Aristotle's answer to the sophistic critics who have been misled by the fact that mathematics does not talk explicitly about the beautiful and the good. Contrary to appearances, he argues that these sciences both say and show a great deal about such causes. For instance, according to him, they show constructions (τὰ ἔργα) and prove theorems (τοὺς λόγους) concerning them. This statement is a little misleading because Aristotle cannot mean that the geometer, for instance, proves by means of constructions that something is beautiful or good. Instead, what he seems to have in mind is that beauty (and hence goodness) is a characteristic feature of the constructions and proofs given by the geometer. This is confirmed when he says that the highest forms of the beautiful are order (τάξις), symmetry (συμμετρία) and definiteness (τὸ ὡρισμένον), all of which belong to the mathematical sciences in the highest degree (μάλιστα). While this might serve as a catalogue of the forms of beauty embodied in Greek art, it is difficult to see how mathematics can be claimed to exhibit these qualities *par excellence*. One can make a strong case for definiteness, however, because the so-called arguments from the sciences use this characteristic of mathematical objects to mark them off as separate from sensible things which are always subject to the more and the less.

Given the antipathy with which the ancient Greeks viewed the indefinite (and the chaotic) in all spheres of life, we can understand why such a characteristic might be enshrined as an aesthetic value. Order is also a closely related value which is reflected in the ancient Greek preference for hierarchies in social and political matters, as well as in theoretical reflection. The order which is embodied in the mathematical sciences is connected with the fact that they begin

with principles (ἀρχαί) which lead them in a successful march to victorious conclusions. Even in the word ἀρχή we find an implicit parallel between hierarchies on the battlefield and in the theoretical sphere; cf. *Posterior Analytics* II.19. Without delving into the sociology of aesthetic value, such parallels may enable us to understand how the order embodied in the mathematical sciences could become paradigmatic for Greek thinkers. In a similar fashion, we might make sense of the historical fact that symmetry was such an important consideration in Greek art and literature. Since it was linguistically connected with the Apollonian maxim 'Everything in measure,' the due proportion which is exemplified in mathematics became a suitable metaphor for self-control in social and political matters.

Whatever its historical background, Aristotle clearly thought that 'the beautiful' (τὸ καλόν) in its highest forms is embodied in the mathematical sciences. But what does this have to do with the ontological status of mathematical objects? Why should Aristotle choose to fight a rearguard action against some unimportant sophistic critics as a conclusion to his discussion of that issue? Consistent with my general approach, I claim that it only makes sense within the broader context of his response to the mathematical cosmology of the Platonists. As far as I can see, the crucial issue concerns the kind of teleology involved in mathematical structures, as distinct from the kind embodied in organic structures like living things. While the good as an end for the latter must be fitted to changing things, the order and definiteness that constitute the goodness and beauty of mathematics are more appropriate for eternal and unchanging things (ἀκινήτοις).

While I recognize that the text of *Metaphysics* XIII.3 provides little evidence that Aristotle is raising this larger issue here, I think there are some indications that he intends to raise it in a later treatise. As evidence for this let us examine his argument that the mathematical sciences deal with certain kinds of causes. This appears to be a continuation of his argument against the sophistic critics, but it hardly seems necessary for their refutation. It would appear to be sufficient for this purpose to establish that the mathematical sciences somehow deal with the beautiful (and hence with the good) under some of its forms. The subsequent statement about causes appears to be the beginning of a new train of thought which is then broken off when Aristotle realizes that a broader issue is being raised. Let us see how this impression is created by the text itself.

Here we find a particular conjunction of particles (καὶ ἐπεί γε) which

tends to suggest that a new implication is being drawn from the previous discussion; i.e. that the mathematical sciences deal with causes in a certain sense. It is noteworthy that Aristotle feels it necessary to establish this by means of the following argument: Some forms of the beautiful (like order and definiteness) appear to be causes (αἴτια) of many things. He does not supply examples of these causes, so that we can only conjecture that he might have in mind the ordered beauty of the heavens. However, as a result of the putative fact that these forms of beauty are efficacious in some way, he claims it is obvious that the mathematical sciences must be speaking about this kind of cause; namely, the beautiful as cause in a certain sense. I think it is evident from the carefully qualified way in which Aristotle admits mathematical forms of beauty as causes that he wants to avoid making a broader concession to the Platonists; e.g. that the final cause of the whole universe is expressible in terms of its mathematical structure whose beauty is identical with order and definiteness. Thus I conjecture that, even if this precise discussion is postponed indefinitely, it is touched upon indirectly in *Metaphysics* XII.

Conclusion

Hence I think it is clear that the question about the ontological status of mathematical objects amounts to much more than just a special issue in the philosophy of mathematics. When placed against the background of the *Timaeus,* it is plausible to treat it as a broader cosmological issue about the intrinsic structure and goodness of the universe. Myles Burnyeat (1987) has noted that modern thought overlooks this cosmic aspect of goodness but what he neglects to say is that this may be due to our loss of perspective on the divine, since that was an intrinsic part of cosmological speculation among ancient Greek thinkers. So, even though it is on the right track, Burnyeat's formulation of the disagreement between Plato and Aristotle is not quite accurate. He takes their disagreement to be about where the paradigm cases of being and goodness are to be found; i.e. whether in the hierarchy of natural kinds studied by theology and natural philosophy (Aristotle) or in the hierarchy of abstract structure revealed by mathematics and dialectic (Plato).

Perhaps it is simply a matter of emphasis but I want to underline the fact that mathematics and dialectic constitute Plato's 'theology'

insofar as both sciences deal with the divine cosmos. In the *Timaeus,* for instance, the divine craftsman applies to his world-making all the conceptual tools of dialectic and mathematics. In this respect, therefore, I disagree with Burnyeat when he claims that the central books of the *Republic* contain the research program which is the primary target for Aristotle's criticism in *Metaphysics* XIII & XIV. However, I agree that the dispute between Plato and Aristotle about mathematical objects boils down to the question of which sciences should be taken as more fundamental for understanding the world and its goodness. But it seems to me that Plato's position on this cosmological question is set out more clearly in the *Timaeus* and that Aristotle recognizes this when he persistently attacks the doctrine of this dialogue by name.

Yet Burnyeat is essentially correct when he asserts that the difference between Plato and Aristotle can be reduced to a choice as to which sciences are going to determine one's metaphysics. This, I take it, is what lies behind the dispute about mathematical objects and whether or not they are independent substances. As evidence for this, I would point to the third question raised in *Metaphysics* XIII.1, which still remains to be settled after the question about the mode of being of mathematical objects. Specifically, this is the question of whether or not numbers and Forms are the substances and principles of things. As I have already shown, this is identical with what Aristotle considers to be a very difficult aporia that still remains to be resolved. Yet in spite of the fact that he explicitly formulates this question in XIII.1 and indicates that it will occupy the greater part of the discussion, it is difficult to find a clear and comprehensive treatise on it such as we have in XIII.1–3. This whole issue will be taken up in my next chapter.

CHAPTER SIX

THE PERFECTION OF THE COSMOS

It is now time to bind up several strands of my argument. Using Aristotle's aporetic framework once more, we can connect the aporia about whether Numbers and Forms are the substances and principles of things with the aporia about whether One and Being are the substances of things. These are both related to the general aporia about whether or not there is any other kind of substance besides sensible substances. While Aristotle denies that mathematical objects are supersensible substances, he holds that there is at least one such kind of substance; i.e. an unmoved mover. But it is within a physical context that he argues for the Prime Mover as an ultimate cause of motion in the universe.

In exploring this network of related problems, therefore, I will first examine the aporia about whether or not Numbers and Forms are the substances and principles of things. Despite the polemical character of his discussion, one can reconstruct Aristotle's answer from *Metaphysics* XIII.6–8, by relating it to his views on one as a measure in XI.1–2. Subsequently, with reference to Books XIV and XII, I will explore his affirmative answer to the question of whether or not there is any kind of separated substance apart from sensible substances. My main purpose is to clarify Aristotle's reasons for positing one kind of supersensible substance (i.e. an unmoved mover), while denying that Forms and Mathematicals are such substances. This brings us back to the cosmological question about the ordering principles of the universe which is raised by the *Timaeus*. In combining craftsmanship with geometry, Plato proposed an ideal of order and harmony that is made intelligible through mathematics. Despite the attractions of this ideal, Aristotle finds it inadequate as a teleological explanation of the sensible universe, and so he replaces the Platonic Demiurge with his Prime Mover as both a cosmological and a metaphysical first principle.

I. *Form Numbers*

Once more let us take our bearings from the guiding question of his inquiry which Aristotle sets out in *Metaphysics* XIII.1; i.e. whether or not there is besides sensible substance any kind of immobile and eternal substance.[1] As we have seen, his initial inquiry is shaped by the differences of opinion among his predecessors about two proposed kinds of supersensible substance, Forms and mathematical objects. Thus in XIII.1–3 he asks whether or not mathematical objects (taken simply as such) exist and if so how they exist. Similarly, in XIII.4–5, he considers claims made for the existence of Forms (as such), though in a perfunctory manner, with an explicit reference to the so-called 'published works.'[2]

Most likely, the reference is to *On Forms* where Aristotle claims that the arguments for the existence of Forms fail to go through. Here he merely repeats some of these objections, while adding others that cast doubt on the existence of such separated entities. For instance, he raises (1079b12–15) a difficulty about how Forms can contribute anything either to eternal or perishable sensible things, since they cannot serve as causes of movement or change in them. Furthermore, it would appear that Forms do not help us to gain knowledge of these sensible things because they are not the substance (οὐσία) in them nor do they contribute as constituents to their being (τὸ εἶναι). In general, therefore, Aristotle insists that the separation of Forms renders them useless as causes in the sensible world and so they fail as objects of science.[3] When considering the rationale for positing Forms in XIII.4, Aristotle seems to have in mind an earlier Platonic theory which does not raise the question of their identity with numbers. This question is connected with the subsequent inquiry about whether the principles of things are numbers and Forms.[4]

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[1] πότερον ἔστι τις παρὰ τὰς αἰσθητὰς οὐσίας ἀκίνητος καὶ ἀΐδιος ἢ οὐκ ἔστι, *Met.* 1076a10–12.

[2] Ross (1924) ii, 408–10 gives a list of parallel references in the Corpus together with a review of different interpretations that have been offered for this much-discussed phrase. He himself thinks that the reference here may be to an attack on the Ideas by Antisthenes and by sophists like Polyxenus, who is reputed to have been the inventor of the so-called 'third man' argument.

[3] See Fine (1987) for a useful analysis of Aristotle's criticism of Plato's Forms as causes.

[4] εἰ αἱ οὐσίαι καὶ αἱ ἀρχαὶ τῶν ὄντων ἀριθμοὶ καὶ ἰδέαι εἰσίν, *Met.* 1076a30–31. Berti (1987) cites the hendyads αἱ οὐσίαι καὶ αἱ ἀρχαὶ as evidence that this problem deals

Although this third inquiry is not completed in any self-contained treatise like XIII.1–3, one can still piece together from a number of different texts the general shape of Aristotle's answer to the leading question.[5] In XIII.6, for instance, one aspect of the inquiry is introduced in the following passage:

> Having stated our position concerning the Forms, it is well now to examine what follows from the statements concerning numbers made by those who say that the numbers are separated substances and the first causes of things.[6]

Such an introduction implies that the proposed inquiry follows naturally after the previous one about Forms, so perhaps it refers to the general sequence of questions already set out in XIII.1. There he proposed to consider first the mode of being of mathematical objects as such, without asking whether or not they happen to be Forms and whether or not they are the principles and substances of things.[7]

In that initial proposal, Aristotle postponed the question of whether or not these objects are the principles and substances of things. Thus, having examined the claims for the existence of Mathematicals and Forms, it is quite natural for him to return to that postponed question, as he does when he refers to the claim that numbers are the first causes of things.[8] Yet it is rather strange that he should propose to examine the implications of a claim that numbers are separated substances (οὐσίας χωριστάς), given that the claim has already been rejected for mathematical objects including numbers, so his emphasis must be on the additional claim that they are the first principles of things. This is exactly the claim about mathematical objects and Forms that was not considered in the first two inquiries.

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with the existence of Form Numbers (XIII.6–9), since Plato treats these as formal causes or principles of other things.

[5] Annas (1976) 78–81, 162–3 thinks that the so-called 'third inquiry' does not begin properly until Book XIV and that the rest of XIII consists of arguments against platonist theories of mathematical objects, and does not contain challenges to basic principles. However, I agree with Berti (1987) that XIII.6–8 is not a digression (as Annas thinks) but rather begins the 'third inquiry' outlined in XIII.1.

[6] *Met.* 1080a12–14: tr. Apostle (1966).

[7] πότερον ἀρχαὶ καὶ οὐσίαι τῶν ὄντων ἢ οὔ, *Met.* 1076a24–25.

[8] τῶν ὄντων αἰτίας πρώτας, 1080a13–14. Tarán (1978) 83 also thinks that the third question listed in XIII.1 is taken up again in XIII.6 and following. In denying that this is the case, Annas (1976) goes against Aristotle's formulation of his leading question, which covers the possibility that Form Numbers might be the substances and principles of things, according to Plato.

In XIII.6–8, however, Aristotle concentrates on the implications of the first claim that numbers are self-identical and independent substances. Using this claim as an initial assumption, he draws out some possible implications as follows:

> If a number is indeed a nature and if its substance is not some other nature but is itself just a number, as some say, it is necessary that either (1) there is something first in it, something else that follows, etc., these being distinct in kind, and this priority begins immediately, with units, so that no unit is comparable with any other unit; or (2) all units are merely successive and any of them is comparable with any of the others, as mathematicians speak of the mathematical number, for in this number no one unit differs from another; or (3) some units are comparable but others are not; for example, if Two is first after One, and Three follows Two, and so on with the other Numbers, and the units within each Number are comparable (for example, the two in the first Two are comparable with each other, the three units in the first Three are likewise comparable, and so on with the rest of the Numbers), but the units of Two Itself are not comparable with those of Three Itself, and similarly for any two such Numbers (and so, mathematical number is counted thus: one, and then two, the latter resulting by the addition of another unit to one, and three results by the addition of another unit to two, and similarly with the other numbers; but these Numbers are counted thus: One, and then Two, the latter being composed of units distinct from One, and then Three, without including Two as a part, and so on with the other Numbers; or (4) one kind of number is the first kind we stated, a second is the one mathematicians speak of, and a third is the one we have named last.[9]

This passage contains a typical Aristotelian attempt at a complete review of all the possibilities, which are later matched up with the views of predecessors.[10] Such a combination of the logical and the factual is typical of his method of dialectical inquiry, which itself is designed to produce a persuasive proof. For instance, if all the logical possibilities deduced from a particular claim can be shown to produce difficulties then the claim itself is undermined. Here the claim, taken quite literally, is that number is some nature whose substance

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[9] *Met.* 1080a15–37: tr. Apostle (1966).

[10] Ross (1924) ii, 426 describes as *a priori* this classification of views, and hence suspects Aristotle of doing injustice to actual views in order to fit them into his ready-made scheme. Tarán (1978) claims that Ross himself goes astray in the classification of number he infers from the whole passage, especially when he accuses Aristotle of confusing incomparable numbers whose units are all comparable with mathematical numbers. Still I think that Ross describes Aristotle's dialectical practice very well.

is nothing other than itself.[11] Aristotle's statement of the claim shows some linguistic similarity to the formulation of the aporia in *Metaphysics* III about whether One and Being are nothing else than the substance of things.[12] In any case, the point is that numbers are not to be regarded as attributes of some other subject but as self-subsistent Forms whose being is nothing else than number.

Keeping in view this claim about Form Numbers made by the Platonists, Aristotle outlines an exhaustive set of logical possibilities both for numbers themselves and for the units which constitute numbers. The first option is: (I) that there will be a 'first number' which is different in kind from the next number. Such an eidetic differentiation between the numbers (e.g. Two and Three) is applied in turn to the units, so that any unit is non-combinable (ἀσύμβλητος)[13] with any other unit. Thus one of the implications of treating numbers as self-subsistent entities is that one cannot use them for ordinary calculation because their units are not combinable. Here Aristotle seems to have in mind a later Platonic theory of Form Numbers, which is usually regarded as being part of the so-called 'unwritten doctrine.'[14]

However, we cannot exclude the possibility that this strange theory of number is introduced by Aristotle himself simply as a logical implication that follows from giving numbers the ontological status of independent substances, especially since he later concedes that none of his predecessors adopted the even stranger view that *all* units are non-combinable.[15] So both might be listed simply for the sake of logical completeness. By contrast, the second option actually corresponds to the way mathematicians treat their units: (II) that all units are directly successive (εὐθὺς ἐφεξῆς) and that all units whatever are

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[11] εἴπερ ἐστὶν ὁ ἀριθμὸς φύσις τις καὶ μὴ ἄλλη τίς ἐστιν αὐτοῦ ἡ οὐσία ἀλλὰ τοῦτ' αὐτό, *Met.* 1080a15–16. This claim may be implicitly referring to the reported Academic distinction between *kath' hauto* terms, which pick out independent items, and *pros ti* terms that pick out dependent or relative items; cf. Alexander, *in Metaph.* 83.24–26; Xenocrates Fr. 12 Heinze; *Divisiones Aristoteleae* 39–41 Mutschmann; Simplicius, *in Phys.* 247.30–248.15.

[12] οὐχ ἕτερον τί ἐστιν ἀλλ' οὐσία τῶν ὄντων, *Met.* 996a7.

[13] Robin (1908) 272 n1 thinks it makes no difference whether one translates this word into French as 'incomparable' or as 'inadditionnable,' though he expresses a preference for the latter because it captures the link with the consequent differentiation of Form Numbers. But I have chosen 'non-combinable' because of its numerical connotations in English, since I cannot think of any word with the right ontological nuance.

[14] See Gaiser (1963) 539–40 who cites XIII.6 as a source for the unwritten doctrine.

[15] Cf. *Met.* 1080b8–9.

combinable (συμβληταί) with all others. With respect to this option, Aristotle explains that in mathematical number no unit is in any way different from another.[16] The language here is so reminiscent of Plato's descriptions (*Rep.* 526A & *Phil.* 56D–E) of how mathematicians treat their units that both must be referring to the same feature of Greek arithmetic. Thus the second logical possibility preserves the phenomena and is given a preferred status by Aristotle as a possible view of number.[17]

The same can hardly be said for the third option: (III) that some units are combinable whereas others are not. This introduces the rather bizarre view that the units *within* each Form Number are combinable with each other but that the units which constitute different Form Numbers are not interchangeable. Thus, for instance, the units in what Aristotle calls 'the first two' are combinable with each other but not with the units which constitute 'the first three.' It seems that he is referring here to Form Numbers and insisting that, on account of their separation as independent substances, their units are also separated in such a manner as not to be combinable with each other. This is why he distinguishes between the normal mathematical way of counting units which are combinable and some eidetic manner of counting that goes as follows: after One another Two is counted which does not include the first One and, similarly, a different Three is then counted which excludes the previous number Two. However, if we treat this simply as an anticipation of our modern distinction between cardinal and ordinal numbers, we may be missing the point.[18]

The important thing for the purposes of Aristotle's subsequent objection is that he takes Form Numbers also to be constituted by units which are not combinable with each other outside the particular number they constitute. Such mutual exclusivity with respect to their units is thought to follow from the claim that such numbers are

[16] ἐν γὰρ τῷ μαθηματικῷ οὐδὲν διαφέρει οὐδεμία μονὰς ἑτέρα ἑτέρας, *Met.* 1080a22–23.

[17] I think that the identification of this option with mathematical number is confirmed by the final lines of the passage (a35–37), whether we take these to be outlining a fourth possible view or to be summarizing the previous options.

[18] Tarán (1978) accepts the view of Cherniss (1944 & 1945) and Cook-Wilson (1904) that Aristotle has simply misunderstood Plato's 'advanced' concept of number. But this view is rightly called into question by Burnyeat (1987) because there is no evidence that any ancient thinker grasped the 'advanced' concept attributed to Plato. Annas (1976) points out that Plato held an arithmetical view of Form Numbers as constituted from 'pure' units which are unique to each number, since each Form is itself unique.

self-subsistent Forms which are prior and posterior to one another. Perhaps this was Plato's attempted solution to the intractable problem about the unity of number, which is composed of units and which was traditionally defined in terms of these elements. In other words, he proposed to explain the unity of Form Numbers by giving their ordinal character precedence over their cardinality, especially with regard to the combinability of their constituent units.

This aspect of the claim becomes more explicit when Aristotle supplies a brief but exhaustive survey of possible modes of being for number:

> Again, the kinds of numbers listed must (A) all be separated from things, or (B) none of them be separated but exist in the sensible objects, not existing in them, however, in the manner we first considered, but in the sense that sensible objects consist of numbers as their constituents, or (C) some of them be separated but others not separated.[19]

Once again, a logical schematism dictates the structure of this survey in the following way: either (i) *all* numbers are separated or (ii) *none* are separated or (iii) *some* are separated and *some* are not. Qualifying the second option, he rules out the possibility that numbers can be *in* sensible things as independent substances, presumably because this applies only to geometrical objects that have relative position.[20]

It seems that Aristotle has an eye to the historical facts when setting out the possible modes of being for number, as we can see from his matching of the logical with the factual in the subsequent passage:

> These, then, are of necessity the only ways in which numbers can exist. And of those who said that the One is a principle and a substance and an element of all things, and that a number is made out of the One and of something else, almost everyone spoke of numbers as existing in some one of these ways, except for this, that no one said that all the units are noncomparable. And this happened with good reason; for no other way is possible besides the ones stated.[21]

Given its emphasis on necessity (ἐξ ἀνάγκης), the first statement can be read as a logical claim about the completeness of the previous list

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[19] *Met.* 1080a37–b4: tr. Apostle (1966).

[20] If some thinker held that only geometrical objects are *in* sensible things, as XIII.2 seems to indicate, then Aristotle may here be specifically excluding the possibility that numbers are similarly *in* things. Perhaps this is why he focuses on the objection about two solids competing for the same place.

[21] *Met.* 1080b4–11: tr. Apostle (1966).

of possible ways for number to exist. The relevance of this survey to the leading aporia is established by the reference to those who made a principle of number (i.e. the One) a substance and an element of all things.

After the above introduction, Aristotle outlines the various historical opinions about different kinds of number. This is consistent with his dialectical procedure, since he has already (1080a15 ff.) outlined all the possible modes of being for number as a self-identical and independent entity. So now Aristotle continues with his original project (1080a13–14) of examining the consequences for numbers which follow from the claim that they are separate substances and the first causes of existing things. But the opinions of the Platonists differ from those of the Pythagoreans on the separation of numbers, though not on their being causes of sensible things:

> Thus, some say that both kinds of numbers exist, those of one kind which have the prior and the posterior being the Forms, and those of the second kind being the mathematical numbers and that the latter are distinct from the Forms and the sensible objects; and both kinds are separate from the sensible objects. Others say that only mathematical numbers exist, and that of all things these are first and are separate from sensible objects. The Pythagoreans, too, posit numbers of only one kind; and they say that these do not exist separately, but that the sensible substances consist of these. For they posit the whole universe as constructed out of numbers, but not numbers consisting of units in the usual sense, for they believe that units have magnitude. But these thinkers are unable to say how the first One was constructed so as to have magnitude. Another thinker says that numbers of the first kind alone exist, those of the Forms; and some say that mathematical numbers are the same as these.[22]

This passage is typical of Aristotle's schematic summaries in that it tries to give a comprehensive review of all the historical opinions about number as an independent entity, while also differentiating the Platonists from the Pythagoreans on the question of separating numbers from sensible things.

The first view is usually attributed to Plato, who is reported to have posited both Form Numbers and mathematical numbers. From the above passage we gather that it is characteristic of Form Numbers to have a prior and a posterior (τὸ πρότερον καὶ ὕστερον) among them, whereas we can presume that mathematical numbers do not

[22] *Met.* 1080b11–23: tr. Apostle (1966).

have this ordinal character. In *Metaphysics* I.6 (987b14–18), Aristotle describes the latter kind of numbers as 'being many alike' (πόλλ' ἄττα ὅμοια), whereas the corresponding Form for each is single (ἓν ἕκαστον). Both kinds of number, however, share the common features of being eternal and immovable (ἀΐδια καὶ ἀκίνητα).[23] Such features differentiate them from sensible numbers and also separate them from sensible things. Since both kinds of number are said (1080b13–14) to be separate, it is clear that the priority and posteriority of Form Numbers is not a direct result of their being separate (χωριστούς) from sensibles. On the other hand, however, Form Numbers have non-combinable units presumably because they each have a unique place in a serial order (i.e. prior and posterior).

Besides those who accepted Plato's view about two kinds of number, Aristotle reports (1080b14–16) on others who posited only mathematical number as the first of all things and as separated from sensible things. This view is usually attributed to Plato's successor, Speusippus, who carried on the Pythagorean tradition in the Academy. In the above passage, therefore, it is quite appropriate that the view of Speusippus should be linked with the Pythagorean view that there is only mathematical number. According to Aristotle, however, the crucial difference is that the Pythagoreans do not separate numbers but rather claim that sensible substances are composed of them.[24]

This appears to be the same as one of the views mentioned previously when Aristotle outlined the possible modes of being for numbers. By contrast with a separated mode of being, he canvassed the possibility that numbers are not separate but in sensible things as constituents.[25] In the present passage, moreover, Aristotle suggests that the Pythagoreans treated numbers as the material constituents of sensible things when he reports their claim that the whole universe is constructed out of numbers.[26] The claim is clarified further when he points out (1080b19–20) that these numbers do not consist of abstract units (μοναδικῶν) because the Pythagoreans assumed the units

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[23] Cf. *Phaedo* 103E–104B for a Platonic discussion of the eternal attributes of number. David Lachterman (1989) 117–8 correctly links the multiplicity-in-similarity of Intermediates with their image character, as distinct from Forms which are imageless.

[24] πλὴν οὐ κεχωρισμένον ἀλλ' ἐκ τούτου τὰς αἰσθητὰς οὐσίας συνεστάναι φασίν, *Met.* 1080b17–18.

[25] ὡς ἐκ τῶν ἀριθμῶν ἐνυπαρχόντων ὄντα τὰ αἰσθητά, *Met.* 1080b3–4.

[26] τὸν γὰρ ὅλον οὐρανὸν κατασκευάζουσιν ἐξ ἀριθμῶν, *Met.* 1080b18–19.

to have magnitude. Such an assumption is consistent with treating numbers as material constituents, yet it is more likely that the Pythagoreans were thinking of the formal structure of sensible phenomena like musical harmony.[27]

As an objection against the Pythagorean view of numbers, however, Aristotle raises the question of how the first One was composed so as to have magnitude.[28] This implies that the Pythagoreans did not address this difficulty but simply assumed their basic unit to have magnitude. Presumably Aristotle's objection would be that they are not entitled to assume this much, given that the usual concept of a unit is of something indivisible. Such an objection might be derived from his distinction between discrete and continuous quantity in the *Categories* and in *Metaphysics* V. If one accepts this distinction then it is difficult to see how 'the first One' can have magnitude. While earlier Pythagoreans were probably unaware of this problem, Speusippus may have tried to resolve it by positing διάστημα as a material principle of continuity along with the point as a formal principle.[29]

The original Pythagorean project of constructing the whole world out of numbers clearly illustrates the cosmological significance of Greek mathematics.[30] According to Aristotle's report in *Metaphysics* I.9 (989b29 ff.), the aim of the so-called Pythagoreans was to understand the visible order of the sensible heaven in terms of numbers, and he is referring to the same goal here when he says that they construct the whole universe (τὸν ὅλον οὐρανόν) out of numbers. Furthermore, since they considered mathematical principles to be the principles of all beings, their views are relevant to the aporia in XIII.6. Apparently, they were deeply impressed by the discovery of a numerical structure in the harmonic intervals that limit the indefinite continuum of sound. Thus the nature (φύσις) of all things appeared to be like numbers and, since they regarded numbers as first in all of nature, the Pythagoreans assumed that the elements of number are the elements of all things and that the whole heaven is a harmony and a number.[31]

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[27] Cf. Burkert (1962), Szabo (1978).

[28] ὅπως δὲ τὸ πρῶτον ἓν συνέστη ἔχον μέγεθος, ἀπορεῖν ἐοίκασιν, *Met.* 1080b20–21.

[29] Cf. Tarán (1981) 45.

[30] Klein (1968) ch. 7 argues that treating mathematics as a self-sufficient science involves ignoring the ontological difficulties that determine both its problems and its historical development..

[31] τὰς τούτων ἀρχὰς τῶν ὄντων ἀρχὰς ᾠήθησαν εἶναι πάντων, *Met.* I.5, 985b25. Cf. *Met.* 985b33–986a3.

It is precisely because of their cosmological assumptions that Aristotle includes the Pythagoreans in his review of opinions of those who hold that numbers are the primary causes of existing things. Although the general purpose of such a dialectical review is to prepare for the refutation of these opinions, Aristotle delays bringing forward difficulties for the Pythagoreans until later (XIII.8). Thus, in the above-quoted passage, he merely hints that their first principle is incoherent because they posit the first One as having magnitude. While this objection focuses on the difficulty of seeing how a continuous magnitude can be produced from something indivisible, it seems to neglect the second principle which he attributes elsewhere (*Met.* 987a13 ff.) to the Pythagoreans. This is the principle called the indefinite (τὸ ἄπειρον) which, along with the One, might serve as a principle for number.

These would appear to be what Aristotle has in mind when he says (986a15 ff.) that the Pythagoreans treated numbers as the principles of things in two distinct senses; i.e. as matter (ὡς ὕλην) and as attributes or dispositions (ὡς πάθη τε καὶ ἕξεις). In that passage he explains that the elements (στοιχεῖα) of number are the Even and the Odd, of which the latter is the finite (τὸ πεπερασμένον) whereas the former is the indefinite (τὸ ἄπειρον). By assuming corresponding principles for each one of the three dimensions, however, later Pythagoreans like Speusippus and Xenocrates found ways of generating magnitude from the One.

This is relevant to Aristotle's brief survey of a range of different opinions about geometrical objects like lines, planes, and solids. For instance, some Platonists claim that such mathematical objects (τὰ μαθηματικά) are distinct from the so-called 'things that come after the Forms' (τὰ μετὰ τὰς ἰδέας).[32] Ross (1924 ii, 429) thinks that this phrase comes from a later Platonic theory of Forms as numbers, which denies that geometrical objects are Forms because they are not numbers. Given the obscure reports we have about this theory, however, it is difficult to say whether this was an implication drawn by Plato or part of an Aristotelian objection against Form Numbers; cf. *Met.* 992b13–18. Within the context of a review of difficulties in XIII.9, for instance, Aristotle refers (1085a7–8) to lines, planes, and bodies, as 'the genera posterior to number' (τῶν ὕστερον γενῶν τοῦ ἀριθμοῦ), indicating perhaps that geometrical objects were described by the Platonists as 'the things after the numbers' (τὰ μετὰ τούς

[32] Cf. Isnardi-Parente (1987).

ἀριθμοὺς) in virtue of some schema of priority. But it is unclear whether they took such priority relations to imply that the Line Itself or the Plane Itself were not Forms, and indeed Aristotle himself may have drawn this implication from the identification of Forms and Numbers.

Be that as it may, such an understanding clearly informs his distinction (1080b25–26) between strict Platonists and those who follow Speusippus in speaking about mathematical objects in a mathematical way (μαθηματικῶς). From Aristotle's point of view, the important difference is that the latter do not identify Forms with numbers because they deny the existence of Forms.[33] What he means by 'speaking mathematically' emerges from the further distinction (1080b30) between Speusippus and Xenocrates. Even though the latter accepts the existence of mathematical objects, according to Aristotle, he fails to treat them in a mathematical manner because he does not accept two basic theses: (1) that every magnitude is indefinitely divisible into other magnitudes; and (2) that any two units make a dyad. The first thesis (which is the basic principle of continuous magnitude) is denied by the doctrine of indivisible lines that is usually attributed to Xenocrates, even though Aristotle also ascribes it to Plato (*Met.* 992a20). The second thesis is contradicted by the theory of non-combinable units which is held to follow from the positing of Form Numbers by Plato. However, Aristotle accuses Xenocrates of not treating mathematical objects in a mathematical way, as he claimed that Forms and numbers have the same nature.[34]

The final summary at the end of XIII.6 reveals that Aristotle's complete review of opinions about number is guided by some leading question to which his predecessors are seen as giving answers:

> All those who say that the One is an element and a principle of things posit numbers as consisting of units taken in the usual sense, except the Pythagoreans who, as we said before, say that units have magnitude. It is evident, from what has been said, in how many ways it is possible to speak of numbers, and that all the views taken concerning numbers have been stated. But all these doctrines are impossible, and some are perhaps more so than others.[35]

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[33] Tarán (1981) 308–11 shows that Speusippus rejected any kind of Form Numbers; cf. *Met.* 1083a20–24.

[34] Tarán (1981) 310 gives a similar analysis of the views of Xenocrates. Krämer (1964) 204 ff. also reads this whole passage as distinguishing the views of Xenocrates from those of Plato and Speusippus.

[35] *Met.* 1080b30–36: tr. Apostle (1966).

Whether or not he is correct, Aristotle treats all of the surveyed opinions as sharing the assumption that the One is an element and a principle of things.[36] This means that the whole inquiry into number comes under the guiding aporia about whether or not One and Being are the substances of things. But, as I have already shown, the aporia itself has a broader range of concerns than the mathematical cosmology of the Academy, since it takes up the Parmenidean issue about the uniqueness of Being that Aristotle resolves through his notion of its *pros hen* structure. The issue is also relevant to his postulation of a Prime Mover as the teleological principle of unity in the universe.

Within this broader perspective, however, the inquiry into number may be taken as serving a negative purpose for Aristotle, who wants to show that all mathematical cosmologies are fundamentally mistaken. This might help to explain why the views of his predecessors are forced into schemata of logical possibilities, rather than being treated on their own merits. His acute interest in the number theory of his predecessors becomes more intelligible if we assume that such metaphysical and cosmological issues lie behind it. Therefore, when Aristotle declares here that all such theories are impossible (ἀδύνατα), we may take him to be implicitly rejecting the mathematical cosmology which is linked to Greek number theory. In XIII.9 (1086a14–16) when he gives a summary analysis of what is wrong with such theories, he asserts that their assumptions and principles are false, and that this explains why they do not agree but rather conflict with one another. But our natural expectation that a correct account of the principles of mathematics will follow is disappointed in XIII.9 when he makes an abrupt transition to a different set of problems about the principles and elements of things; i.e. whether such principles are universal or are like particular things. As Berti (1987) has pointed out, these problems correspond to a different set of aporiae in *Metaphysics* III than those which guide the discussion in XIII.1–9.

II. *The difficulties about Form Numbers*

After outlining the views of his predecessors on number in *Metaphysics* XIII.6, Aristotle reviews the difficulties that arise for each of their positions. Thus in XIII.7 he returns to the question of whether units

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[36] ὅσοι τὸ ἓν στοιχεῖον καὶ ἀρχὴν φασιν εἶναι τῶν ὄντων, *Met.* 1080b31–32.

are combinable or non-combinable, since this was an issue that arose from treating numbers as natures that are independent and unique. Once again, he sets out all the logical possibilities for non-combinable units as follows: (i) that any unit whatever is non-combinable with any other unit whatever; (ii) that the units in each Form Number (e.g. the Two Itself) are non-combinable with units in another. In general, with respect to the units composing number, there are three logical possibilities: (i) either *all* units whatever are non-combinable with each other, or (ii) *some* units within each Form Number are combinable with each other, while *some* units in different Forms are not combinable with each other, while *some* units in different Forms are not combinable with each other, (iii) that *all* units whatever are combinable with each other.

Within his review of the difficulties in XIII.7, the third possibility is discussed first in a way which suggests that Aristotle himself accepts it as being true about number. What he says is that if all units are combinable and without difference then there is only mathematical number, and Forms cannot be numbers (1081a5–7). The final remark shows that Aristotle thinks that the Platonic theory of Form Numbers is undermined by a true conception of number. Another significant fact is that he does not introduce any difficulties against the possibility that all units are combinable, but rather uses this mathematical view of number to make difficulties for the theory of Form Numbers.[37] For instance, given that there is only one Form of Man, it seems impossible to identify it with any particular collection of units, since there is nothing to differentiate one collection as unique like a Form (1081a8 ff.).

Aristotle next (1081a17 ff.) considers the difficulties that arise for the first possibility (i) that all units whatever are non-combinable with each other. Significantly enough, his immediate objection is that a number composed of such units cannot be a mathematical number, since the proofs in that science are only applicable to numbers which are composed of undifferentiated units (1081a19–21). This fits with Plato's reports on mathematical number, and from it we can infer what is meant by non-combinability.

But Aristotle pays more attention to a second objection; i.e. that

[37] Annas (1976) 165 claims that Aristotle wants to understand non-combinability in terms of combinability, and so he first discusses the possibility that all units are combinable.

if all units are non-combinable then the numbers composed from them cannot be Form Numbers (1081a21 ff.). Given that Plato did not assume that all units are non-combinable, however, it is difficult to see how the objection tells against his view of Form Numbers. Indeed, as Ross (1924 ii, 435) notes, Aristotle pays a disproportionate amount of attention to the view that all units are non-combinable, given that the view had no supporters. However, I think this may be explained as an attempt at logical completeness, which is an integral part of his dialectical method. Thus, at the end of a long discussion, Aristotle winds up his consideration of the difficulties as follows: if every unit differs from every other unit, these and other similar difficulties follow of necessity (1081b33–35). In addition, although none of the Platonists spoke of their units as being non-combinable in this way, he insists that their principles can plausibly be taken to imply such a view (1081a35–b1). For instance, he argues, if there is a first One, then it is reasonable that there should be priority and posteriority among the units; just as there should be among the Dyads if there is a first Dyad (1081b1–10).

The whole point of the objection is that the Platonists should accept an ordered series of units (which they do not), if they were to follow their principles consistently. The objection depends on the application to units of the authentic Platonic view that Form Numbers constitute an ordered series. But Aristotle's argument trades on an ambiguity in talk about 'the first One,' which can be taken to refer to the first unit, just as easily as to the first number. Subsequently, he attributes to the Platonists the thesis that there is a first One, though he admits that they do not posit a second or third One (1081b9–10). This admission should raise our suspicions about Aristotle's dialectical strategy here, since the logic of an ordered series would require a second and third unit, if the Platonists had really posited the unit as first in such a series. On the contrary, however, they seem to have distinguished between mathematical and Form number on the grounds that mathematical units do not constitute an ordered series.

One possible explanation for Aristotle's rather odd strategy is that he is trying to undermine the Platonic principles by exploiting their ambiguity. Given the different ways of interpreting the One, Aristotle seems to be arguing that this principle could be taken to imply that all units are non-combinable (by virtue of forming an ordered series), even though the Platonists did not draw this conclusion. Similarly,

the positing of a 'first two' could be taken to imply that there is a second and a third two; i.e. an ordered series of twos. While this implication undermines the uniqueness of the Form of Twoness, it also yields the absurd consequence that none of these twos can be added to one another to yield four, just as in the case of the non-combinable units.

Aristotle's subsequent objection (1081b10 ff.) is more radical in concluding that, if all units are non-combinable, then Form Numbers like the Two Itself cannot even exist. The objection seems to be directed against some Platonic method of producing Form Numbers that contravenes the ordinary method of counting by addition, whether the units are undifferentiated or differentiated from each other. For instance, two is generated by adding one to another one, while three is generated by adding another unit to this pair. Obviously, Aristotle takes this feature of counting to be definitive for numbering in general, since it is used as a basis for criticising the Platonic method of generating Form Numbers from the Dyad and the One. For example, even though two is clearly part of four in the case of mathematical number, the Platonists cannot accept that the Two Itself will be a part of four, since this would imply that another Two is added to make four (1081b22–24). Along with undermining the uniqueness of the original Two, it would appear to be incompatible with the Platonic 'generation' of Form Numbers like Four from the first Two and the Indefinite Dyad. From Aristotle's brief and obscure description, it seems that the Indefinite Dyad acts by doubling the original Two so as to yield Four (1081b21–22). In general, therefore, the Indefinite Dyad functions as a principle of multiplicity through doubling, while the One serves as a principle of unity by setting a limit to the indefinite multiplication of the other.

Thus Aristotle's objections are directed against the Platonic principles of generation for number, which are taken to imply that all units are non-combinable, even though the Platonists did not hold this view. So we find him concluding his dialectical *tour de force* by outlining an insoluble aporia in which the Platonists are enmeshed as a result of their principles for number:

> Again, how will there be other 'Triads' and 'Dyads' besides the Triad Itself and the Dyad Itself, and in what manner will they consist of prior and posterior units? Now all these are fictitious, and there cannot be a first Dyad and then a Triad Itself. Yet there must be, if indeed the One and the Indefinite Dyad are to be elements. Thus, if

> the results are impossible, the principles cannot be those which they put forward. If all the units are different, then, these and other such results necessarily follow.[38]

The rhetorical questions here serve to recall previous objections. For instance, the question about other Triads and Dyads refers back to the objection against the Platonic generation of numbers, which argued that either there must be two other Twos besides the Two Itself or else the latter will be part of four (cf. 1081b21–24). The second question refers back to the difficulty about how Form Numbers can consist of non-combinable units (cf. 1081a25 ff.). Thus, without further argument, he asserts that these views are absurd and fictitious, since the difficulties concerning them have already been explored. Yet he insists that such views follow necessarily from positing the One and the Indefinite Dyad as elements of number. Clearly, the whole point of his elaborate set of objections against a view that nobody held is to undermine such Platonic principles, both as elements of number and as ontological principles.

Finally, he considers the second possibility (ii) that units are combinable within but not between Form Numbers: a view which is attributed to some Platonists. Aristotle insists that this view does not escape the difficulties associated with non-combinable units, but rather accumulates additional ones. In the Ten Itself, for instance, he claims that there are ten units of which it consists, as well as the two fives from which ten comes to be (1082a1–2). From this he argues that the units in the Ten Itself must also differ from each other, since this is not any chance (τυχών) number, nor is it composed of any chance fives; just as it not composed of any chance units (1082a2–5). Therefore the whole position is inconsistent because the conclusion that the units in the Ten Itself are different from each other conflicts with the initial assumption that they are internally combinable.

The general strategy of Aristotle's objection seems to be as follows. First, he assumes that the Form Numbers (e.g. Ten Itself) are specifically different and from this infers the specific difference of their constituent units. Next, he argues that one runs into difficulties whether one assumes that these units are specifically the same (and hence combinable) or different (and hence non-combinable). Finally, he appeals to the Platonic principles for the generation of Form Numbers in order to confirm that Ten Itself, for instance, is not

[38] *Met.* 1081b27–33: tr. Apostle (1966).

composed of any chance Fives.[39] Here I think that Aristotle is contrasting such specifically different numbers and units with those commonly used in arithmetic, which he often refers to as 'any whatsoever' (ὁποιαιοῦν ὁποιαισοῦν); cf. 1081b34, 1081a2, 1080a20. The latter do not differ specifically from one another, and so they are combinable with each other in all of the operations of arithmetic; e.g. $5 + 5 = 10$.

Thus I would suggest that the whole objection makes more sense when we see such ordinary mathematical intuitions about number operating in the background. For instance, this would explain Aristotle's assumption that Ten Itself must be composed of ten units and of two Fives. So the difficulties arise from the counter-intuitive result that the ordinary operations of arithmetic cannot be applied consistently to the Form Numbers. For example, the two Fives which compose the Ten Itself must be specifically different (and hence non-combinable with each other) if they are themselves Form Numbers. Since these two Fives are not combinable with each other, they cannot be added to yield Ten Itself and this goes against our ordinary mathematical intuitions.

But the second objection against inaddible numbers has wider implications for the general concept of number as composed of units, which was shared by Plato and Aristotle. It shows that Aristotle's objections are guided by the leading question of whether or not numbers are separated substances and the first principles of things (cf. 1080a13–14). If numbers are such substances then they must each have a principle of unity that combines their parts into a substantial whole, just as the soul does for a living being. In *Metaphysics* XIII.2 (1077a20 ff.) Aristotle made this objection against claims for the substantiality of geometrical objects such as lines and planes. Now he makes a similar objection against the putative Platonic claim that numbers are separate substances:

> Again, how is it possible for the Dyad to be a nature besides its two units and for the Triad to be a nature besides its three units? For this may be either when one thing shares in another, as a white man shares in whiteness and in a man, or when one is a differentia of the other, as a man in the case of being and animal and two-footed. Moreover,

[39] Ross (1924) ii, 437 notes the obscurity of Aristotle's talk about any 'chance' number, while Annas (1976) 171 conjectures that 'not just any' is his scornful way of describing the Form number made up of units combinable only within that number.

> some parts are one by contact, others by being blended, still others by position; yet none of these ways can apply to the units to form the Dyad or the Triad; but just as two men are not a unity apart from both, so must it be with the units. And the fact that units are indivisible is no cause for any difference in them; for points too are indivisible, yet a pair of them is nothing other apart from the two.[40]

It seems clear that the initial rhetorical question is directed against what Aristotle takes to be a key implication of positing Form Numbers; namely, that the Dyad is 'some nature' apart from its two constituent units.

Thus his objection homes in on the problem of what unifies a number composed of units, which was a problem that also worried Plato; cf. *Theaet.* 205E, *Parm.* 132A, *Phil.* 14B. With regard to number, the crux of the problem is to find an appropriate principle of unity such that its parts (i.e. the units) actually constitute a substantial whole, and not simply a collection or heap. Therefore, the general strategy of Aristotle's objection is to canvass different principles of unity, in order to show that number cannot share in any of them and so cannot be a separate substance, as the Platonists claimed.

Initially, he introduces two different kinds of unity between parts, which may be called accidental and essential unity, respectively. Significantly, he describes the principle of accidental unity as 'participation' (μεθέξις), by contrast with differentiation as the principle of essential unity. In illustrating accidental unity, Aristotle uses his favourite example of 'white man' which is a compound of a substantial subject and an accidental attribute; cf. *Met.* 1037b14–18, 1077b4–11. Prior to being compounded, the subject and attribute did not have any essential relationship to each other, and so their unity must be explained in terms of some principle like participation. However, the unity of biped animal in the definition of man cannot be explained in this way, since animal cannot participate in two-footedness and its contraries at the same time; cf. *Met.* 1037b10 ff. Thus the principle of essential unity for parts of the definition is to be found in differentiation (1082a19).

But, with regard to the units which compose a number, it seems that neither participation nor differentiation can function as principles of unity, presumably because of the lack of a suitable subject-predicate structure in number. Therefore, Aristotle canvasses other

[40] *Met.* 1082a15–26: tr. Apostle (1966).

types of unity which might serve to unify the parts of Form Numbers: (i) unity by contact; (ii) unity by blending; (iii) unity by position. Although he does not illustrate these kinds of unity here, we may draw on *Metaphysics* V.6 for illustrations. For instance, a bundle of sticks is unified by contact, and this sort of unity by virtue of continuity is called one *per se* rather than *per accidens*. In any case, it is clear that unity by contact is not appropriate for number, since an arithmetical unit has no position; cf. *Met.* 1016b25. For the same reason it is clear that unity by position is not appropriate for number, though it may be suitable for the unity of a house built from stones. Finally, unity by blending is not suitable since it belongs to such things as the mixture of honey with wine that constitutes mead.

Therefore Aristotle concludes that none of these types of unity can belong to the units from which the Two and the Three Itself are composed. Indeed, he argues that there seems to be no substantial unity apart from the two units, just as there is nothing else apart from two individual men. Furthermore, he refuses to accept that the indivisibility of the units explains why they are differentiated within Form Numbers and hence are non-combinable outside of these numbers. Drawing on the parallel with points, he argues that they are also indivisible, yet a pair of them does not constitute a separate entity beyond them. Thus the basic thrust of Aristotle's objection is to produce an aporia about how a Form number like Twoness can be unified as a self-subsistent entity. It is noteworthy that, contrary to people like Cook-Wilson (1904) who attribute an 'advanced' concept of number to Plato, Aristotle consistently assumes as self-evident that the Form Numbers will be composed of units.

Furthermore, the difficulties raised against non-combinable numbers are firmly based on ordinary mathematical intuitions, as the following objection shows:

> In general, to posit units as being different in any manner is absurd and fictitious (by 'fictitious' I mean that which is forced to agree with a hypothesis); for neither with respect to quantity nor with respect to quality do we observe one unit as differing from another unit, and one number must be either equal or unequal to another number, this being so in all cases but most of all when the ultimate parts of a number are indivisible units. So if one number is neither greater nor less than another, then it must be equal to it; and in numbers, things which are equal and do not differ at all are believed to be the same. If not,

> neither will the 'Dyads' in Ten Itself be without difference, though they are equal; for what reason will one have in saying that they are without difference?[41]

Just as in *Metaphysics* XIII.2 (1077a14 ff.), Aristotle makes a transition here to a new set of difficulties based on our ordinary assumptions about the truth of the matter. From such a perspective, he describes the notion of non-combinable units as 'fictitious' and as being driven by an hypothesis, rather than conforming with the ordinary view of units which treats them as homogenous both in quantity and quality. Aristotle is on firm dialectical ground here because Plato also recognized that mathematicians postulate units as being equal to each other in every way without the slightest difference; cf. *Rep.* 526A3–4. This lends some weight to the charge that other units are being fictitiously postulated in conformity with the hypothesis of non-combinable numbers. But what Aristotle finds most absurd about Form Numbers is that the basic axioms of arithmetic cannot be applied to them, so that the very meaning of 'number' has been altered.

III. *The One as a principle of number*

But Aristotle's own positive views on units and number are quite difficult to excavate from under the layers of dialectical objections in *Metaphysics* XIII.6–9, without some map that tells one where to dig or what to recover from the rubble of collapsing Platonic positions. For that purpose we must use X.1–2 and V.6 in our search for clues as to why Aristotle rejected Plato's theory of Form Numbers as an inadequate account of Greek arithmetic. A brief look at his treatise on time in *Physics* IV will also be in order.

The connection between the question about number and the guiding aporia outlined in *Metaphysics* III is clearly established at the beginning of *Metaphysics* X.2 as follows:

> With respect to the substance and nature of oneness we must inquire, as we did in the discussion of difficulties, what oneness is and how we should consider it, whether it is a substance of a sort, as the Pythagoreans said first and Plato later, or rather there is some nature underlying it, in which case we should make this more known and speak more in

[41] *Met.* 1082b1–11: tr. Apostle (1966).

> the manner of the physical philosophers; for one of these says that the one is Friendship, another that it is Air, another that it is the Infinite.[42]

Here the opposing views, along with the illustrative examples, are exactly the same as those which constitute the aporia in *Metaphysics* III (1001a9 ff.) about whether being and unity are the substances of things or not. On the one hand, Plato and the Pythagoreans held that the substance of the One is nothing else than being one; whereas the natural philosophers posited some other underlying thing such as Friendship which is more familiar (to us) as the cause of unity in all things. These groups represent the two extremes of logical and physical inquiry which Aristotle himself wishes to avoid, while extracting elements of truth from each in working out his own views on unity and on number.

On the one hand, the Platonists posited One Itself as an independent substance whose nature is nothing other than unity; i.e. as a self-identical Form. Aristotle's objection begins (1053b16 ff.) from the claim that no universal can be a substance, which he has defended in *Metaphysics* VII.13–15. Since being and unity are the most universal predicates, it follows that unity cannot be an independent substance or a 'one over many' such as the Platonists held it to be. Just like being, unity is a common predicate shared by all things that are unities, no matter how they are unified. Nor can unity be a genus for the same reasons that being cannot be the genus of anything. On the basis of these parallels, Aristotle declares (1053b25) that being and unity have equally many senses. This thesis about the convertibility of unity and being has important implications for his views on the unit and on number.

In the first place, it means that unity will have a different meaning in each of the categories, just as in the case of being. But, despite the multivocity of being and unity, Aristotle insists (1053b27) that one ought to inquire in general (ὅλως) what unity is (τί τὸ ἕν) in the same way that one must inquire about what is being (τί τὸ ὄν). Although the question seems to beg a Platonic answer in terms of a Form of One, he rejects (b28) as uninformative any account of the nature of unity in terms of unity itself. By contrast, he finds it more useful to seek a different account of unity for entities in different categories. For instance, in the case of a quality such as color, the

[42] *Metaphysics* X.2, 1053b9–16: tr. Apostle (1966).

unit will be a color like whiteness, since (in Aristotle's chromatology) the other colors are generated from white and black.

Thus, to generalize the point, if all things were colors then they would be a certain number (ἀριθμός τις), as the Pythagoreans and some Platonists said. But what they failed to recognize, according to Aristotle, is that this would still be the number *of* something (τινῶν) such as colors, and that the unit would be one *something* (τι ἕν) such as whiteness. Similarly, if all things were tones then they would be a number, as the Pythagoreans were fond of saying, but a number *of* quarter-tones; so that their substance would not be simply a number, and the one in this case would not be simply unity but rather it would be a quarter-tone; cf. 1053b34 ff.

This important corrective to the Pythagorean claim about numbers being the substances of things constitutes part of Aristotle's own solution to the aporia in *Metaphysics* III about the nature of the one and of number. Even though he ignores the contrary views of the natural philosophers, he retains their insight that unity (and hence number) is always a predicate of some other substratum. In every category unity is a certain nature (τις φύσις) but the nature of unity is never just unity itself; cf. 1054a9 ff. Just as with qualities like color we must seek unity in one color, so also in the category of substance we must seek unity in one substance. But if one is a *pros hen* equivocal like being, then Aristotle's general concept of unity and number will depend on whether any of the categories yields a primary sense. In order to clarify this matter, we must consider the many senses of one as they are outlined in *Metaphysics* X.1 and V.6.

For this task an important distinction is that introduced (1052b1–3) by Aristotle himself between two different kinds of question: 1) What sort of things are said to be one? (2) What is the essence of the one and what is its definition? In relation to the first kind of question, Aristotle outlines four different headings under which things are said to be one primarily (πρώτως) and *per se* (καθ' αὑτο):

> The term 'one,' then, has these many senses: what is continuous by nature, and the whole, and the individual, and the universal. All these are one in view of the fact that they are indivisible, some in motion, and others in the thinking of them or in their formula.[43]

[43] *Met.* 1052a34–b1: tr. Apostle (1966).

One noteworthy feature of this summary is that Aristotle uses his distinction between the particular (τὸ καθ' ἕκαστον) and the universal (τὸ καθόλου) to characterize the difference between the third and fourth headings under which things are said to be one. But, in relation to the argument in X.1, his most important conclusion is that the things under each heading are all said to be one by virtue of being indivisible (τῷ ἀδιαίρετον εἶναι), either with respect to motion (τὴν κίνησιν) or with respect to thinking (τὴν νόησιν) or with respect to formula (τὸν λόγον). From his survey of things that are said to be one, there has emerged this common characteristic which will enable him to give a general account of the essence of unity itself. Thus Aristotle's answer to the first question about what sort of things are said to be one yields an answer to the second question about the essence of the one and its general definition. Even though the second question is prior in the order of being, the first question is primary in the order of inquiry.

In fact, the key to Aristotle's procedure can be found in the reasons which he gives for distinguishing between these two types of question about the one (1052b1 ff.). Each thing will always be called one if it falls under any of the headings given above, whereas 'the being of one' (τὸ ἑνὶ εἶναι) will sometimes mean to be one in some one of these senses, but sometimes its meaning will be closer to that of the name 'one,' which is potentially each of the senses outlined. His comparison with terms like 'element' and 'cause' suggests that he has in mind transcendental predicates that have different meanings in different categories. Thus, in response to the second type of question, we may either state a definite thing of which the term is predicated or give a completely general formulation that covers all senses of the term. For instance, we may say that fire is *an* element but the essence of fire and that of element are not the same, otherwise there would be only one element instead of the traditional four. In Aristotle's terms (1052b13–14), fire may be an element by its nature but 'element' means (σημαίνει) that so-and-so belongs to something; i.e. that something else is constituted from it as a primary constituent.

With this general formula of the meaning of 'element' as his model, Aristotle now offers a similar account of the meaning of 'one':

> Similarly with 'cause,' 'one' and all such terms. Thus, too, to be one is to be indivisible, and this is to be a *this* and separable by itself either in place or in kind or in thought, or, in addition, to be a whole and indivisible. Most of all, it is to be the first measure of each genus, and

most properly, of quantity, for it is from this that it has been extended to the others.[44]

From the initial survey of things that are called one *per se,* we can see the grounds for his claim that the most general meaning of 'one' is to be indivisible. However, it is not so easy to understand why he explicates this as being 'a this' (τόδε ὄντι) and as being separable by itself (ἰδίᾳ χωριστῷ) in different ways. Quite apart from possible corruptions in the text, it is difficult to decide whether Aristotle defines the essence of unity as 'being indivisible' or as 'being the first measure of a number.'[45]

Given his survey of the many ways in which things are called one, however, Aristotle cannot be offering 'being the first measure of a number' as a general definition covering all these senses because the concept of measure does not even appear in many of them. But the notion of indivisibility does appear under each of the four headings and so 'being indivisible' has a better claim to being such a general definition of the essence of unity. With some effort we can also see the supplementary parts of the definition as summarizing the various guises under which indivisibility makes its appearance for each heading. For instance, things that are continuous by nature are also 'thises' in some sense and separable in place, while the individual thing is a 'this' in its primary sense and it is also separable in being.

Having set out his general definition of the essence of unity, Aristotle next (1052b18) introduces a special sense (μάλιστα) of one as 'the first measure of each genus.' But even within that special sense, a privileged position is given to the first measure of quantity as the most proper (κυριώτατα) meaning of unity because, according to Aristotle, it is from this (ἐντεῦθεν) that the term has been extended to other measures. At the end of X.1 (1053b4–5), he concludes that to be one especially (μάλιστα) according to the name is to determine a certain measure (μέτρον τι), most properly (κυριώτατα) of quantity and then of quality.

Though this conclusion appears to suggest that 'being a certain measure' is the essential definition of unity, I agree with Donald Morrison (1983) that the whole passage shows that 'being indivisible' is really the essence of unity. For instance, it declares (1053b7–8)

[44] *Met.* 1052b14–20: tr. Apostle (1966).

[45] Commentators tend to be divided on the issue; e.g. Aquinas (*in Metaph.* X.1, 1936–7) thinks it is the first, whereas Ross (1924) ii, 281 holds it to be the latter.

that the one is indivisible, either simply (ἁπλῶς) or qua one (ᾗ ἕν). If Allan Bäck (1979) is correct about its technical use, the 'qua' locution here suggests that indivisibility is an explanatory cause of unity that might be used as a middle term in a demonstrative syllogism, and so 'being indivisible' is the essence of unity. By contrast, 'being a certain measure' is more akin to a nominal definition of unity, as the phrase 'according to the name' (κατὰ τό ὄνομα) suggests. Drawing on other parallels in the Corpus, Morrison (1983) claims that this phrase indicates that Aristotle is giving the etymological origin of a word, rather than its current literal meaning. This claim seems to be supported by the remark (1052b19–20) that, from this original meaning, the one was extended to its other applications.

However, *pace* Morrison, I think that Aristotle tries to do more here than merely to indicate the commonsensical and original use of the word. His usual practice is to look for some metaphysical significance in such well-established linguistic usage, and he seems to find it in the coincidence that both unity and indivisibility take their primary meanings from the category of quantity. In this context, Aristotle's clarification of the primary and most proper sense of unity is worth noting:

> Now a measure is that by which a quantity is known; and a quantity qua quantity is known either by a unity or by a number, and every number is known by a unity. Thus, every quantity is known qua quantity by a unity, and that by which, as first, quantities are known is that which is one. And so a unity is the principle of a number qua number.[46]

When taken along with the underivative sense of unity given in *Metaphysics* V.6, this passage might be thought to indicate that the essence of unity is to be a measure of quantity. But in both passages the one is said to be the principle (ἀρχή) of some number, so it is likely that Aristotle would explain this fact in terms of indivisibility.

This is confirmed in V.6 when he says (1016b23–24) that, although the one is not the same in all genera, in every case the one is indivisible (ἀδιαίρετον) either in quantity (ποσῷ) or in kind (εἴδει). Following an Academic schema of priority, Aristotle explains (1016b24–31) that what is indivisible according to quantity is called a unit (μονάς) if it is indivisible in every dimension and has no position, a point

[46] *Met.* 1052b20–24: tr. Apostle (1966).

(στιγμή) if it is likewise indivisible but has position, a line (γραμμή) if it is divisible only in one dimension, a plane (ἐπίπεδον) if it is divisible only in two dimensions, and a body (σῶμα) if it is divisible in all three dimensions. Now this explanation appears to contain an implicit contradiction with respect to body which is said to be divisible (διαιρετόν) in every dimension, while it is also classified as being indivisible (ἀδιαίρετον) according to quantity. For all the other mathematical objects named, it is possible to claim that they are indivisible under some dimension but this is not the case for body. So how can it serve as a unit of measure by which we know three-dimensional quantities?

Although no answer is forthcoming in *Metaphysics* V.6, it is possible to reconstruct one from Aristotle's claim in X.1 that the notion of measure is extended to other genera from its primary use in reference to the unit as a principle of number. Deriving from this (ἐντεῦθεν), a measure in other genera is said (1052b24–25) to be something by which as first (ᾧ πρώτῳ) they are known and so the measure of each of them is a unity. For instance, length is known by some length as a measure and, similarly, depth is measured by some depth as a unit, even though both are divisible. Referring to length, width, depth, weight and speed as things to be measured, Aristotle explains how something that is divisible (strictly speaking) can serve as a unit of measure and hence be treated as indivisible:

> Now in all these, the measure and the principle is something which is one and indivisible, since even in lines we treat the length of one foot as indivisible. For everywhere we seek as a measure something which is one and indivisible; and this is something which is simple either in quality or in quantity.[47]

The crucial point here is illustrated by the example of a foot being used as a measure of length, since it is obviously divisible in length but is being treated *as a unit.* The point is that, even though it is divisible, it imitates in some respect the paradigm unit (of number) which is one and indivisible in every respect. This is what Aristotle means by saying that the measure sought in every case is something simple (τὸ ἁπλοῦν) either in quality or in quantity. Insofar as it is simple, it imitates the unit that is used to measure number, which itself is absolutely simple.

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[47] *Met.* 1052b31–35: tr. Apostle (1966).

Such an interpretation is supported by Aristotle's proposed criterion (1052b35–36) of accuracy in measurement; i.e. wherever it seems impossible to add or subtract, there one has an accurate (ἀκριβές) measure. According to this criterion, therefore, the measure of a number is judged to be the most accurate because a unit is posited as indivisible in all respects (πάντῃ ἀδιαίρετον). But in all other genera the measures can only imitate such a measure because in the case of something comparatively large, like a stade or a talent, the addition or subtraction of something might escape notice more easily than in the case of something small. This theory about the origin of measurement is used by Aristotle to explain (1053a5–8) two general facts about measures. One is that everyone uses as a measure the first thing with respect to sensation from which nothing can be subtracted without being noticed. Such general usage conforms to the criterion of accuracy in measurement already given, and so supports his theory.

The second supporting fact is that all who use these measures think that they understand the quantity of something only in this way. This fits nicely with Aristotle's previous (1052b20) account of the general sense of measure as that by which a quantity is known. It also 'saves the phenomena' of measurement in related genera like motion which is measured in terms of a simple motion and in terms of the fastest motion which takes the least time (and thereby is one and indivisible). In astronomy, for instance, scientists posit as a principle or as a measure some such unit when they take the uniform and fastest motion of the heavens as a standard for judging the motions of other things. Similarly, in music one takes the quarter-tone as a measure because it is the least interval, while the syllable serves as a measure for voice. Thus Aristotle concludes that the one in all these different genera is *not* something common (κοινόν τι) but rather that it is said in many ways, as *Metaphysics* V.6 had already indicated.

Having discussed the unit as a principle of number, we are now in a position to understand Aristotle's concept of number as this is reflected in the different descriptions of number given in the Corpus. Although there is no extant treatise on number, Aristotle seems to have accepted the traditional Greek definition of it as a plurality of units or of indivisibles. For instance, he sometimes describes number as a limited multitude (*Met.* 1020a13), or as a plurality of units or of indivisibles (*Met.* 1053a30, 1085b22, 1088a4–8), or as a plurality measurable by one (*Met.* 1036b23–4, 1057a2–3, 1088a4–8). All of

these descriptions of number are consistent with the general description of it as a kind of quantity that is potentially divisible into parts which are countable but not continuous.

What is common in all cases is the tendency to define number in terms of its parts as a collection or aggregate of units. But this cardinal concept leaves unresolved the problem of what makes any number itself a unity, given that it is not continuous because its parts do not touch at a common boundary; cf. *Cat.* 4b25–31. Thus it is necessary to supplement it with the ordinal concept of number, according to which number has an inherent order because one thing is counted before two things, and two things before three, and so on. Here the concept of number is closely tied to the notion of counting, and the unity of a count is prior to its parts because number is not conceived of as a collection of parts. In contrast to the cardinal concept of number as an aggregate of units that are discontinuous by nature, the ordinal concept depends on a characteristic measure which is used to count in a definite order of succession; cf. *Phy.* 227a2–4, a19–21. The latter suggests a notion of number as a sequence (of prior and posterior) that is counted out, while the former suggests an aggregate of things that are collected in the process of counting.

These alternative concepts are relevant to the question about the unity of numbers, which is central to the problem of defining any number. If it is defined by the units into which it can be divided, then it would be defined in terms of its potential parts, which seems to contravene the proper order of definition. This problem is broached already by Plato when he considers how numbers and syllables can be regarded as unities if they are naturally defined by their parts; cf. *Theaet.* 202D ff. Arguably, this is the problem which is being resolved when he reportedly distinguishes between Form Numbers, whose units are only combinable internally, and mathematical numbers whose units are promiscuously combinable. But Aristotle rejects such a concept of number as something separate from its constituent units, and so he must provide an alternative account of the unity of number. Yet it is difficult to find such an account in his treatment of mathematical number, since he consistently emphasizes that its units cannot be differentiated with regard to how they are counted. However, if we turn from this cardinal notion of number towards the ordinal notion, perhaps we can discover some principle of unity in the activity of counting that uses the unit as a standard of measure.

In the absence of any formal discussion of number, we must depend

on Aristotle's treatise on time at *Physics* IV.10–14 where he defines it as the number of motion with respect to a before and after. There is an ancient puzzle deriving from Strato of Lampsacus as to how something continuous like motion can be characterized in terms of number, especially if the 'now' is the unit which is supposed to provide the measure for the temporal continuum. This puzzle is not easy to resolve, but one approach would be to clarify the character of the 'now' which is to serve as the unit measure for motion. Obviously the 'now' as persistent present does not give any basis for counting, so it must be some 'now' that is always different and that has a definite ordering. Since Aristotle draws a parallel between time and a line, it is plausible to think of 'nows' as being successive points on a time line. But this parallel has its limitations because a line is not constituted from points, and so they cannot function as measures of its length; just as the indivisible 'now' that forms the boundary between past and future cannot serve as a measure of time.

Yet Aristotle insists on the analogy between the 'now' and a unit, so perhaps he has in mind a segment of time that is marked by an end-point and thereby differentiated from the previous and successive 'nows.' However, since time is a continuum, a soul is required to actualize these potential divisions by noticing them, just as a geometer actualizes the divisions of a line. Thus, in marking out these segments of changes, one counts out the end-points which function like units in counting. Just as one may treat a point either as a single entity moving along a line, or as always being a different end-point of a segment; so also for the 'now' in time. If the 'now' is taken by the soul as a single moving point then there is no perception of change, but if it is taken as different then change is perceived. This is the reason why Aristotle raises the question of whether or not time would exist if no soul existed, since nothing can be numbered without soul; cf. *Phy.* 223a22–29.

Since this passage is frequently cited as evidence that, like the modern intuitionists, Aristotle makes number depend on a definite counting procedure carried out by the human mind, it should be examined carefully within its proper context to see whether such a view is espoused here. First, we should note that the passage contains an implicit distinction between number and that which is numbered, which roughly corresponds to Aristotle's general distinction between that by which we count and that which we count (219b5–11). The distinction is important because he says that number in the

sense of what has been numbered or what can be numbered may depend on a counting soul (223a22–25). In other words, the existence of a countable group of horses depends on the existence of a possible counter who can count the horses. After a careful analysis of the whole argument, Mignucci (1987) concludes that it is the impossibility of the existence of someone doing the counting that entails the impossibility of there being countable things. If he is correct then Aristotle is not committed to saying that the number by means of which we count is dependent on a counting soul, but only that the possibility of counting things depends for its realization on a soul that can count. Such a commitment does not make him an intuitionist in the modern sense, as it is compatible with realism about number, especially about the number by which we count.

In fact, it is arguable that this sort of number is Aristotle's substitute for Plato's Form Number, stripped of the ontological baggage which gave rise to the difficulties already discussed. Although it might appear that such a concept of number has been rejected by Aristotle, it turns out that he needs some version of it to solve the problem about the unity of a mathematical number which is constituted by completely undifferentiated units. Since he treats these units as being analogous to matter, it is reasonable to suggest that number must have a formal aspect that is captured in definition. There is some evidence to suggest that Aristotle thinks of mathematical objects as having a form/matter structure like physical objects; cf. *Metaphysics* VII.10–11. If that is the case, then it is the form rather than the matter which is the cause of unity in the composite number that is counted, whether that be a collection of units or a group of horses.

Therefore, it is plausible to identify this formal aspect as the number by which one counts units or horses in the right sequence (ordinals) which yields the correct aggregate (cardinal). This might help to clarify why Aristotle says (224a2 ff.) that the number of sheep and of dogs is the same if they are equal, but that they are neither the same decad nor the same ten things. Subsequently (224a12 ff.), he explains that the numbers are the same because one does not (formally) differ from the other by a differentia of number, yet they are not the same ten because the things of which they are predicated differ (materially). This whole passage can therefore be understood in terms of Aristotle's distinction between the ordinal numbers by which we count, and which are reflected in definition, and the cardinal numbers which are counted either in units, or horses, or dogs.

Finally let us consider whether the one is predicated in a purely analogical fashion or whether there is one central meaning that gives a focus for all the other uses of one. There is *prima facie* evidence in favor of such a focal meaning for 'one,' since Aristotle often indicates that it is convertible with 'being' which he treats as a *pros hen* equivocal. In *Metaphysics* IV.2 he says (1003b23–24) that being and unity have a single nature (μία φύσις), though they are not elucidated by a single account (ἑνὶ λόγῳ). His example is that 'one man' and 'being a man' have the same referent, so that 'one man exists' does not mean something different than 'a man exists.' Subsequently, with special reference to the unity of substance, he argues (1003b33–34) that there are as many kinds of being as there are of unity.

Similarly, in X.2 (1053b20–21 & b25), he declares that 'unity' and 'being' have equally many (ἰσαχῶς) senses and that these are the most universal of all predicates. By way of conclusion to that chapter, he summarizes (1054a13 ff.) the reasons why being and unity have the same meaning in some sense (πως). First, they both follow the categories equally and are not confined within any one of them. Second, as he says in IV.2, nothing more than 'man' is predicated either by saying 'one man' or 'man exists.' Third, just like 'to be,' 'to be one' is nothing other than to fall under some one of the categories. So there appear to be good reasons to think that both predicates will have the logical structure of *pros hen* equivocals with substance anchoring the central meaning in each case.

But there are also reasons for doubting that these two transcendentals are so parallel in structure as to have the same focal meaning. In the first place, unity seems to have an essence for which Aristotle offers a general definition, whereas being does not appear to have a single essence and so he uses the logical device of a *pros hen* equivocal to provide a single theoretical object for the science of metaphysics. Secondly, while the primary sense of being falls under the category of substance, the definition of unity suggests that its primary meaning is to be sought under the category of quantity. In fact, this is confirmed in X.1 (1052b15 ff.) when unity is defined in terms of indivisibility as the first measure in each genus but chiefly (κυριώτατα) in quantity. It is also consistent with V.6 (1016b18 ff.) where the essence of one is said to be the principle (ἀρχή) of some number, since this is the paradigm example of an indivisible measure by which we know things in every genus. Thus, although being and unity have just as many senses, it would appear that they have different primary meanings.

In spite of these differences, however, I think that Aristotle treats both unity and being as having the logical character of *pros hen* equivocals.[48] For instance, he identifies a primary sense of unity under the category of quantity from which the meaning of 'one' is extended to all the other categories of being. Similarly, in the case of being, he locates its primary meaning in substance and proceeds to discuss substance as if it were the essence of being; cf. *Metaphysics* VII.1. Of course, Aristotle does not hold that there is a single essence of being that is common to all the categories of being. At best, all of these senses of being have merely analogical sameness. By contrast, he does seem to suggest by offering a single definition of unity that it has a single essence which is the same in all the categories. But closer inspection reveals that this amounts only to sameness by analogy because the unit for numbers, for instance, has nothing except the name in common with the unit for colors. Given the primacy in meaning of the unit for number, however, this will be analogy by attribution rather than proportional analogy. In response to the leading aporia about being and unity, Aristotle's answer is that neither of them is a separate substance nor the essence of anything else. These conclusions about unity and being have wider implications for his own conception of supersensible substance, which I will now examine.

IV. *Principles for the supersensible realm*

Metaphysics XII introduces an inquiry about substance (Περὶ τῆς οὐσίας) which is justified in the following terms: "because it is the principles and causes of substances that are being sought."[49] While it is clear that some kind of inquiry into substance is being proposed, both its precise scope and character remain obscure. After some preliminary remarks, Aristotle tries to justify his inquiry with reference to the primary place of substance in the universe. In general terms, the argument is that substance is the first thing in the universe, no matter how we conceive the All (τὸ πᾶν) to be structured. If we take the universe to be a kind of whole (ὡς ὅλον τι), for instance, Aristotle claims (1069a19 ff.) that substance would be the first part (πρῶτον

[48] Donald Morrison (1994) leans heavily on *Metaphysics* IV.1 in arguing that for Aristotle unity and being are not merely convertible but have one and the same nature. But this would also mean that they have the same focal meaning, and there is less textual evidence to support this claim.

[49] τῶν γὰρ οὐσιῶν αἱ ἀρχαὶ καὶ τὰ αἴτια ζητοῦνται, *Met.* 1069a18–19.

μέρος).[50] And, even if we should assume the universe to be serially ordered (τῷ ἐφεξῆς), substance would still be prior to quality and quantity.

While Aristotle appears to be relying on his own categories here, he does offer two independent criteria by which the primacy of substance may be judged. First, speaking without qualification (ἁπλῶς), none of the things which follow substance can be called 'beings' (ὄντα), otherwise one would have to give the same status to negations like not-white and not-straight (since we also say that these exist). The important distinction here seems to be that between speaking of 'being' in a qualified way (ὄν τι) and speaking of it in an unqualified way (ὄν ἁπλῶς). Since the latter is appropriate only for substance, Aristotle uses a linguistic criterion to distinguish substance from the other categories. Yet this is not a criterion of *priority,* unless we appeal to the priority of substance over the other categorial beings. By contrast, the priority criterion of non-reciprocal dependence is implied by the brief statement that none of the other categorial beings is separated or separable (χωριστόν). Whichever meaning of χωριστόν one adopts, it is clear that substance may be argued to be prior because it can exist without the other categorial entities, whereas they cannot exist without it.

At this point in *Metaphysics* XII.1, Aristotle calls upon the ancients to witness (μαρτυροῦσι) to the primacy of substance and its principles. Deftly distancing himself from their views on substance, he appeals to their work (ἔργον) of investigating the principles and elements and causes of substance. It was characteristic of the natural philosophers (to whom Aristotle is referring) to posit as substances particular kinds of body, like fire and earth, rather than body in general. By contrast, contemporary thinkers who pursue their inquiries dialectically posit universal genera as being more substantial; cf. *Met.* 1069a25–9. What is here called 'logical inquiry' (τὸ λογικῶς ζητεῖν) may correspond to the so-called 'flight into the *logoi*' described at *Phaedo* 99D ff. The assertion that this tradition regards the more universal genera as being more substantial is consistent with Aristotle's

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[50] See *Metaphysics* V.26 (1024a1 ff.) where Aristotle distinguishes between an 'all' (πᾶς) and a 'whole' (ὅλος) in which the order of the parts does make a difference. An example of the latter is a tragedy, which as a whole has a beginning, a middle, and an end; cf. *Poetics* 7, 1450b26–31. Thus when Aristotle compares the Speusippean universe to a bad tragedy, he means that it is not a whole in this sense but is rather a series of episodes.

own report (*Met.* 987b1–4) about Socrates; i.e. that he paid more attention to definition and to the universal (τὸ καθόλου), while neglecting physical inquiry. Yet it was probably Plato who turned the Socratic preference for dialectical inquiry into a full-scale metaphysical investigation of the principles and causes of being. The goal of Socratic-Platonic inquiry was to discover the good both in theory and in action, whereas this was not so important in the tradition of physical inquiry. By contrasting these two traditions, Aristotle not only gives the frame of reference for his own investigation of substance but also serves notice that he will not neglect the question about the good of the whole universe, which was so central to the Platonic tradition.

Since his topic is substance, Aristotle first distinguishes between three different kinds of substance which he assumes to exist. Thus, even though there is no explicit reference here to a guiding aporia, an appropriate one might be the fourth aporia listed in *Metaphysics* III.1 (995b13–18): i.e. whether only sensible substances exist or whether there are other kinds of substance apart from these. This possibility is supported by the correspondence between the opinions about substance listed in III.2 and XII.1 (along with VII.2). On the one hand, according to Aristotle (1069a30–32), everyone agrees that there are eternal (ἀΐδιος) substances such as the heavenly bodies, and perishable (φθαρτή) substances like plants and animals. With respect to these, the task for inquiry is to grasp their elements (στοιχεῖα), whether they be one or many. But, on the other hand, there is an unchanging (ἀκίνητος) kind of substance which some people say is separated (χωριστή). With reference to substances of that description, Aristotle outlines three different opinions: (i) that there are two different kinds, Forms and Mathematical Objects; (ii) that these two kinds have a single nature; (iii) that there is only one sort, i.e. Mathematical Objects.

Since these opinions are attributed elsewhere to Plato, Xenocrates and Speusippus, the connection between Books XII and XIII & XIV seems to be established, though not the order of inquiry. In fact, XII begins with a review of the principles of sensible substance and then discusses supersensible substance.[51] Since Book XII does not contain any systematic attempt to refute the views of the Platonists about

[51] Oehler (1969) takes the two parts of Book XII to reflect the general structure of first philosophy as an investigation of sensible and supersensible substance in that order. Thus he sees XII.1–5 as dealing with natural substances and their principles in the same manner as *Metaphysics* VII–IX, i.e. as a preliminary inquiry to the

supersensible substance, my conjecture is that Books XIII & XIV undertake that negative task which is neglected in XII. Of course, there is little textual evidence that Aristotle himself had any such systematic conception of the task to be accomplished in the last three books of the *Metaphysics* as we have it. One might also object that Book XII proceeds in a dogmatic rather than in an aporetic manner, which suggests that he is not consciously following any aporia in his presentation. While this is undeniable, I still think it is possible to interpret XII as proposing solutions to some aporiae set out in III and explored in XIII & XIV.

Near the end of XII.1, for instance, Aristotle briefly tries to assign the different candidates for substance to the corresponding sciences which should be concerned with them. He indicates that sensible substances would be part of the subject-matter of physics because they undergo change of one kind or another. Since he has previously distinguished between sensible substances that are perishable and those that are imperishable, Aristotle presumably should say that the latter kind of substance is dealt with by astronomy, which is the most physical of the mathematical sciences. But what he does say (1069b1–2), in a very cryptic fashion, is that the immovable kinds of substance will be the subject-matter of other sciences if there is no principle common to (all of) them (εἰ μηδεμία αὐτοῖς ἀρχὴ κοινή). It is difficult to know how to interpret this last conditional clause, though it may be important for understanding the place of Book XII in Aristotle's *Metaphysics.*

Let me explore the following possibilities. If it means that there is no common principle shared by both sensible and supersensible substances, then the implication would seem to be that these different kinds of substance should be investigated by different sciences. With regard to the aporia about 'the science being sought' (ἡ ἐπιζητουμένη ἐπιστήμη) in *Metaphysics* III (995b10–13), therefore, one possible solution is that more than one science is required to deal with all substances. Another possible answer, however, is that there exists some common principle which would provide a basis for a single science of all kinds of substance; i.e. the science of being qua being which is

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treatment of supersensibles. This interpretation is quite compatible with mine, except that I give more emphasis to the negative dialectical task of eliminating the competing claims for Forms and Mathematicals as supersensible substances. For a more recent analysis of the structure of *Metaphysics* XII, see Helen Lang (1993).

proposed in *Metaphysics* IV. A third possibility is that although there are special sciences like physics and astronomy, which treat different kinds of substance under different aspects, there could still be one higher science which deals with all kinds of substance under a single aspect. This might be the point of the aporia in Book III when it goes on to ask whether all the sciences are 'akin' (συγγενεῖς) or whether one of them should be called 'wisdom' and the rest something else. Presumably, the same point would hold if we substituted first philosophy or theology for 'wisdom' here. By appealing to the aporia in III, one can read the cryptic passage near the end of XII.1 as recalling all these questions and possibilities. Therefore, I suggest that it makes a great deal of sense to take the rest of XII as constituting a search for unifying principles, and that the role of the Prime Mover becomes more intelligible in terms of Aristotle's whole inquiry into substance.

But, in line with his own aporetic method, we should first review Aristotle's criticism of the principles for supersensible substance posited by members of the Academy. This criticism is located in *Metaphysics* XIV yet, as Jaeger has already noted, there are many striking parallels with XII. In XIV.1, for instance, Aristotle rejects as inadequate the previous consensus that contraries alone serve as principles for both changing and unchanging substances. Appealing to his own analysis of change in *Physics* I, he argues (1087a36 ff.) that everything which comes into being from contraries must itself belong to a substratum. Thus the contraries also belong to the same substratum, which would function as a prior principle. In any case, it is obvious that a substance has no contrary, and this fact was also noted in the *Categories* (3b24–7). Therefore, Aristotle concludes (1087b3–4), something other than a contrary must be the principle of everything in the proper sense, and this is the basis for his subsequent criticism of Academic principles.

In reporting different views of these principles, Aristotle seems to be using his own terminology when he says that one of the contraries is being treated as matter. For instance, to supply matter for the One, some people (Platonists?) posit the unequal as being the essence of plurality; whereas another (Speusippus) simply treats plurality itself as matter, since numbers are 'generated' from it along with the One as their form. Thus all seem to agree on the identity of their formal principle, while disagreeing about how the material principle of number is to be understood. According to Aristotle, Plato's

material principle is a Dyad consisting of the great and the small, even though he identifies it as a single thing (i.e. the unequal) because he fails to differentiate what is one in definition from what is one in number. As Annas (1976, 195) suggests, the point of Aristotle's complaint here seems to be that no single thing can be both great and small, despite Platonic attempts to define the material principle of number in this way.

In general, therefore, Aristotle is unhappy with Academic formulations of the elements of number because, with the exception of Speusippus, most of them fail to posit genuine contraries. For instance, the great-and-small (however understood) is not contrary to the one; nor is the situation improved by those who posit the exceeding-and-exceeded as a more universal material principle for numbers and magnitudes. Aristotle thinks the most plausible view is that of Speusippus, who makes one and plurality the elements of number; yet even this view is inadequate because it implies that the one will be few (since fewness is the opposite of plurality). But if all of the Academic views fail to provide true contraries, one might ask whether Aristotle is correct in interpreting their principles in such terms, as it seems that he may be misrepresenting the views of his predecessors because of his own presuppositions.

In criticising the view that the One is a principle, for instance, Aristotle begins (1087b33 ff.) with a brief summary of his own view that one always means a measure of something. Thus he emphasizes that in every instance of 'the one' there is something else which is the underlying subject; e.g. a quarter-tone is a measure in a musical scale, while a foot is a measure in magnitude. In general, as he explained in *Metaphysics* X.1–2, the measure is indivisible in kind for qualities, and indivisible in perception for quantities. The central point of his talk about an underlying subject emerges when Aristotle insists (1088a3–4) that 'the one' does not have the ontological status of an independent substance, despite the priority given to it by the Academy as a principle of supersensible entities like numbers and magnitudes.

Apparently in support of his own ontological view, Aristotle appeals (1088a4 ff.) to the meaning of 'the one' as a measure of some plurality, and to the meaning of number as a measured plurality or as a plurality of measures. In the light of such semantics, it is reasonable that the one should not be a number, since it is a principle and a measure which cannot itself be 'measures' (μέτρα). Here Aristotle seems to rely on the Greek convention that 2 is the first number,

though he himself sometimes counts 1 as the first number. In general, he claims, the measure must always be some one and the same thing applying to all the cases involved; e.g. horse is the measure for horses, and man for men. It is possible to count a man, a horse, and a god, if we take 'living being' as the measure. But there are difficulties about counting items from different categories, according to this convention, even though 'a man,' 'white,' and 'walking' can somehow be counted, perhaps as a number of categories. Still Aristotle denies (1088a12–13) that these can constitute a number (in the proper sense) because they might all belong to the same substratum which is numerically one. Despite its strangeness to our modern eyes, there is evidence elsewhere (*Phy.* 248b19–21) that he takes seriously the view that number terms have different meanings according to the type of items being counted; e.g. sheep numbers differ from cattle numbers.

With regard to the second principle proposed by Plato, Aristotle claims (1088a15 ff.) that it is highly implausible, whether one posits the unequal as a single principle or the great-and-small as the Indefinite Dyad. His first objection is that these are properties and accidents, rather than the subjects which underlie numbers and magnitudes. For instance, many and few are properties of numbers, while great and small are properties of magnitudes; just as even and odd, smooth and rough, straight and curved are also properties of their respective subjects. Aristotle's objection seems to depend upon assuming a subject-predicate dichotomy, just as in his previous criticism of contraries as first principles.

In a similar fashion, Aristotle objects (1088a21 ff.) that Plato is mistaken in positing the great-and-small as a principle, since it must be a relative with less of a claim to reality than quality or quantity. Hidden within this straightforward appeal to his own categories, perhaps there is an implicit appeal to the Academic distinction between *kath' hauto* and *pros ti* entities, since that would give the objection a better dialectical basis. Be that as it may, Aristotle argues that the relative (i.e. the great-and-small) is a characteristic of quantity, and not of matter, since there is some other subject for the relative in general along with its parts and forms. The important ontological point is that relatives like the great-and-small, the many-and-few, have a dependent mode of being as belonging to something else rather than an independent mode of being as self-subsisting entities. We recall that this is also how Aristotle answered the aporia from *Metaphysics* III about whether One has an independent or dependent mode of being.

As additional evidence that a relative is least of all substantial, Aristotle cites (1088a29 ff.) the fact that relatives are not subject to change in any of the usual ways, like generation and corruption, alteration, growth, or locomotion. In the case of relatives, by contrast, a thing will be greater or less or equal without itself undergoing change, if another thing that is compared to it changes in quantity. For Aristotle the association of such accidental change with relatives is a sign that they have less claim to reality than other categorial entities that suffer essential change; cf. *Phy.* 225b11–13. It is significant that a similar 'change test' is used in *Metaphysics* III to undermine the substantiality of mathematical entities.

Continuing his appeal to modes of generation, Aristotle objects that the matter of each thing must be that kind of thing potentially; whereas a relative is neither potentially nor actually a substance. Therefore, he argues, it is absurd and impossible to posit what is not a substance as an element of substance and so prior to it, since all the other categories are posterior to substance. Once again, the direct appeal to Aristotle's own categories may vitiate the argument as a dialectical objection, unless it also contains an implicit appeal to Academic categories.

In *Metaphysics* XIV.2, however, Aristotle raises a general question about whether it is possible for eternal things to be composed of elements. Perhaps, as Annas suggests (1976, 199), he is overloading a casual Platonic suggestion with meaning, since he wants to emphasize the ontological difference between eternal and corruptible substances. Thus, by taking the Platonic language of generation from elements literally, he can mount an effective dialectical offensive against what he regards as a mistaken approach to eternal entities. We know from *De Caelo* I.10 that the question of whether such language is to be taken literally or metaphorically was one which divided Aristotle from more orthodox Platonists like Xenocrates. Behind the dispute about language, however, lies the central issue of whether Plato's mathematical cosmology gives the correct picture of the universe, and especially of the relationship between sensible and supersensible substances. Therefore, the subsequent objections made in XIV.2 against talk of the principles as elements provide a natural introduction to Aristotle's own views in *Metaphysics* XII on the true nature of supersensible substance.

For instance, he argues (1088b15 ff.) that if eternal things can be composed out of elements then they will contain matter, since every-

thing constituted from elements is a compound. But, whether it is eternal or generated, every such compound comes into being from its constituents which must somehow have the potential for becoming that thing. However, what is potential can either be actualized or not. So even if it is true to claim that numbers exist forever, still they could fail to exist just like anything else having matter. Therefore, given the Platonic view of principles as elements, Aristotle argues that numbers cannot be eternal because the material element implies that they can fail to exist. While we may question Aristotle's application of the principle of plenitude to the 'material' principle of number posited by Plato, yet it is consistent with his literal interpretation of all Platonic talk about 'generation.'[52]

We might also wonder why the same principle cannot be applied to his own claim in *De Caelo* that the heavenly bodies are eternal substances constituted from a special kind of imperishable matter popularly called *aither*. This latter question is relevant for understanding the conceptual development that takes place in *Metaphysics* XII with the introduction of the Prime Mover. Indeed, the way is prepared for such a development with the general conclusion drawn by Aristotle (1088b25–28) from the above objection in XIV.2. Assuming as universally true that no substance is eternal unless it is (pure) actuality, and that elements always constitute the matter of a substance, it follows that no eternal substance has elements present in it from which it is constituted. In other words, an eternal supersensible substance like the Prime Mover must be absolutely simple; i.e. it cannot have aspects differentiated as matter and form.

From his own (superior) perspective on supersensible substance, Aristotle critically reviews (1088b28 ff.) mistaken proposals for the first principles of such substances. For instance, he reports that some people posit the Indefinite Dyad as an element along with the One because they are aware of the difficulties associated with positing the Unequal as a principle. Presumably, he means the difficulties which he had previously raised about positing a relative as a principle of

[52] In *Met.* I.6 (987b21 ff.) Aristotle reports that for Plato the One 'begets' the numbers as from a matrix (ἐκμαγεῖον), namely, the great-and-small as matter. Rist (1989) 193 infers from the language that Aristotle has in mind Plato's *Timaeus* as a source for this quasi-sexual account of the generation of numbers. This would give further weight to his objection because he regards Plato as being confused about whether the Receptacle provides some matter for generation or merely the placeholder for generation.

substance. But he insists that they avoid only the difficulties arising from making such a relative entity an element, since all the other difficulties apply to their principles also, whether they use them to produce Forms or mathematical numbers.

In his general diagnosis of where the Platonists went wrong in their search for the principles of supersensible substance, Aristotle identifies (1088b35 ff.) as the basic cause of their error an old-fashioned approach to the problem. Thus they accepted that all existing things would be simply one thing, Being Itself, unless one could refute the Parmenidean argument against plurality based on the impossibility of not-being. So they thought it necessary to show that what-is-not (i.e. not-being) exists in some way, in order that a multiplicity of things can emerge from being and from something else. If Aristotle is referring to Plato's *Sophist*, Annas (1976, 201) suspects a misunderstanding because the arguments canvassed here are not to be found in that dialogue, although it quotes the same Parmenidean passage. But perhaps Aristotle is simply giving an historical reconstruction of Plato's motivation for positing being and not-being as principles of plurality and change.[53] In any case, I think that Aristotle is more anxious to display the advantages of his own solution to the aporia discussed in *Metaphysics* III.4 (1001a4 ff.) as to whether One and Being are the substances of things.

Appealing to the different categorial meanings of 'being,' Aristotle asks (1089a7 ff.) what kind of 'one' all existing things can possibly be, if there is no such thing as not-being (as Parmenides claims). He finds it absurd or (better) impossible for a single kind of thing to be the cause of one thing being a 'this,' of another being a 'such,' and of yet another being a 'so much.' Thus, by appealing to categorial language, Aristotle can claim that there is not merely a single sense of 'being' or of 'one' but rather many, and perhaps as many as ten. Conversely, there are just as many senses of 'not-being.' So, for instance, 'not-man' means being a 'not-this,' just as 'not-straight' means being 'not-such.' Given the many senses of being and not-being, the question about the principle of plurality for all existing things becomes more pressing. It was to this question that Plato is taken to be

[53] Ian Mueller (1987) 248–49 thinks that Aristotle is using his own terminology when he refers to one of Plato's principles as 'not-being,' especially when he identifies it with the great-and-small. Mueller also thinks that Aristotle is picking up misleadingly on Plato's *Sophist* (256D–E) where the stranger says that change is genuinely a not-being; i.e. not identical with being.

proposing not-being in the sense of falsity as an answer; cf. *Soph.* 237A, 260B, 263B.

In an interesting aside, Aristotle reports (1089a21 ff.) that it used to be said that one has to assume something false, just like geometers do when they assume a line to be foot long when it is not. But he denies that this is actually the case, because geometers do not use false assumptions as premises in their reasoning. With regard to the other side of the parallel, he also denies that things come into being or perish into non-being in the sense of falsity. It would appear, however, that falsity is the appropriate sense of not-being for mathematical hypotheses as premises for demonstration. In fact, Aristotle concedes as much by drawing the parallel in the first place, though he denies that the geometer makes use of false assumptions in his reasoning. So, for instance, in the so-called 'ekthesis' of a Euclidean theorem, the geometer says 'Let ABC be a right-angled triangle' and proves that some attribute belongs to it, even though the particular diagram may not have been perfectly right-angled.[54] Yet this makes no difference to the theorem because the attribute belongs to the universal right-angled triangle that is inadequately represented through the particular diagram. All of this is quite consistent with Aristotle's views in XIII.3 (where a similar example is used) about the ontological status of mathematical objects, but what does it have to do with the principle of plurality in mathematics?

One clue is to be found in the subsequent objection at XIV.2 (1089a25 ff.) where Aristotle declares that existing things do not come into being from not-being in the sense of falsity. According to his analysis, Plato failed to see that not-being has many other senses, especially as what is potential, which are more important for understanding change under the categories of substance, quality, quantity, and place. However Aristotle thinks (1089a31 ff.) it is obvious that the Platonists are introducing the wrong principle of plurality because they take mathematical entities, such as numbers, planes, and bodies, as the substantial beings that are multiplied through the principle. While he disagrees with them about these entities being substances, what he finds most absurd is the choice of a principle which is clearly confined to one category of beings, whether these be substances or quantities.

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[54] Ross (1924) ii, 476 notes a distinction between *protasis* and *ekthesis* in Euclid, but concedes that *protasis* was used to cover both in Aristotle.

Thus he criticizes the Indefinite Dyad or the great-and-small as a general principle of plurality because it cannot explain the multiplicity of colours or flavours or shapes, unless these be also reduced to numbers. He suggests (1089b2 ff.) that, if they had approached the question from a more general perspective, they would have seen that the same thing (i.e. matter) is responsible for plurality in all the categories, but only in an analogous fashion. This provides us with another parallel to the discussion in *Metaphysics* XII (1070b16–27), which identifies matter as the single analogous principle of plurality for every category of being. It also suggests a possible significance for Aristotle's talk elsewhere about 'intelligible matter' as a principle of plurality for mathematical objects. But I will return to this topic from another angle in my final chapter.

The link which I have already suggested with the aporia about One and Being is reinforced here (1089b4 ff.) when Aristotle continues his analysis of the peculiar Platonic aberration of positing a single (and univocal) principle of plurality. He accuses those who search for the contrary of Being and One of making the same mistake when they posit the relative or the unequal as such a principle. Appealing to his own categories, once again, he claims that the relative is not the contrary of One but rather one sort of being, just like substance or quality. Aristotle thinks (1089b8 ff.) that the Platonists could have avoided their mistake if they had asked themselves how there can be many unequals besides the Unequal (as principle), rather than how there can be many units besides the first One. In other words, they should have supplied a principle of plurality for relatives, since they talk about many different types of relative like the great-and-small, the many-and-few, the long-and-short, and so on. I tend to agree with Annas (1976, 204) that this is a particularly striking (and effective) objection to the Platonic practice of simply positing many different versions of the second principle of mathematical entities.

Aristotle himself is quite happy to concede (1089b15 ff.) that for each thing one must assume something which is potentially that thing, but the Platonists made a fundamental mistake in positing a relative as being potentially a 'this' or a substance. For his analysis of this basic mistake, Aristotle seems to be relying on his own categories, as Annas (1976, 204) claims; yet he may be referring to the Academic distinction between *kath' hauto* and *pros ti* categories of being. For instance, he declares the Platonic mistake to be that of treating as potentially a substance what has no independent being (καθ' αὑτό);

i.e. what is a relative (πρός τι). If I am right then there is an extra dialectical bite to Aristotle's objection when he argues (1089b18 ff.) that this is just one sort of being and not potentially One or Being, nor even the negation of One or Being. Thus, on the assumption that the Platonists were trying to explain plurality among entities, Aristotle may be entitled to upbraid them for failing to account for a multiplicity of entities both within the same category and in different categories.

With regard to plurality within the other categories besides substance, Aristotle sees (1089b24 ff.) an additional problem because they are not separable, and so it is through the multiplicity of an underlying subject that qualities and quantities are multiple. From his brief and imprecise statement here it is unclear whether he means that individual qualities like red, for instance, are unique to one particular substance or that they can recur in many particulars. This imprecision also leads to uncertainty about the exact ontological status of individual mathematical objects, and about their principle of plurality; cf. *Metaphysics* VII.10–11. In XIV.2 (1089b27–8), however, Aristotle insists that there ought to be a distinct type of matter for each category of entity, except that it cannot be separated from substances. In my final chapter, I will explore the compatibility of such statements with his concept of intelligible matter as the appropriate principle of plurality for mathematical objects.

IV.1. *Supersensible substance*

At the beginning of *Metaphysics* XII.6 Aristotle establishes a link with the division made in XII.1 (1069a30 ff.) between sensible (αἰσθητή) and unchanging (ἀκίνητος) substance, along with a sub-division in the realm of sensible substance between eternal (ἀΐδιος) and perishable (φθαρτή) substances. The same distinctions recur in XII.6 when Aristotle says (1071b3–4) that there are two kinds of physical substances (φυσικαί) and one kind of unchanging substance (ἀκίνητος). It is the latter to which he refers when he also explains that "one ought to speak about this because it is necessary for there to be some eternal and unchanging substance."[55] In comparison with his discussion of the source of eternal motion in *Physics* VIII, the argument given here in *Metaphysics* XII for the necessary existence of

[55] περὶ ταύτης λεκτέον ὅτι ἀνάγκη εἶναι ἀΐδιόν τινα οὐσίαν ἀκίνητον, *Met.* 1071b4–5.

immobile substance is more metaphysical in character.[56] For instance, it begins with the assertion that substances are 'the firsts' of all things.[57] As already noted, one implication of the priority of substance is that all other categories of being are dependent on it in a non-reciprocal fashion.

Such an implication is what enables Aristotle to take the next step in the argument; i.e. that if all substances are destructible then everything is destructible. But, he claims (1071b6–7), it is impossible for motion either to be generated or to be destroyed, since it always existed. Here Aristotle is relying upon his long argument for the eternity of motion in *Physics* VIII.1–3 just as, with respect to the related claim about the eternity of time, he summarizes what he has already established in his physical treatises. Thus his metaphysical inquiry in *Metaphysics* XII is remarkable for the way it uses physical conclusions so readily as starting-points for reflection. In the present passage, for instance, he claims (*Met.* 1071b9–10) that motion is continuous (συνεχής) just like time, since either they are the same or time is an attribute of motion. This claim is obviously taken from the *Physics,* although it is here related to the question about the action of an unmoved mover in the universe. This is the point of his conclusion (1071b10–11) that the only motion which can be continuous in the way necessary for eternality is circular locomotion.[58]

But Aristotle has not yet clinched his argument for the existence of an immovable mover, since the eternal motion of the universe might be due to the continuous circular motion of the first heaven if it were self-moving in the manner of Plato's World-Soul.[59] The fact that Aristotle himself seemed content with such a postulate in *De Caelo* renders even more pressing the question of why he needs to posit an unmoved mover.[60] A revealing hint may be found in his

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[56] Manuwald (1989) has argued convincingly for a complementarity in Aristotle's approach to the unmoved mover in physical and metaphysical treatises.

[57] αἵ τε γὰρ οὐσίαι πρῶται τῶν ὄντων, *Met.* 1071b5. See *Metaphysics* VII.1 & XII.1 for clarification.

[58] It is relevant to note that *Metaphysics* V.6 (1016b16–17) holds a circle to be more unified than a line because it is a whole and complete (ὅλη καὶ τέλειος). This claim underlies Aristotle's argument for the priority and unity of circular motion as the characteristic activity of the primary moved movers, which are imitating the absolute immobility of their unmoved movers.

[59] Klaus Oehler (1984) ch. 2 draws attention to the fact that *Met.* 1071b3–11 has only established the existence of the first heaven as the subject for continuous circular motion.

[60] Cherniss (1944) app. X points out that if *aither* is in motion without having the potentiality of motion and rest, then the prime mover does not cause its motion but

argument (*Met.* 1071b12 ff.) for the priority of actuality over potentiality, which begins with the general claim that there will not be (eternal) motion even if there is something which is capable of moving or acting but is not actually doing so (μὴ ἐνεργοῦν). The reason given for this claim is that whatever has such a potentiality (τὸ δύναμιν ἔχον) may not be exercising it (μὴ ἐνεργεῖν). Whether or not Aristotle assumes some version of the so-called 'principle of plenitude,'[61] he holds that the notion of potency implies non-activity at some time and this is sufficient to undermine the eternality of motion. One might see this as an Aristotelian version of the Parmenidean problem about generation (or motion) from not-being, reformulated in terms of potentiality and actuality.

Plato had also wrestled with the problem, of course, but Aristotle insists (1071b15) that it does not help to posit eternal substances (οὐσίας ἀϊδίους) *unless* there is in them some principle capable of causing change; cf. *Met.* 988b2. Still, he argues (1071b17), even if there were such a principle in Forms or in some other substance besides the Forms (e.g. the World-Soul or the Mathematicals), it would not be sufficient as a cause of (eternal) motion unless it were at work (εἰ μὴ ἐνεργήσει). Now he takes the argument one step further by considering the nature of the cause of eternal motion, which itself requires constant rather than intermittent activity. Even if the principle is now at work, as long as it has some potentiality it will not be sufficient as a cause of eternal motion because potential being might not exist at some time or other. Thus Aristotle concludes that the principle of eternal motion must be such that its essence is activity.[62]

Even though Plato posited eternal movement, Aristotle objects (1072a2) that he cannot make soul the principle of this movement because it is posterior and simultaneous with the universe (ὕστερον γὰρ καὶ ἅμα τῷ οὐρανῷ).[63] The phrase which Aristotle uses to refer to

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rather the direction of its motion, since this is the only potentiality for actualization it possesses. Thus *aither* would be a self-mover.

[61] Lovejoy (1936) 55 thinks that Aristotle's concept of an unmoved mover precludes such a principle and he cites the present passage (1071b13) among others (1003a2 & 1047b3 ff.) from the *Metaphysics* as evidence in support of his claim. By contrast, Hintikka (1968) argues that Aristotle is committed to a fairly strong version of the principle, on the grounds that he does not make a clear distinction between logical and physical possibility. Whether or not this is the reason, it is clear that Aristotle's notion of the complete actuality of the unmoved mover precludes any element of potentiality.

[62] δεῖ ἄρα εἶναι ἀρχὴν τοιαύτην ἧς ἡ οὐσία ἐνέργεια, *Met.* 1071b19–20.

[63] There seems to be an implicit contradiction between attributing the idea of eternal motion to Plato and Aristotle's report at *Physics* VIII.1 (251b18 ff.) that Plato

soul, i.e. 'that which moves itself' (τὸ αὐτὸ ἑαυτὸ κινοῦν), seems to be taken either from Plato's *Phaedrus* (245C–246A) or from the *Laws* (894C–896B) where we find him positing soul as the eternal principle of all movement. However, as we have already seen from the *Timaeus* (34B–C), Plato makes a point of explaining that the World-Soul is subsequent to the World-Body only in the order of exposition, whereas it is prior in generation and in excellence (καὶ γενέσει καὶ ἀρετῇ προτέραν). So why does Aristotle claim that Plato cannot posit soul as the principle of eternal movement? His objection seems to be that there is an inconsistency in the Platonic way of speaking about the soul. Whereas a principle should be prior to that of which it is a principle, Plato appears to speak of soul as something posterior and simultaneous with the universe. Aristotle may be simply referring to verbal inconsistencies in the *Timaeus* but I suspect there is a deeper point to his objection which is important for the development of his own concept of an unmoved mover.[64]

In order to elucidate this point we must turn briefly to *Physics* VIII.2, where he rehearses and rejects some arguments against his own position that motion is eternal.[65] All such objections try to render plausible the view that motion may at one time exist, though it did not exist at all prior to that time. Based on the experience of living beings, the third argument (252b17 ff.) claims that at one time they are completely at rest, while at another time they can set themselves in motion even without external stimulus. Thus, in contrast with inanimate things which need such an external mover, 'we' say that it is the animal itself which causes itself to be in motion.[66] Here the argument draws (252b26–27) an exact parallel between the 'microcosm' of the animal and the 'macrocosm' of the universe, and I think it is reasonable to assume that Plato was also influenced by the evidence about living things when he postulated a World-Soul as the

alone held time to be generated since he said that both time and the universe come into existence simultaneously. If time is an attribute of motion (251b28) then it seems that Plato must say that motion also had an absolute beginning.

[64] One possible construal of Aristotle's motivation is that for him (but not for Plato) soul is essentially related to some kind of body, and so contains an element of potency; whereas Nous is separable from all physical structures and so qualifies as a pure actuality.

[65] A parallel passage is *Physics* VIII.6, 259b1–16 where he also argues that, although animals appear to be completely autonomous self-movers, they are continuously being moved by their environment.

[66] τὸ δὲ ζῷον αὐτό φαμεν ἑαυτὸ κινεῖν, *Phy.* 252b22–23.

source of motion in the universe. Hence Aristotle's response to such an argument should provide us with some significant pointers for his own move away from that cosmological view.

He begins his defense by conceding (*Phy.* 253a8 ff.) that the greatest difficulty for his thesis that motion is eternal is indeed posed by the fact that living things seem to generate motion by themselves. However Aristotle insists (253a11–12) that this impression is false because we see (ὁρῶμεν) that there is some natural part of the animal which is always in motion. Furthermore, he claims, the cause of this constant motion is not the animal itself but perhaps the environment (τὸ περιέχον).[67] Ross (1936, 691) thinks that Aristotle has in mind such motions as growth, decay, and respiration, which are caused in animal bodies by their environment. This seems to be confirmed by the subsequent (253a14–15) clarification that, when we say that the animal moves itself, we are not referring to all kinds of motion but only to locomotion. This means that some of the lower functions of the soul, along with their resulting motions, are not self-caused but rather need an external cause.

In addition, Aristotle suggests, many of the higher functions may also have external causes, even if only indirectly through the lower functions. Since some motions in the body (e.g. growth, respiration) must be caused by the external environment, there is nothing to prevent these motions from causing thought (διάνοια) or desire (ὄρεξις) to move; thereby moving the whole animal. The illustration which Aristotle gives (253a15–19) is that of dreaming, which depends on his theory that in sleep animals are deprived of normal sense perception and hence that their dreams must be the result of other kinds of motion in the body.[68] From my point of view, however, the crucial step which he takes in his rebuttal of the third argument is the claim that many motions of the animal soul are not self-caused but rather need an external mover. Within its specific context in the *Physics,* the result of this claim is that the objection against the eternality of

[67] Cf. also *Phy.* 259b1–16. What Furley (1989) 125 finds particularly striking about both passages in the *Physics* is the way in which intentional action is assimilated to mechanical movements, by contrast with its teleological treatment in *De Anima* and *De Motu Animalium.* He thinks this is due to Aristotle's limited purpose of showing that animal motions do not exemplify an absolute beginning of motion, and that such motions could not be explained on the assumption that there was no prior motion; cf. *DA* III.10.

[68] Cf. *De Insomniis* 3, 460b28 ff.

motion based on evidence about living things fails. At a more general level, however, the analogy between the microcosm of the living animal and the macrocosm of the Living Animal (such as we find in the *Timaeus*) is shown to be inadequate for a universe that is eternally in motion. Thus, returning to *Metaphysics* XII, we can see the deeper reason why Aristotle rejects so strongly the Platonic move of positing soul as the principle of eternal motion in the universe.[69]

Aristotle now resolves the previous (1071b23–24) aporia in a characteristic fashion when he says (1072a3–4) that in one sense it is correct to think that potency is prior but that in another sense it is not. His obscure reference[70] to another extended treatment of the question may be to *Metaphysics* IX.8, for instance, where a solution is spelled out in terms of different senses of priority. There Aristotle claims (1049b10–12) that actuality is prior to potentiality in definition (λόγῳ), in substance (οὐσίᾳ) and in time (χρόνῳ). Yet, with respect to temporal priority, potentiality is prior to actuality and this should cover the phenomena mentioned in Book XII. The crucial distinction in IX.8 is that between specific and numerical identity. If one considers the same species (e.g. man) over a period of time then actuality is temporally prior to potentiality, since it is a fully realized member of the species that generates another. But when one studies a single individual in the process of being actualized, one finds previous stages (like the embryo state) which may be called potential with reference to the fully mature state.

Only in this limited sense, therefore, is potentiality temporally prior to actuality; whereas actuality is prior in all of the important metaphysical senses. Yet this single and unimportant sense can save the appearances in XII.6 because the individual is more obvious to sense perception and its temporal development from potentiality to actuality is most familiar to us. Ordinary experience also tells us that some potencies are never actualized (e.g. the acorns may be eaten by the pigs), whereas it does not seem that there is any actualized thing which is without a prior state of potentiality. Thus, by the criterion

[69] Further insight into these reasons may be obtained from Aristotle's analysis of self-movers in *Physics* VIII, which shows that one aspect must be an unmoved mover if an infinite regress is to be avoided.

[70] Despite the obscurity of the reference, I think that Ross (1924) ii, 371 is clearly wrong in accepting that the reference is to the previous passage (i.e. 1071b22–26) where the aporia is raised, since this passage merely sets up the difficulty and does not even hint at a solution.

of non-reciprocal dependence, one might infer that potentiality is ontologically prior to actuality. It must be some such conclusion that constitutes the aporia in XII.6 because it opens up the possibility that nothing will exist, given that any potency whatever may not be realized. So Aristotle's solution is to concede that potency is prior to actuality in a special temporal sense, while insisting that actuality is prior in all of the key metaphysical senses.

When seen in the light of his typical manner of treating an aporia, Aristotle's subsequent procedure in *Metaphysics* XII also becomes more intelligible. Instead of elucidating the solution which he draws from elsewhere, he proceeds (1072a4 ff.) to 'save the phenomena' by showing that the *endoxa* of his predecessors confirm (μαρτυρεῖ) the view that actuality is prior. Anaxagoras, for instance, posits Nous as the first principle of motion in the universe and, according to Aristotle (1072a5), Nous is an activity. In relation to his own concept of an unmoved mover, it is significant that he elsewhere (*Met.* 989b15–16) praises Anaxagoras for conceiving of Nous as alone unmixed and pure. Here in XII.6, however, Aristotle is simply appealing to the Anaxagorean concept as supporting evidence for his own view that actuality is prior. Similarly, he cites the Empedoclean principles of Friendship and Strife as examples of actualities that are posited as principles of motion. Even people like Leucippus, who posit eternal motion, testify to the primacy of actuality because, as Aristotle sees it (1071b32–33 & 1072b7–8), they are positing eternal activity (ἀεὶ ἐνέργειαν). The conclusion which he draws (1072a7–9) from all this supporting evidence is that, since activity is prior to potency, Chaos or Night cannot have existed for an infinite time.

Thus, in XII.7, Aristotle claims (1072a19–21) that all the difficulties which arise from positing Night or 'all things together' as the origin of the cosmos have been resolved by his previous account, which makes activity prior to potency. This metaphysical account has its correlate on the cosmological level in the thesis that there is something which is always moved with an unceasing motion that is circular.[71] Aristotle finds (1072a22) this thesis convincing because it is confirmed not only by the argument (λόγῳ) but also in fact (ἔργῳ). By using the traditional contrast between *logos* and *ergon* here, he may be referring to the way in which the theoretical arguments of his cosmology seem to be supported by the empirical observations of

[71] καὶ ἔστι τι ἀεὶ κινούμενον κίνησιν ἄπαυστον, αὕτη δ' ἡ κύκλῳ, *Met.* 1072a21–22.

astronomy; e.g. that the sphere of the fixed stars is observed to move constantly around a stationary earth. Such a concordance of theory and observation lends firm support to Aristotle's conclusion (1072a23) that the first heaven must be eternal.

Building on this conclusion, he now constructs a terse metaphysical argument for the existence of an unmoved mover, which is distinct from the first heaven. Textually, the first step in the argument (1072a23–24) is the most difficult to reconstruct because it seems to infer from the previous conclusion (i.e. that the first heaven is eternal) that there is something which moves it (ἔστι τοίνυν τι καὶ ὃ κινεῖ). Next (1072a24), Aristotle characterizes the first heaven as an intermediate (μέσον), since it is both a moved thing (κινούμενον) and a moving thing (κινοῦν). It has been noted by some scholars[72] that this characterization depends on a tripartite division outlined in the *Physics* (VIII.5, 256b14 ff.) between things that are simply moved, those that are both moved and move other things, and those things which simply move other things without being themselves moved.

I think that this fact is crucial for understanding how Aristotle, without any argument in *Metaphysics* XII, simply asserts that there is something which moves and is not moved.[73] When we compare this with the conclusion reached in the *Physics* passage, the parallel is striking. There, after positing the aforementioned tripartite division with respect to motion, Aristotle concludes (256b23–24) that it is reasonable (εὔλογον), even though not necessary (ἀναγκαῖον), that there should be a third thing which causes motion but is immovable (ὃ κινεῖ ἀκίνητον ὄν). In this connection he praises Anaxagoras for describing Nous as 'unaffected' (ἀπαθῆ) and 'unmixed' (ἀμιγῆ), when the latter posits it as the principle of motion. The reason for such unusual praise is found in Aristotle's subsequent (256b26–27) explanation that it is only when Nous is immobile that it can cause motion in this way and only when it is unmixed can it rule. Thus he is partially indebted to Anaxagoras for his conception of the activity of an unmoved mover.

In *Metaphysics* XII.7, however, from the physical claim that there is something which moves other things without itself being moved, Aristotle draws the metaphysical implications that it is an eternal

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72 For instance, Manuwald (1989) has noted the parallels and has argued that the *Physics* passage (256b14–27) must be out of place where it stands in the middle of an analysis of self-movers.

73 τοίνυν ἔστι τι ὃ οὐ κινούμενον κινεῖ, *Met.* 1072a24–5.

thing, a substance, and an activity.[74] This constitutes an answer to the leading questions for his inquiry, namely, whether or not there is a supersensible substance and what is its mode of being. Yet his more immediate concern is to find a model for the action of an unmoved mover that will avoid an unwanted implication of motion through contact; i.e. that the mover is itself moved by reaction.[75] Without an alternative to the mechanical model of motion, he cannot justify the postulation of an unmoved mover.

While this problem appears to be unresolved in *Physics* VIII,[76] *Metaphysics* XII.7 proposes a model based on the way an object of desire moves the faculty of desire without itself being moved:

> And this is the way in which the object of desire or the intelligible object moves, namely, without itself being moved. Of these, the primary objects are the same; for the object of desire is that which appears to be noble, and the primary object of which is that which is noble.[77]

Notice that in this alternative model both an object of desire (τὸ ὀρεκτόν) and an intelligible object (τὸ νοητόν) can move other things without themselves being moved. Indeed Aristotle thinks that such objects are identical in the case of the firsts (τὰ πρῶτα) for each class of object; i.e. whatever is supremely desirable is also of the highest intelligibility.[78] The crucial point needed for his subsequent argument, however, is that the beautiful (or the noble or the fine) is also prior to our desire for it, since it is by becoming conscious of the beautiful that we are led to desire it. The next step is to establish the identity of the primary object of desire with the primary intelligible object.

The first move depends on the explicit claim that the intellect is moved by the intelligible object, which seems to be an assumption from Aristotle's psychology. The next move involves a distinction

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[74] ἀΐδιον καὶ οὐσία καὶ ἐνέργεια οὖσα, *Met.* 1072a25–26.

[75] Cf. *GA* 768b16–25 which says that every moving agent, with the exception of the first mover, is reciprocally moved by that which it moves. There is a reference to a treatise on 'acting and being acted upon' which is not extant but which also seems to be mentioned at *GC* 324a25 ff.

[76] Manuwald (1989) gives some persuasive reasons as to why Aristotle does not specify the type of causality exercised by the first mover, since his argumentative strategy is simply to establish that there must be such an unmoved mover, even if one correctly posits a self-mover as the cause of motion.

[77] *Met.* 1072a26–28: tr. Apostle (1966).

[78] Menn (1992) has recently argued convincingly that the positive essence of the Prime Mover may be described in terms of Nous and Good-Itself (but not the Form of the Good).

between an object of cognition which is intelligible in itself (καθ' αὑτή) and one which is not. Here he seems to be presupposing a division within the realm of intelligibles between a positive and negative series of things, roughly corresponding to the distinction between being and non-being.[79] Within the positive series, he claims (1072a32), substance is primary (ἡ οὐσία πρώτη) and, within the category of substance, that is primary which has a mode of being that is simple and in activity (ἡ ἁπλῆ καὶ κατ' ἐνέργειαν). In *Metaphysics* VII substance is said to be prior in every important sense and, even within the category of substance itself, substantial form is given both ontological and epistemological primacy.[80]

In XII.7, however, a new step in this hierarchy is made with the introduction of a kind of substance apart from sensibles which is immaterial, utterly simple, and a pure activity. Aristotle even takes the trouble to explain (1072a32–34) that the simplicity he attributes to such primary substances is not the same as unity, because that signifies a measure (μέτρον) whereas simplicity indicates a mode of being (πὼς ἔχον). Therefore, within the series of things which are intelligible *per se*, absolute primacy is given to the kind of substance which is completely simple and in a state of pure activity.

In XII.7 Aristotle also introduces value principles within a metaphysical context when he proposes to show that the final cause may exist in immovable things.[81] While one might take this cause to be mainly a principle of being, we must not neglect its axiological connotations in view of his previous talk about noble and choiceworthy objects. But the explicit problem here is that the final cause seems to be incompatible with immobile things because it involves movement towards a goal. In order to avoid this difficulty, Aristotle distinguishes (1072b2–3) between two senses of final cause where we say something is good for the sake of (i) someone (τινί) or (ii) something (τινός). In the first case, the attainment of a goal brings a change to the person or thing that had not previously reached it. Thus it would appear to be an immanent type of final causality which is incompatible with the unchanging nature of eternal active substance. But the second sense of final cause, which is thought by Aristotle to be com-

[79] Cf. *Met.* IV.2 & IX.9 for corresponding table of opposites that may have been Pythagorean in origin.

[80] Cf. *Met.* VII.1, 16, & 17. See also IX.8, 1050b2 ff.

[81] ὅτι δ' ἔστι τὸ οὗ ἕνεκα ἐν τοῖς ἀκινήτοις, *Met.* 1072b1–2.

patible with unmoved things, is rather more difficult to understand. We may conjecture that he is referring to some goal or end, existing actually in the unchangeable realm, to which other things respond. Such an objective end seems to be what Aristotle has in mind when he explains (1072b3–4) that this final cause moves as being loved (κινεῖ δὲ ὡς ἐρώμενον), whereas other things move by being moved (κινουμένῳ δὲ τἆλλα κινεῖ).[82]

Here he is appealing once more to his previous (1072a26–27) analogy with the way in which the object of desire or the intelligible object moves our thinking, without itself being moved. This appears to be the only way in which an unchanging object can be posited as a final cause. By contrast, as he explains (1072b4–5), if something is moved then it can be in another state. This has important consequences for his model of the universe by comparison with that of Plato. For instance, the circular locomotion of the sphere of the fixed stars is no longer deemed adequate as the ultimate source of motion in the universe. Even though this primary locomotive exists as an activity (ἐνεργείᾳ), Aristotle insists (1072b5–6) that qua moved (ᾗ κινεῖται) it can be in another state with respect to place (κατ' τόπον) though not with respect to substance (κατ' οὐσίαν). This means that the unmoved mover cannot even be accidentally moved like the World-Soul which shares in the circular locomotion of the World-Body; cf. *Tim.* 34A ff. Since Aristotle has already established to his own satisfaction that there is some mover which is itself an immobile being and a pure activity, he concludes (1072b8) that it cannot be in another state.

Having attributed this static mode of being to the unmoved mover, however, Aristotle must now determine its characteristic action in the universe. When he claims that among all changes locomotion is primary, he is drawing once more upon conclusions reached in the *Physics* (260a26–b29). In VIII.9 (265a13–b16), for instance, he gives a detailed argument not only for the primacy of locomotion but also for the priority of circular locomotion over other kinds of locomotion. In *Metaphysics* XII.7, therefore, he merely indicates (1072b8–10) that this is the kind of motion caused by an unmoved mover. Some of the reasons for this latter claim must also be adduced from the *Physics,* since they are not elucidated in XII.7.

First, there is a striking coincidence of priorities: the priority of

[82] Here I am following Jaeger's reading of κινουμένῳ instead of Ross's κινούμενα.

circular locomotion among all changes, the priority of simple and eternal substance among all beings, and the priority of the most noble thing among all choiceworthy things. This wonderful coalescence of all absolutely prior things within the Prime Mover gives an almost mystical sense of cosmic unity. In addition, Aristotle assumes that a completely simple substance like an unmoved mover can cause only one simple motion.[83] But circular locomotion is simpler and more complete than any other kind of motion; cf. *Phy.* 265a15 ff. What makes it particularly appropriate as the effect of an absolutely simple eternal activity is that it is the only kind of motion which can be one, continuous, and eternal; cf. *Phy.* 267b10 ff. & *Cael.* 286a9–12.

Aristotle might have finished at this point because he has reached an absolute first principle that is more intelligible by nature. But his philosophical method requires him to make this solution more intelligible to us by 'saving the phenomena.' First, he tries to make it plausible in terms of human experience by comparing the eternal activity of divine being to the best life that we live only for a short time.[84] The unmoved mover has already (1072b10–11) been said to exist nobly (καλῶς), insofar as it is a self-determining first principle, so the new aspect of the divine life is its pleasure (ἡδονή). The identification of pleasure and activity in the divine is introduced to support the claim (1072b15–16) that it can live the best life forever, whereas this is impossible for us. Though this contrast is not explained, we may guess that it depends on the distinction between first and second actuality not being applicable to divine being, since that kind of being is utterly simple by contrast with composite and destructible beings like man.

Another way of formulating the same contrast might be to say that, whereas pleasure and activity are distinguishable for us, they are indistinguishable for the divine; cf. *EN* 1153a14, 1175a15. Yet the deeper reason why Aristotle fixes on pleasure as a human experience, which is also appropriate to the divine, is that it has degrees of perfection extending all the way from the lowest pleasures to the

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[83] By contrast with the Presocratics and the Platonists, all of whom posit opposites as their principles, Aristotle thinks that the unmoved mover (as a primary substance) and its unique effect (i.e. circular locomotion) excludes all opposites from the supersensible realm which is therefore eternal.

[84] διαγωγὴ δ' ἐστὶν οἵα ἡ ἀρίστη μικρὸν χρόνον ἡμῖν, *Met.* 1072b14–15. The word διαγωγή used in the present context for the activity of the divine may mean both noble activity and pleasure.

highest. Furthermore, unlike other human activities whose goal is external and hence destructive of them, the activities associated with the highest pleasure are those whose goal is internal (ἐντελής) and which are therefore self-fulfilling (ἐντελέχεια); e.g. in human seeing one simultaneously sees and has seen; cf. *Met.* 1048b23.[85] Aristotle must have in mind this close connection between pleasure and such activities when he remarks (1072b17) that "it is because of this that being awake, sensing, and thinking are most pleasant," since all of these are self-fulfilling activities. Among these human activities, of course, thinking is the highest and hence the most appropriate for divine being.

According to Aristotle, however, divine thinking has special characteristics which are not shared by human thinking, except perhaps in its highest form as contemplation. This shared form of human and divine thinking seems to be what he has in mind when he says (1072b18–19) that thinking *per se* is of the best *per se* and that thinking in the highest degree is of the best in the highest degree. It would appear that thinking in itself is being distinguished here from ordinary human thinking which depends on sense and imagination. Such divine thinking has for its object not any apparent good but absolutely the best and most noble thing, which is none other than the unmoved mover itself. Thus qua intellect this mover thinks itself when it participates in thinking, since something becomes intelligible through touching and thinking. The result of divine thinking, therefore, is that the intellect and the intelligible object are identical. This situation is described in *Metaphysics* XII.9 (1074b34) as 'thought thinking itself' (νόησις νοήσεως νόησις). In XII.7, however, he simply explains (1072b22) that what is capable of receiving the intelligible object and the substance is the intellect (νοῦς).

By implicit contrast with the human intellect which is capable of receiving all intelligible forms, Aristotle claims (1072b23–24) that the actual possession of intelligible objects appears to be divine (θεῖον) and that the contemplation of such objects is the most pleasant and the best activity. The first claim might be justified in terms of the priority (and hence superiority) of activity over capacity, along with the previous identification of pure activity with the unmoved mover. But the second claim implicitly appeals to three hierarchies whose first members coincide in that wondrous fashion which Aristotle found

[85] Cf. Kosman (1984) & Kahn (1985).

so aesthetically appealing. Thus the highest activity of the mind is the best and the most pleasurable. Appealing to our intermittent experience of this privileged activity of contemplation, Aristotle concludes (1072b24–26) that if the divine exists in this way eternally then this is cause for wonder (θαυμαστόν). But since the mode of being of the divine is better, this is even more wondrous (θαυμασιώτερον). If philosophy begins in wonder, it also seems to culminate in wonder at the mode of being of the divine, in which we can briefly participate through our own activity of contemplation.

Predictably enough, Aristotle also maintains that life belongs to the divine because, as he explains (1072b26–27), the activity of the intellect is life (ἡ γὰρ νοῦ ἐνέργεια ζωή). But God just is that activity and his activity is in itself a life that is the best and is eternal. Here Aristotle switches from the neuter to the masculine in his talk about the divine and perhaps this is significant, given that he subsequently connects his conception of the divine with more popular conceptions: "We say that God is a living being which is eternal and best; so life and continuous duration and eternity belong to God, for this is God."[86] Aristotle tries to save the phenomena of ordinary speech about the divine, in which we say (φαμεν) that God is the best among all living beings precisely because he has eternal existence.

In the background here we may spy the traditional Homeric division between divine and mortal beings, the latter of which are doomed to die no matter what the quality of their lives. By contrast, the divinities are conceived of as immortal and deathless, despite traditional myths about their 'generation' from higher divinities. Perhaps this is why Aristotle adds continuous duration (αἰὼν συνεχής) to eternity (ἀΐδιος) in his description of the mode of being of the divine. While the first characteristic tends to suggest that God exists in time, the second implies that He is outside time. This is analogous to the tension in Aristotle's conception of the unmoved mover at *Physics* VIII.10 (267a21 ff.), where he appears to claim, on the one hand, that it is located at the circumference of the first heaven yet, on the other hand, that it is not in any place because it is without parts or magnitude.[87]

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[86] *Met.* 1072b28–30: tr. Apostle (1966).

[87] Even though it seems strange to us, Manuwald (1989) 56 insists that Aristotle acknowledges the localization of an immaterial principle either in the part or the whole; cf. *MA* 9. Thus Manuwald thinks (1989) 61 that the description in *Physics* VIII.10 of the Prime Mover as indivisible, partless and without magnitude does not

The fundamental problem underlying such a tension, I think, is how to define the relationship of the divine to the universe. While its role as first principle of motion requires it to have some causal efficacy in the universe, yet if it is to remain unmoved itself it cannot work through any mechanical action which would cause it to suffer an equal reaction. Whereas this problem is not tackled in the *Physics*, Aristotle offers a resolution in *Metaphysics* XII by treating the unmoved mover as a teleological cause that moves things in the same way as an object of love or an intelligible object moves the lover or the thinker. However, if the unmoved mover is a divine being whose eternal activity is thinking then Aristotle's conceptual model of thought requires that he specify the object of such thinking. Now he cannot claim that God thinks the same intelligible forms which stimulate our thinking because that would imply that the mover is itself moved, unless he eliminates the receptive mind from the divine. This problem is resolved in *Metaphysics* XII.9 when Aristotle says that the divine thinks itself.

In XII.7, however, after elucidating his own conception of the divine, Aristotle makes some revealing polemical remarks (1072b32 ff.) about people like Speusippus who assume that the most noble and the best are not in the starting-point. He declares flatly that such people 'do not think rightly' (οὐκ ὀρθῶς οἴονται), yet he rehearses (1072b32–34) their distinction between initial and final causes in a way that makes their position appear similar to his own. While conceding that the principles (ἀρχαί) of plants and animals are causes (αἴτια), they hold that the beautiful or the noble (τὸ καλόν) and the complete (τέλειον) are in what comes from these principles (ἐν τοῖς ἐκ τούτων).

On the face of it, this seems to correspond with Aristotle's distinction between the final cause (which comes later in time for the individual) and the formal plus material causes, which are those elements 'out of which' the composite is completed and perfected. So why doesn't he acknowledge the contribution of these thinkers to his conception of the final cause? By all historical accounts, the Pythagoreans were fascinated by the 'perfection' of the number ten which is the sum of the first four numbers and is therefore, in a sense, produced out of them.[88] Speusippus is also credited with a book on the Pythagoreans, in which he dwells especially on the perfection of

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decide the question of its immanence or transcendence.

[88] Cf. *Theologoumena Arithmeticae* 55 (Ast), & Aetius, *Placita* i. 3; Dox. 280.

the highest number, ten.[89] Presumably, he held that the good only shows itself in one of the later grades of substance; cf. *Met.* 1028b21 & 1091a30 ff.

From such a historical perspective, we can see that what Aristotle rejects is the mathematical approach to the beauty and perfection of the universe. In his rebuttal, by way of contrast, he appeals to a biological model of perfection and completeness, even though he does not accept the Platonic view of the universe as a Living Creature. At the level of species, however, he insists that the seed comes from other things which are prior and complete.[90] This means that what is first (τὸ πρῶτον) is not the seed but something complete like a man. To understand Aristotle's argument here (1073a1 ff.), one must keep in mind that he is assuming the species form to be eternal and fixed. Therefore, even though the seed is prior in time to this individual man, the order of priorities is reversed for the species. For Aristotle this is attested by the fact that a man is generated by another full-grown man who provides the seed. Thus, at the species level, man is prior in time to the seed and this involves priority in definition and in substance; cf. also *Met.* 1050a4 ff. On the cosmological level it also implies that actuality or a state of completion is prior in every important sense to potency, whereas Speusippus gives priority to some potentiality.

Thus, after saving the phenomena by correcting his opponents and by deepening the popular conception of the divine, Aristotle draws some final conclusions from the discussion of XII.6 & 7 as follows:

> It is evident from what has been said that there exists a substance which is eternal and immovable and separate from sensible things. It has also been shown that this substance cannot have any magnitude but is without parts and indivisible. For it causes motion for an infinite time, but no finite thing has infinite potency. Since every magnitude is either infinite or finite, this substance cannot have a finite magnitude

[89] Cf. Diels (1889) i, 303.20. In *Metaphysics* XIV.4 Aristotle discusses the relationship of the Platonic elements and principles with the Good. Some Platonists are reported as identifying the Original One with the Original Good, whose essence consists primarily in its unity; whereas others (Speusippus) found difficulties in that identification and so claimed that the good only emerged later as a product of the principles. According to Aristotle's analysis (1091b1–2), however, the difficulties do not arise from ascribing the good to the principles, but rather from making the One a principle in the sense of an element and from making number come from the One. This supports my claim that Aristotle is criticizing the mathematical cosmology of the Platonists as vitiating their teleological approach to the universe.

[90] τὸ γὰρ σπέρμα ἐξ ἑτέρων ἐστὶ προτέρων τελείων, *Met.* 1072b35.

> because of what we said, and it cannot be infinite in view of the fact that there exists no infinite magnitude at all. Moreover, it cannot be affected or altered; for all the other motions are posterior to locomotion. It is clear, then, why these facts are so.[91]

The first part of this conclusion clearly picks up the leading question of the whole inquiry and gives it a positive answer; i.e. that there is some supersensible substance. Whereas in *Metaphysics* XIII he denies that Forms and Mathematicals are such substances, here he asserts (1073a4) that the unmoved mover exists as a substance which is eternal and immovable and separated (κεχωρισμένη) from sensible things. Yet, despite its separation from the sensible universe, the unmoved mover serves as a final cause of its motion, but separated Mathematicals or Forms can have no such function.

Thus, in its role as first principle, the transcendent Prime Mover completes the ordering of the universe, whereas the Platonists (in Aristotle's view) are mistaken in setting up separate Forms and Mathematicals because they cannot really be separated from the sensible world. In the present passage, we should hear a polemical note when he specifies the characteristics of divine substance by contrast with sensible substance. For instance, he claims (1073a5–7) to have shown that this kind of substance cannot have any magnitude but rather that it is without parts and indivisible (ἀμερὴς καὶ ἀδιαίρετος). Such a claim is rather unexpected, in view of the complete absence from XII.6 & 7 of any discussion of the partlessness and indivisibility of the Prime Mover. Even if we take these characteristics to be directly implied by the completely simple mode of being that is attributed to divine substance (cf. 1072a32–34), it is still undeniable that their meaning is parasitic upon the contrasting physical characteristics that belong to sensible substances. Therefore I think it is more plausible to see this claim as being a summary of his discussion in *Physics* VIII.10, which is introduced here partly for polemical purposes.

The connection with the *Physics* is firmly established by the subsequent (1073a7–8) clarification that divine substance moves (something) for an infinite time, whereas no finite thing has an infinite potency (δύναμιν ἄπειρον). While there is very little in *Metaphysics* XII.6 & 7 to render this explanation intelligible, *Physics* VIII.10 elaborates upon the same theme. The stated purpose (266a10–11) of that chapter is

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[91] *Met.* 1073a3–13: tr. Apostle (1966).

to discuss the claim that the unmoved mover necessarily has no parts and no magnitude. In support of this claim, Aristotle adduces (266a12–13) the principle that nothing which is finite can cause a motion for an infinite time. We can see immediately from the Greek that this principle is identical with that to which Aristotle appeals without further argument in *Metaphysics* XII.7. In the *Physics* discussion (266a13–22), however, we do get a dialectical defense of the principle which grounds the claim that the Prime Mover has no magnitude, by eliminating the possibility that it might have finite magnitude. The other possibility that its magnitude might be infinite is also rejected (1073a10) on the grounds that such an infinite magnitude does not exist.

Now this latter assertion also depends on *Physics* III.4 (202b30 ff.) for its detailed discussion of the different ways in which the infinite can and cannot exist. Briefly stated, the basic reason why the possibility of an infinite magnitude is rejected by Aristotle is that he opts for a finite universe over against the infinite universe of the Atomists. Of course, he offers (*Phy.* 205a9 ff.) what he considers to be convincing arguments for this choice in terms of the existence of natural places in the universe. Yet the plausibility of such arguments clearly depends on the presupposition that the universe is a finite sphere with an absolute center and a unique circumference.

Thus Aristotle's espousal of a finite universe may be taken as a basic presupposition of his whole approach to cosmology and metaphysics, which are closely intertwined in *Metaphysics* XII.[92] One of the important implications, for instance, is that the only possible motion that can be both continuous and eternal is circular locomotion. Therefore this is designated as the unique effect of an utterly simple Prime Mover, which itself is not directly involved in the mechanical motion in the universe. Furthermore, this entity is identified as a divine being which, as eternal and pure activity, is the highest form of being of which we can form some idea only through the experience of our own highest activity; i.e. pure contemplation. Such an ordering shows significant differences from that espoused by the Pythagoreans and those Platonists who developed their own mathematical cosmology.

[92] Cf. Furley (1987b). By contrast, as Solmsen (1960) has noted, Aristotle's physics is not so closely related to his cosmology, even though his views about a finite and eternal universe do have clear implications for such things as the infinite and the void.

V. *Astronomy and metaphysics*

After showing that there exists a supersensible substance as an unmoved mover, Aristotle devotes *Metaphysics* XII.8 (1073a14 ff.) to the question of whether there is one or many supersensible substances and, if the latter, how many. Ever since Jaeger (1923, 342 ff.) argued on grounds of style and content that this chapter represents a later insertion into a book consisting largely of Aristotle's Platonizing theology, there has been uncertainty about how this chapter fits into the general plan of XII. Jaeger has claimed that chapters seven and nine fit smoothly together to form a continuous train of thought, which is interrupted by the intrusive eighth chapter. Given the philological arguments for treating XII.8 as an insertion, few scholars have questioned this claim even though Jaeger's whole developmental hypothesis has come under general attack.[93]

If we follow the guidance of Aristotle's own leading questions, however, the treatment of XII.8 as an 'insertion' is inappropriate because it assumes the rest of XII to form one continuous treatise. But if, as I have argued, the leading question of the book has been answered by the end of XII.7, then it is plausible to read the subsequent chapters as taking up other problems or objections which are loosely associated with that leading question.[94] Such a reading is supported by the fact that each of these chapters begins with a question which is either described as an aporia or has the characteristic form of one. For instance, at the beginning of XII.8 (1073a14–15), Aristotle asks whether one ought to posit a single such (supersensible) substance or many and (if the latter) how many. He then surveys the opinions relevant to the question, proposes his own solution and subsequently tries to save the phenomena. In other words, XII.8 may be seen as a self-contained treatise which follows the standard aporetic schema.

[93] A notable exception is Düring (1966a) 214 ff. who resists Jaeger's developmental theory by his own plausible attempt to find a unity in *Metaphysics* XII. More recently, Helen Lang (1993) has given a convincing argument for a unity of theme in Book XII, including the controversial Chapter 8.

[94] Grayeff (1974) 167 actually suggests that these chapters might be additions by later lecturers who are answering such objections and also responding to the particular demands of their audience. Though this suggestion indirectly supports my interpretation, I do not find it historically plausible. By contrast, Lang (1993) gives a comprehensive reading of *Metaphysics* XII, which integrates chapters 8 to 10 into the whole argument of the book, while accepting that they deal with subsidiary questions.

Hence I will examine the main lines of argument in this little treatise from the perspective of my own hermeneutical question; i.e. why does Aristotle reject a mathematical ordering of the universe? When we consider XII.8 & 10 together, I think we will find some grounds for his own cosmological stance. In this regard I see a great deal of significance in the fact that, after introducing the leading question in XII.8, the only opinions which he reviews are those of the mathematical Platonists. One might explain this as being due to the fact that the leading question is about supersensible substances, and that the Platonists are the only thinkers reported to have posited such substances. But Aristotle complains (1073a15–17) that they said nothing clear about the number of these substances, which strongly suggests that they did not address the question. Indeed he concedes as much when he says (1073a17–18) that the doctrine of Forms says nothing specific about this question, which is not surprising given that the Platonic dialogues are more concerned with what kinds of attributes of sensible things must be posited as Forms in order to escape confusion and contradiction.[95]

So how does the question about the number of Forms arise? A clue is to be found in Aristotle's remark (1073a18) about people who speak of the Forms as numbers (ἀριθμοὺς . . . λέγουσι τὰς ἰδέας). Here he lists (1073a18–21) two conflicting opinions about the number of Numbers, and such a conflict seems to be the source for his own question. On the one hand, they speak about numbers as if they were infinite whereas, on the other hand, they talk as if numbers were limited to ten. He complains, however, that no serious proof is given as to why there should be just so many Numbers. Here Aristotle seems to be manufacturing an aporia by emphasizing the opposition between the finite and the infinite, since these can be treated as compatible characteristics that simultaneously belong to numbers.[96] For instance, the view that numbers are an indefinite multitude is not incompatible with giving some special status to the first ten numbers in a decadic system of counting. So it is not clear why Aristotle makes such a fuss about this. Perhaps he is motivated by the fact that the decad plays a major role in the mathematical cosmology of

[95] Cf. *Phd.* 100B ff., *Rep.* 523A ff., *Parm.* 131A ff.

[96] Elsewhere (*Phy.* III.4–8) Aristotle himself accepts that numbers are both finite and infinite, though in different senses; so he may be unhappy with the lack of qualification in the claims of the Platonists.

Speusippus, who becomes Aristotle's special target in XII.10.

Thus Aristotle proposes (1073a22 ff.) to discuss the question from a vantage point already established; i.e. that the principle and the primary being is unmoved both *per se* and *per accidens*.[97] The double qualification applied here to the Prime Mover (i.e. that it is neither moved *per se* nor *per accidens*) is consistent with its description as impassible and unalterable at the end of XII.7. Whatever brief explanation of these characteristics is given there depends implicitly upon a schema of priority among kinds of change, which Aristotle has already established in the *Physics*. Thus he explains that, since all other kinds of change are posterior to locomotion, the divine is both unaffected and unaltered because it is completely unmoved with respect to place.[98] Unlike the World-Soul of Plato's *Timaeus*, it cannot even be located anywhere in the world because that might make it susceptible to being moved *per accidens* through being carried, for instance, along with the circular locomotion of the first heaven.[99]

The hierarchy of motions is also important for the next step (1073a25) of the argument in XII.8; i.e. that the divine is responsible for the first motion, which is both eternal and one. For Aristotle's argument the crucial point here is the single and unique nature of the motion caused by the unmoved mover. But, since there are many other eternal and continuous motions besides the diurnal motion of the first heaven, the logic of the argument requires other unmoved movers. And this is exactly the conclusion of Aristotle's subsequent argument, which is worth rehearsing because of its unusual fullness.

First, he repeats (1073a26) the logical fact for which he claims necessity: (1) what is moved must be moved by something. Then he combines this with other logical and cosmological facts which he claims to have established; i.e. (2) that the first mover is unmoved *per se*, and (3) that an eternal motion is caused by an eternal being, and (4) that a single motion is caused by one being. This listing of necessary facts constitutes the 'logical' preamble to the argument. Then, using

[97] ἡ μὲν γὰρ ἀρχὴ καὶ τὸ πρῶτον τῶν ὄντων ἀκίνητον καὶ καθ' αὑτὸ καὶ κατὰ συμβεβηκός, *Met.* 1073a23–24. Cf. *Physics* VIII.6, 258b10 ff.

[98] This conclusion in *Physics* VIII.6, 259b16–28 seems to represent a conceptual advance over *De Caelo* II.2, 285a29 which seems to admit that the heavens are self-moving bodies with their own internal principles of motion.

[99] By contrast, Manuwald (1989) thinks that a completely immobile and immaterial principle of motion may still be located in the heavenly sphere which rotates in the same place; cf. *Phy.* VIII.9, 265b1.

the verb ὁρῶμεν, Aristotle introduces (1073a28–31) some 'physical' facts about the universe: (5) Besides the simple motion of the universe (which is caused by the first and unmoved substance), there are also the eternal motions of the planets. With an explicit reference to the physical treatises, Aristotle explains (1073a31–32) that a body in circular locomotion has eternal and unceasing motion. Here the important point for his argument is that the motion of each of the planets be single and continuous, especially in the sense that it is without stopping and starting.

But it was notorious among ancient astronomers that, as their name suggests, the planets 'wander' back and forth along the ecliptic. Thus, if appearances are to be trusted, the motion of the planets in two different directions cannot be treated as single and continuous; cf. *Physics* VIII.8 & 9. So we must assume that Aristotle is treating the appearances as non-veridical phenomena which must be 'saved' by postulating a combined set of circular locomotions of spheres for each planet. In fact this is similar to Plato's approach in the *Timaeus,* where astronomers are challenged to save the phenomena of planetary motion.[100] It is hardly surprising, therefore, that Aristotle appeals to the results of astronomy in XII.8, even though he is ostensibly discussing a metaphysical question.

After citing all the foregoing logical and physical facts, he concludes (1073a32–34) his rather elaborate argument as follows: (6) It is necessary that each of these locomotions be caused by something which is unmoved *per se* and an eternal substance. The necessity of this conclusion seems to be logical for the most part, since it follows from steps 1–4 in the argument. Given the physical fact that there are other eternal circular locomotions in the universe besides the diurnal motion of the fixed stars, Aristotle concludes that each of them must have its own unmoved mover because one simple motion is caused by a simple being. But it does not follow from the argument that all the other unmoved movers are supersensible substances of the same kind as the Prime Mover. Aristotle may have been aware of this weakness in the argument because he immediately (1073a34–35) proceeds to argue that these other eternal movers are substances on the grounds that the stars by their very nature are eternal substances.

Elsewhere (e.g. *Met.* VII.2, 1028b8–13), when Aristotle gives a list

[100] Cf. Vlastos (1975) & van de Waerden (1954).

of the most obvious substances, the heavenly bodies are prominent because they serve as paradigm examples of sensible substances that are individual, separate and eternal. But we should notice that the move to supersensible substance is based on an appeal (1073a35) to a criterion of priority according to which the mover is prior to the moved thing. Indeed, it is not just priority with respect to motion (cf. *Met.* 1018b20–21) which is relevant here but also the priority with respect to actuality that belongs to the unmoved mover. Thus Aristotle concludes (1073a36) his argument with the assertion that whatever is prior to substance must itself be a substance. This establishes that the unmoved movers are substances and, presumably, that they are supersensible because they are not observable. I think we must also take Aristotle to be convinced that they are of the same kind as the Prime Mover, since they perform the same function with respect to the planets as it does for the first heaven.[101]

So from these arguments there emerges a general principle for answering the leading question of whether there is one or more supersensible substances and, if the latter, how many. The answer which Aristotle gives (1073a36 ff.) is that there are just as many such substances as there are circular locomotions for the heavenly bodies. Like the Prime Mover, their nature is eternal and they are unmoved *per se,* while also being without magnitude. The latter characteristic of immaterial being has already been shown to belong to the Prime Mover at the end of XII.7 (1073a5–11) and, presumably, it is now being attributed to the other unmoved movers because they also move things for an infinite time.

Aristotle defers to the science of astronomy in connection with the subsidiary question of just how many unmoved movers exist, since he thinks that the answer to this question depends on how many eternal circular locomotions there are. Thus he seeks an answer from another science besides philosophy:

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[101] Ps.-Alexander (*in Metaph.* 706.32) insists that, contrary to what the language of *De Caelo* (292a20 ff.) implies, the moving causes of the planetary motions are not identical with the souls of the planets. Yet, if we apply here the teleological model for the action of the Prime Mover on the first heaven, it is obvious that the planets would not move at all unless their souls desired to imitate the immobility of their unmoved movers. Thus, while the souls of the planets are immanent in them, their final causes transcend them just as the divine transcends the first heaven. Ross (1924) ii, 384 points out, however, that Aristotle nowhere speaks explicitly about the souls of the planets, even though he ascribes life and action to them; cf. *Cael.* 292a20 & 286a9–12.

> Now as regards the number of locomotions, this should be the concern of the mathematical science which is closest to philosophy, and this is astronomy; for it is this science which is concerned with the investigation of sensible but eternal substances. While the others, such as arithmetic and geometry, are not concerned with any substances.[102]

While this passage clearly presupposes the sort of differentiation of the sciences according to subject-matter which one finds in the *Posterior Analytics,* it also blurs these distinctions somewhat. For instance, he describes astronomy as the mathematical science which is 'most at home' (οἰκειοτάτη) in philosophy and justifies this description in terms of their shared objects of study. Among all the mathematical sciences what brings astronomy closest to philosophy is the fact that it studies the heavenly bodies which are eternal sensible substances. By contrast, other mathematical sciences like arithmetic and geometry are not concerned with any substance (περὶ οὐδεμιᾶς οὐσίας).

If this remark about the ontological status of the objects of arithmetic and geometry is a reference to the discussion in XIII.2–3, it would support my argument about the logical order of these treatises. Still one cannot overlook some important differences between the detailed discussion in Book XIII and these isolated remarks in XII.8 where, for instance, there is no trace of the 'qua' locution that was an essential part of Aristotle's differentiation of the sciences both in *Metaphysics* VI & XIII and in *Physics* II. Yet perhaps such differences can be explained away in terms of the different concerns in XII.8, where he considers the mathematical sciences only from the point of view of the proximity of their subject-matters to that of theology or first philosophy. Furthermore, Aristotle seems to have in mind only a specific question; namely, how many supersensible substances exist. Since this is a question involving quantity, it makes sense for him to look towards the mathematical sciences for an answer, and especially to astronomy which concerns itself with the mathematical aspects of heavenly bodies. Such an appeal to the conclusions of astronomy seems to confirm the significant dependence of Aristotle's metaphysics upon his cosmology, particularly in *Metaphysics* XII.

I do not propose to discuss the details of the astronomical systems of Eudoxus and Callippus, which are summarized by Aristotle in XII.8. From my point of view it is more important to notice the relationship between his theology and contemporary astronomy. For

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[102] *Met.* 1073b3–8: tr. Apostle (1966).

instance, in support of his claim that there are many locomotions of the heavenly bodies (and therefore many unmoved movers), Aristotle appeals (1073b8–10) to the hypotheses of the astronomers who attribute more than one locomotion to each of the planets. As to exactly how many locomotions there are, he leaves this an open question which is tentatively answered for present purposes but which may receive a better answer through further inquiry. He expresses (1073b10–17) respect for all views, along with a willingness to be persuaded by the most accurate accounts (τοῖς ἀκρίβεστεροις). In fact, after rehearsing the accounts of Eudoxus and Callippus, Aristotle puts forward (1074a1 ff.) an alternative account which tries to preserve the phenomena (τὰ φαινόμενα . . . ἀποδώσειν) by positing a counteracting sphere for each of the planets. Depending on whether or not one adds such spheres for the sun and moon, the total number of spheres comes to either 55 or 47 by Aristotle's reckoning.

But no metaphysical significance seems to be attached to the exact number of such spheres with circular locomotion which are required to save the phenomena more precisely, since Aristotle treats that as something to be ascertained by the mathematical science of astronomy. For the purposes of his own theology the best estimate is sufficient, since it is reasonable (εὔλογον) to assume that there are just as many unmoved principles. Yet, perhaps anticipating an objection, Aristotle goes on (1074a17 ff.) to argue for that assumption in the following way: (1) If there is no locomotion (in the heavens) which does not contribute to the locomotion of a star, and (2) if every nature and every substance which is impassive (ἀπαθῆ) and which has achieved the best by itself ought to be considered an end, then (3) there cannot be any other nature besides those mentioned and (4) this must (ἀνάγκη) be the number of the substances.

The necessity claimed for the final step must be logical, since Aristotle has already (1074a15–17) assigned to astronomers the determination of what is empirically the case with respect to the number of circular locomotions of spheres. The argument here concludes that, whatever that number turns out to be, there must be an equivalent number of unmoved movers which are the teleological causes of these circular locomotions. Behind this argument lies an implicit principle of economy which excludes any causes that do not contribute to the motions of the heavenly bodies.

By way of conclusion to the self-contained treatise that constitutes XII.8, Aristotle refers to the ancient myth about the divinity of the

heavenly bodies in a way that throws light on his own cosmological and philosophical inquiry. He talks (1074b1 ff.) about a myth being handed down by the very ancient thinkers to the effect that the stars are gods and that divinity encompasses the whole of nature. Since it reflects fairly accurately Aristotle's own attitude towards the heavens and towards nature as a whole, however, he isolates (1074b9–10) this part of the tradition as something that was divinely spoken. It is interesting to notice that he dismisses the anthropomorphic element of that tradition as a later accretion with merely human purposes, such as the persuasion of the masses or the preservation of law and order. Aside from those humanly motivated aspects of the tradition, however, Aristotle thinks (1074b9) that there is something divinely inspired about the notion which he identifies as primordial (τὸ πρῶτον); namely, the idea that the first substances are gods.[103]

But this reformulation of what he takes to be fundamental in the mythopoetic tradition tells us more about his own thinking than about any historical thinkers. The basic idea extracted from the tradition may also be taken as a summary of his own cosmological and metaphysical conclusions in *Metaphysics* XII; i.e. that there are supersensible substances which are prior to sensible substances and which are divine. Although he reaches these conclusions by quite a different route, still the mythopoetic tradition has a certain bearing on his philosophical method. The fascination with origins and beginnings that is typical of his thinking may be encapsulated in the plausible myth about cycles of artistic and scientific discovery which he introduces at the end of XII.8 (1074b10–14). I think we should take it quite seriously, since we find a similar myth about golden ages and periodic destructions being repeated several times by both Plato and Aristotle.[104] Perhaps it provides a general rationale for Aristotle's method of beginning every inquiry with a review of the opinions of his predecessors, so as to sift out the grain of truth which represents divine inspiration in all human thought. In fact, it would appear that his dialectical method is grounded on an almost mystical belief in the possibility of recovering the truths which have already been discovered and lost by many previous generations.

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[103] See also Verdenius (1960) on traditional elements in Aristotle's religion.

[104] Cf. *Tim.* 22C–23B, *Crit.* 109D ff., *Laws* 676A–677D, *Cael.* 270b19, *Meteor.* 339b27, *Pol.* 1329b25.

VI. *The best order*

If my interpretive hypothesis is correct, then XII.10 may be treated as another self-contained treatise dealing with the order of the universe. Again he identifies the topic for inquiry in the standard form of an aporetic question as follows: whether the nature of the whole possesses the good and the best as something separated and by itself or as the order of its parts.[105] But he also introduces his own view as a third possibility; i.e. that the universe possesses the good and the best in both ways, as something separate and as something integral to the order of the parts. He clarifies (1075a13–15) this view by way of analogy with an army whose goodness consists in its order (ἐν τῇ τάξει) and also in its leader.

This metaphor is highly appropriate for the order of the universe which Aristotle envisions in his cosmology and metaphysics, since the leader is prior to the army in many different senses just as the unmoved mover is prior to the universe. This is indicated very briefly when Aristotle says that the leader does not exist because of the order but rather it exists because of him.[106] This involves an implicit appeal to the Platonic criterion of priority with respect to nature and substance (cf. *Met.* 1019a1–4). But the example of the leader also provides a good illustration of priority with respect to power (κατὰ δύναμιν), since he exceeds the army in power by virtue of the fact that its movement and order depends on his decision (cf. *Met.* 1018b22–26). In addition, this example involves priority with respect to motion (κατὰ κίνησιν) because the leader is a first principle of motion in the army (cf. *Met.* 1018b20–21). Perhaps, as Apostle (1966, 407) suggests, one can also say that the leader is more honorable because the soldiers could attain no good without him. This would correspond to another sense of priority by nature which is described in *Categories* 12 (14b4–8) as the 'most estranged' (ἀλλοτριώτατος) sense of priority. In any case, the coincidence of different senses of priority in the leader of an army gives a most suitable parallel for the relationship between the Prime Mover and the rest of the universe.

When Aristotle first states (1075a16) that all things are ordered in some way (πως), this might be taken as a commonplace of Greek

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[105] ποτέρως ἔχει ἡ τοῦ ὅλου φύσις τὸ ἀγαθὸν καὶ τὸ ἄριστον, πότερον κεχωρισμένον τι καὶ αὐτὸ καθ' αὑτό, ἢ τὴν τάξιν, *Met.* 1075a11–13.

[106] οὐ γὰρ οὗτος διὰ τὴν τάξιν ἀλλ' ἐκείνη διὰ τοῦτόν ἐστιν, *Met.* 1075a15.

cosmology but, when he subsequently explains (1075a18–19) that all things are ordered in relation to one thing (πρὸς ἕν), this is clearly a statement of his own particular view. Just like the leader of an army, the Prime Mover is seen by Aristotle as the focal point of order and unity in the whole universe. Contrary to the Pythagorean and Platonic traditions, however, he insists that this ordering is not mathematical in its essentials, even though ἀριθμός can refer to a military array. The qualitative character of this order becomes much clearer through the second analogy (1075a19–25) with the internal order of a household. Here a contrast is made between the freemen, who have the least room to act at random, and slaves who act randomly for the most part. The point of the contrast is that slaves and animals contribute least to the common good (τὸ κοινόν), whereas the freemen contribute most because all or most of their affairs are ordered.

The implication of the whole analogy for Aristotle's cosmology can, I think, be elucidated as follows: just as the head of a household is related to its subordinate freemen, so also the Prime Mover is related to the other movers and heavenly bodies which move with great regularity and thereby contribute to the order and goodness of the universe. Similarly, just as the freemen are superior to the slaves and animals in the household, so the divine heavenly bodies are superior to sublunary creatures which are corruptible and whose actions are more subject to chance. Thus things in the universe are ordered according to their natures since, as Aristotle puts it, "such is the principle (of action) of each of these as is their nature."[107]

In contrast with the Parmenidean universe, therefore, the Aristotelian cosmos contains diversity and change along with unique and immutable beings. But the whole is unified by an ordering relationship between the parts, just as an army or a household is united as a single entity. Aristotle thinks of the whole universe as being coordinated and harmonized from the Prime Mover down to inanimate things, even though all things act in different ways according to their respective natures.[108] This seems to be what he has in mind when he briefly explains (1075a23–25) the role of those things which contrib-

[107] τοιαύτη γὰρ ἀρχὴ ἑκάστου αὐτῶν ἡ φύσις ἐστίν, *Met.* 1075a22–23.

[108] Kahn (1985) has argued convincingly that the Prime Mover functions in Aristotle's universe as a principle of actuality that is ultimately responsible for each sublunary and superlunary thing moving from potentiality to actuality according to its particular nature. In each case the causality of the Prime Mover does not act to override but to complete the immanent causality of the thing's specific nature.

ute to the whole by being dissolved so that other things may be generated. In a rather obscure conclusion, he indicates that it is by a similar division of labor that all things participate in the whole.[109] Presumably, he is referring back to the nature of each thing as the principle for such a division.

Having set down his own view about how the universe incorporates the Good, Aristotle critically reviews (1075a25 ff.) the opinions of others on the same question. While we are accustomed to finding such a review being used to introduce his own position, we see it here playing a different role which is also consistent with the dialectical method of 'saving the phenomena.' Typically, Aristotle uses the opinions of his predecessors to highlight difficulties which his own view can resolve. Thus, according to his concept of dialectical proof, the objections against the views of his opponents serve to confirm the correctness of his own theories.[110]

I think this is the basic motivation behind the lengthy discussion of opinions with which Aristotle concludes XII.10. In this light we can understand his insistence (1075a25–26) that 'one must not fail to notice' (δεῖ μὴ λανθάνειν) how many impossible and absurd conclusions follow for those who hold views different from his own. We may also assume that Aristotle is interested in the views which are faced with 'the least difficulties' (ἐλάχισται ἀπορίαι) because these would provide the material for his own position. Presumably, that would also explain his interest (1075a26) in what sort of views are put forward by 'subtler thinkers' (οἱ χαριεστέρως λέγοντες). This brief review therefore serves a similar purpose as Aristotle's extended discussion in *Metaphysics* I of the views of his predecessors on the principles and causes of things.

The first opinion which he reports as shared by all thinkers is that all things come from contraries (ἐξ ἐναντίων), though he immediately objects that they are not right to speak either of 'all things' or of 'from contraries' (1075a28–29). Presumably the exception is eternal substance as discussed by Aristotle in previous chapters.[111] The second

[109] καὶ ἄλλα οὕτως ἔστιν ὧν κοινωνεῖ ἅπαντα εἰς τὸ ὅλον, *Met.* 1075a24–25.

[110] Cf. *De Caelo* I.10, 279b6–7, *Nicomachean Ethics* VII.1, 1145b2–8.

[111] Ross (1924) ii, 402 offers a similar conjecture, while referring to 1069a30 and 1071b4. But the most apt parallel is with XIV.1 (1087a29 ff.) where Aristotle claims that everyone makes their principles contraries both for changing and unchanging things. While he accepts opposites as essential principles for changing things, he adamantly refuses them any explanatory role for eternal and supersensible things.

part (1075a29–30) of the objection is based on the alleged failure of these thinkers to explain how the things to which contraries belong can come to be from contraries. The difficulty is that contraries cannot be acted on by each other, as Plato had also noticed in the *Phaedo* (102E–103B). But Aristotle assumes that the problem can be resolved by positing the existence of some third thing (τρίτον τι); i.e. the substratum; cf. *Met.* 1069b6, 1087a29 ff., & *Physics* I. Against the background of such a solution, he criticises (1075a32) the views of those thinkers who posit one of the contraries as matter. He mentions (1075a32–33), for instance, those who make the Unequal the matter for the Equal (τὸ ἄνισον τῷ ἴσῳ) or those who regard the Many as the matter for the One (τῷ ἑνὶ τὰ πολλά).

It is possible that Xenocrates and Speusippus are being referred to in these examples.[112] If this is the case then Aristotle is competing against his closest rivals who proposed other models for the best and most beautiful cosmos. He claims that he can solve the difficulties encountered by these thinkers because he does not make matter contrary to anything.[113] For instance, they run into the additional difficulty of being committed to making everything but the One (which is identified with the Good) fall under the heading of the Bad Itself; cf. 1075a35–36.[114] It is unclear whether all Platonists would be committed to this but there seems to have some basis in the Pythagorean tradition for placing the Many and the Unequal under the rubric of the Bad, as distinct from the Good which encompasses the One and the Equal; cf. *Met.* 988a14. Among the other unnamed thinkers, who are said (1075a36–37) not to consider the Good and the Bad as principles (ἀρχαί), is included Speusippus who is reported elsewhere (1072b30–34) to have held that the most noble and the best things are not in the principle.[115]

[112] Apostle (1966) 407 and Ross (1924) ii, 402 cite the following as parallel passages: *Met.* 1088b28–33, 1087b5, 1089a35–b8, 1091b35–37. But Ross thinks that the second example is better fitted to the doctrine of Speusippus; cf. 1085a9, b5.

[113] ἡ γὰρ ὕλη ἡμῖν οὐδενὶ ἐναντίον, *Met.* 1075a34. In *Metaphysics* XIV.1 (1087a29 ff.), Aristotle argues that some substratum must be prior to any change that takes place between contraries. Therefore, he concludes, something other than a contrary must be the principle of everything in the proper sense.

[114] There is an exact parallel in XIV.4 (1091b35 ff.) where Aristotle argues that if the Good and the One are identical, as some Platonists claim, then they are committed to some absurd consequence; e.g. that all things partake of evil, except for the original One, and that numbers partake of evil in a stronger form.

[115] According to Aristotle's reconstruction in XIV.4 (1091b20 ff.), the identification of the Good with the One led to many difficulties, which Speusippus tried to avoid

But whoever held such a view is obviously mistaken, according to Aristotle, since in all things the good is a principle to the highest degree.[116] Perhaps he is here referring to the Prime Mover, which is the final cause of the whole cosmos, or just to the natural teleology within the universe. In either case, it is very clear that Aristotle insists upon the Good as a principle and thus he praises (1075a38–b1) those thinkers who accept this view. Yet, on the other hand, he finds fault with them for not specifying how it is a principle; i.e. whether as an end (ὡς τέλος) or as a mover (ὡς κινῆσαν) or as form (ὡς εἶδος). This familiar critique of his predecessors (i.e. that they failed to clarify the formal, efficient, and final causes) is based on Aristotle's assumption that he himself has avoided all difficulties about the Good, when he makes the Prime Mover a final cause of order in the universe.

Finally, at the end of XII.10, he criticises those contemporary opponents whose conception of the universe he is most anxious to reject:

> Again, in virtue of what is a number one, or a soul, or a body, or in general, each form or thing? No one says anything at all; nor can any of them say anything, unless they do in the way we do, that it is the mover that makes each one. As for those who assert that mathematical numbers are first and, following these, posit one kind of substances after another with distinct principles for each kind, they represent the substances of the universe as a plurality of unrelated parts (for substances of one kind, whether existing or not, contribute nothing to those of another kind) and with many principles; but things do not wish to be governed badly. "The rule of many is not good; let one the ruler be."[117]

It is clear from this passage that Aristotle considered all of his predecessors to have failed in the attempt at finding a satisfactory principle of unity both for individual things and for the cosmos as a whole. This is what I take to be the point of the question with which he begins the passage. In fact, he seems to think that there is only one possible answer and that this has already been given in terms of a moving principle that unifies soul itself (e.g. Nous) or that unifies

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by making the good posterior and by positing the one as a principle and element of mathematical number only.

[116] καίτοι ἐν ἅπασι μάλιστα τὸ ἀγαθὸν ἀρχή, *Met.* 1075a37. In XIV.4 (1091b17 ff.) Aristotle says that it would be surprising if what is primary and eternal and most self-sufficient did not possess self-sufficiency and self-maintenance as a good. This claim seems to apply especially to the Prime Mover.

[117] *Met.* 1075b34–1076a4: tr. Apostle (1966).

the composite of soul and body or that makes the universe one (i.e. the Prime Mover). Such utter confidence in the power of his own solution leads Aristotle to criticise Speusippus rather harshly as an incompetent cosmologist.

While it is very difficult to reconstruct the latter's mathematical cosmology from Aristotle's brief and unsympathetic presentation, it seems clear that Speusippus gave primacy to mathematical numbers rather than Form Numbers (which he rejected). Throughout Aristotle's various attacks on the Platonists, this view is consistently ascribed to Speusippus, by contrast with Plato who gave primacy to Form Numbers; cf. *Met.* 1090b14 ff. It would appear that Speusippus posited the principles of number (e.g. One and Plurality) as the principles of all things, and it seems that he also used a hierarchy of principles as dictated by the series: number, point, line, plane, and solid. Thus there would be distinct material principles for lines (e.g. the long and the short) and for planes (e.g. the broad and the narrow) and for solids (e.g. the deep and the shallow). I think it must be some such schema that Aristotle has in mind when he reports that these thinkers who give primacy to mathematical numbers also posit subsequent kinds of substance with distinct principles for each kind.[118] We saw from XIII.1–2 (1076a16 ff.) that he considered some Platonists to be positing as substances such mathematical entities as solids, planes, lines and numbers.

In XII.10, however, what he criticises about the mathematical cosmology of Speusippus is the lack of connection between different levels on the hierarchy of numbers, lines, planes, and solids. It is this alleged character of the system which he describes as 'episodic' (ἐπεισοδιώδη) because each kind of substance, whether it exists or not, does not contribute anything to the other kinds.[119] Thus Speusippus is accused (*Met.* 1076a1–3) of fragmenting the substance of the universe (τὴν τοῦ παντὸς οὐσίαν) and of giving it many principles (ἀρχὰς πολλάς). Here I am not so much concerned with the

[118] ἄλλην ἐχομένην οὐσίαν καὶ ἀρχὰς ἑκάστης ἄλλας, *Met.* 1075b38.

[119] οὐδὲν γὰρ ἡ ἑτέρα τῇ ἑτέρᾳ συμβάλλεται οὖσα ἢ μὴ οὖσα, *Met.* 1076a2. Cf. also XIV.3, 1090b14 ff. where Aristotle uses the criterion of non-reciprocal dependence to show that the Speusippean levels of reality do not hold together, unlike the Platonic universe which is coherent because it is based on a schema of priority. In *Poetics* I.8–10 Aristotle discusses the construction of a good plot in tragedy, and describes an episodic plot as one which has neither necessity nor plausibility in its sequence; cf. 1451b34–5.

justice of this accusation as with what it tells us about Aristotle's conception of his own cosmological system. For instance, when he says by way of criticism of Speusippus that "things do not wish to be governed badly," we recall the political metaphor for good order used at the beginning of XII.10. The quotation from Homer (*Iliad* ii, 204) simply confirms Aristotle's point that the best and most beautiful cosmos is that which is united by a single principle (ἀρχή). The play on the word ἀρχή seems quite deliberate because Aristotle thinks that, just as in an army, the leader or first principle is the source of order.[120] For him, therefore, the highest good is not some entity like Plato's Form of the Good but is both the transcendent unmoved mover *and* the inherent order of the universe for which that first principle is responsible.

Conclusion

With particular reference to *Metaphysics* XII, I have tried to clarify some aspects of Aristotle's metaphysics and cosmology, while showing the integral connection of these two types of inquiry. Just like his Platonic protagonists, Aristotle is concerned with the question about the good of the whole and the highest good in the universe, though he rejects the mathematical approach to the problem which was typical of the Academy under Speusippus. Aristotle resists this Pythagorean influence on the Platonic tradition, even though he accepts the challenge of the *Phaedo* to make the world intelligible in terms of the purposes of an ordering Mind. From such a teleological perspective, therefore, we can see that he belongs in the Platonic camp rather than to the tradition of natural philosophy which is embodied in Democritus.[121]

Where Aristotle parts company with Platonism is on the question of how the goodness and order of the universe is to be conceptualized and logically elucidated. From reports about Plato's famous lecture on the Good it would appear that his conceptual model for cosmological order is borrowed from mathematics; e.g. the derivation of structural order from the One (τὸ ἕν) which is identified with the highest good. This priority of the good is reversed by Speusippus,

[120] Cf. *Met.* 1075a12–25, *APst.* 100a13.
[121] Cf. Furley (1987) & (1987b).

who seems to have posited distinct principles for each level of derivation according to the standard mathematical schema: number, point, line, plane, and solid. But it is just such an ordering of the universe which Aristotle criticises as 'episodic' at the end of XII.10 and which leads him to castigate those who turn philosophy into mathematics.

Thus it may have been the Platonic tradition as carried on by Speusippus and Xenocrates which prompted the dispute over whether mathematics or physics should be given priority in cosmological speculation.[122] It is clear that Aristotle's commitment to the ontological primacy of natural over mathematical entities has a decisive bearing on this general question. This is evident, for instance, from the way he begins in XII with a survey of the principles of physical substances, in preparation for the move to supersensible substances through the argument from eternal motion. Since mathematical objects do not have an inherent principle of motion, their principles are rejected in favor of unmoved movers which guarantee the eternal order of the universe by causing an unending cycle of motion. The best that mathematics can offer by way of a principle of unity is proportional analogy, which involves the relationship of different things without a central focus or terminus.

Even though Aristotle uses this sort of analogy in XII.4 to describe the way in which the principles of sensible things are related, he appeals to a different kind of analogy in XII.10 when he addresses the question of how the universe is ordered for the best. There he falls back on the logical and conceptual structure that was attributed to being in *Metaphysics* IV in order to explain how first philosophy can be a single science. This is what was called 'attributive analogy' by Scholastic logicians, since it involves a central or focal meaning for a word like 'health' to which all other uses of the word refer.[123] In the case of being Aristotle claims that its primary and focal meaning is substance and, within the order of substances, that the first substance is the divine. Given his metaphysical framework, therefore, it is quite consistent for him to identify the highest good with the divine being that serves as a principle of order in the universe. And that is precisely what we find him doing in XII.10 when he says (1075a23) that all things have been ordered with reference to one thing (πρὸς ἕν). But the metaphors drawn from the ordering of an

[122] Cf. Gadamer (1986) ch. 3.
[123] Cf. Aubenque (1986).

army or of a household show that the good of the universe is not merely the transcendent principle but also consists in the immanent order which the first mover brings to the whole.[124]

What I have tried to bring out clearly in this chapter is the integral connection between the question about being and the question about the good. The connecting link is the question about whether the ultimate principles of reality are mathematical or physical. By way of answer to the aporia about whether or not One and Being are the substances of things, Aristotle replies in the negative when these are taken to be mathematical principles (as was the case with Platonists like Speusippus) but with a qualified affirmative when they are taken in their primary senses as essential characteristics of an unmoved mover. We have seen how this question is also related to the issue of what kinds of supersensible substance are to be posited so as to complete the cosmos. While Platonists thought it necessary to postulate either Forms or Mathematical Objects as separate from sensible things, Aristotle felt such transcendent entities to be useless because they did not supply principles of eternal motion or of order for the cosmos. Thus he posited one kind of separated substance whose transcendence has an entirely different function in the ordering of the universe from that given to Forms and Mathematical Objects. Whereas these were thought by the Platonists to be the permanent substances of sensible things, Aristotle completely separates the unmoved mover from the realm of change, even while making it the final cause of motion in the universe.

[124] Gadamer (1986) 157–8 takes the conclusion of XII.10 to mean that Aristotle decides against mere order and in favor of a good existing apart for itself. But I find it difficult to accept that both metaphors support a transcendent interpretation of the Prime Mover.

CHAPTER SEVEN

ARISTOTLE'S PHILOSOPHY OF MATHEMATICS

Although Aristotle's reflections on mathematics should be seen within their proper metaphysical and cosmological contexts, one may still ask whether or not he had a coherent philosophy of mathematics. One typical approach is to compare Aristotle's views with some standard views in contemporary philosophy of mathematics. Within such a hermeneutical[1] perspective, one might consider whether Aristotle was a platonist, a logicist, or an intuitionist, as these views are now understood.[2] But the difficulties we encounter in trying to locate him within any of these well-defined modern positions forces us to reconsider their assumptions, among which is the Kantian presupposition that it is through some constructive activity of the human mind that mathematics is possible.[3]

In order to understand Aristotle's problematic, it may be more illuminating to ask: What is it about mathematical objects that enables the mind to know them with the universality and precision that is found in the mathematical sciences? In addition, one may raise both epistemological and psychological questions about Aristotle's theory of scientific knowledge. Thus, although the things to be contemplated (τὰ θεώρηματα) are eternal, one might ask about the contribution of the mind to the 'generation' of theoretical sciences like mathematics with its idealized or abstract objects. This question also defines his position with reference to the Platonic tradition of correlating mathematical objects and the soul as intermediate entities.

I. *Epistemological problems*

Together with the ontological and cosmological problems associated with mathematics, some attention must also be given to Aristotle's

[1] The term 'hermeneutical' here refers to Gadamer's (1960) thesis that there is an inescapable prejudice in all our attempts at interpreting the historically distantiated thinking of the Greeks.

[2] See 'Introduction' in Benacerraf & Putnam eds. (1983)

[3] David Lachterman (1989) makes a convincing case for identifying the Kantian

discussion of related epistemological problems. Having rejected the privileged position among the sciences that Plato gave to mathematics, he should provide an alternative account which will be consistent with the ontological status that he ascribes to mathematical entities. Thus, for instance, the claim that they are dependent on sensible substances would be quite compatible with his empiricism, though it is not obvious how mathematical forms become accessible through sense perception. Indeed it was their conspicuous alienation from sensible things that led Plato to propose mathematics as a discipline for converting the soul from the sensible to the ideal realm in the *Republic*. Aristotle himself concedes that mathematicians treat their objects of inquiry *as if* they were separated from sensible things, while denying the ontological implications which the Platonists drew from this procedure. Yet it remains as one of the phenomena to be saved by his epistemological account of how such objects become accessible to the mind for theoretical inquiry.

In reconstructing Aristotle's account, therefore, we begin with his distinction between the three theoretical sciences whose characteristic modes of knowing are closely allied. If there is an exact correspondence between modes of being and ways of knowing then the objects of theoretical sciences may be distinguished from each other, and from the objects of practical and productive sciences. In this way the mode of knowing that is typical of mathematics must be elucidated by comparison with those of the other theoretical sciences, namely, physics and first philosophy.

According to Philip Merlan[4] the tripartite division of theoretical sciences mirrors the Platonic distinction between Forms, Intermediates and sensibles.[5] He also claims that Aristotle only accepted such a division of the sciences so long as he held the Platonic view of mathematicals as independent substances; so that on rejecting the 'realistic' interpretation of mathematicals, he gave up the tripartition which placed mathematics between physics and theology.[6] Since this conflicts with the textual evidence, however, Merlan suggests that it was due to 'some kind of inertia' that Aristotle retained the three

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assumption about the constructive capacity of the human mind as the distinctive feature of the post-Cartesian approach to the sciences, in contrast to the ancient Greek approach.

[4] Cf. Merlan (1954) 59 ff.

[5] Cf. *Met.* 987b14–18, b28–29, 1028b19–21, 1059b6–8.

[6] Cf. Merlan (1954) 60 & (1946).

branches of knowledge, even though he denied the 'subsistence' of mathematicals.

With this interpretive problem in mind, let us reexamine *Metaphysics* VI.1 which contains his most detailed discussion of the tripartite division of the theoretical sciences. After a brief discussion of how the mode of being[7] of objects studied by the science of physics guides the way in which they are defined, Aristotle introduces (1026a6 ff.) mathematics and theology as the other theoretical sciences. Merlan (1954, 62) thinks that this tripartition is obviously confused because it is based on two principles; i.e. *ratio essendi* and *ratio cognoscendi.* He claims that while 'physicals' and 'theologicals' differ according to their 'modus of existence' this is not the case for 'physicals' and 'mathematicals,' since the latter have no existence of their own; so that they differ only *formaliter.* Merlan holds that 'mathematicals' differ from 'physicals' only in the way they are considered, which he identifies as 'the way of abstraction' (ἐξ ἀφαιρέσεως). Thus, instead of a genuine tripartition, he finds only a dichotomy of objects along the lines of moved / unmoved, with the former class being subdivided between physics and mathematics. While physicals are moved and are being considered as moved, mathematicals are also moved but are being considered as unmoved.

Such a reading, however, isolates this brief passage from its broader context. The immediate context for the apparent tripartition is the discussion of physics as a theoretical science (1025b18 ff.).[8] As distinct from the practical and productive sciences, the objects of physics have an *internal* principle of motion and rest. So physics is a special (rather than a general) theoretical science which conducts its inquiry about particular kinds of being that have the capacity to be moved. This science also deals for the most part with substance in the sense of form which is not separable (οὐ χωριστήν) from matter (1025b27–28).[9]

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[7] By 'mode of being' (πῶς ἐστί) here, Aristotle means the relationship of a form to matter in a composite entity, as distinct from the existence of the composite. While the composite substance may be separated spatially (κατὰ μέγεθος), the form may be separated only logically (κατὰ λόγον) in the case of non-substantial composites but may have real definitional separation in the case of substantial composites where the form is identical with the essence; cf. *Metaphysics* VII.6.

[8] Edward Halper (1989) 4 thinks that Aristotle makes the possibility of physics being a theoretical science depend on it seeking knowledge of essences, by contrast with practical and productive sciences which are oriented towards actions and artifacts, respectively.

[9] I follow Ross (1924) i, 354 in taking 'for the most part' with the verb implicit

Aristotle's subsequent methodological remark elucidates the point of his discussion: "We should not overlook the mode of being of the essence and thus of the definition, since without this the inquiry achieves nothing."(1025b28–30) Here the leading question is about the mode of being of the essence (τὸ τι ἦν εἶναι) that is expressed by a definition.[10] Thus οὐ χωριστήν describes the mode of being of physical forms as not separable (from matter), and this is clarified by the subsequent distinction between the snub (τὸ σιμόν) and the concave (τὸ κοῖλον). Such a distinction is said by Aristotle to hold 'among things defined or among whatnesses' (τῶν ὁριζομένων καὶ τῶν τί ἐστι).[11] The difference between these definienda is that the snub is taken along with the matter, whereas the concave is taken without sensible matter (ἄνευ ὕλης αἰσθητῆς).[12]

Aristotle here (1025b33 ff.) uses snubness as his paradigm case of a physical form whose special relationship to sensible matter is reflected in defining the snub as a concave nose. From this he thinks it is clear how one should inquire about the what-it-is (τὸ τί ἐστι) in physical things and how one ought to define it. In other words, as he puts it (1026a3), the definition of any natural form under the genus of animal or plant must contain some reference to change or motion, since it always has matter (ἀεὶ ἔχει ὕλην). Such a definition reflects the mode of being of the essence insofar as it has a necessary connection with a certain kind of matter; e.g. snubness is always concavity in a nose and the nose is a certain form in flesh and bone.

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in the περί construction. The Platonic justification for differentiating some sciences as theoretical was that they contemplated the eternal Forms or Intermediates, as distinct from the sensible and changing things of this world. But Aristotle makes a science of such changing things possible by distinguishing between the material and formal objects of physics, although physics has only hypothetical necessity because it must deal with forms in matter.

[10] Every essence seems to have a definite ontological status that is reflected in its definition. For instance, the logical connection of a physical form like snubness with sensible matter seems to imply that it cannot exist like a Platonic Form that is numerically distinct from particular sensible composites of form and matter. Similar implications may follow from the connection of mathematical form with intelligible matter in mathematical particulars. However, a substantial form like the human soul may be independent in definition from matter, precisely because it is identical with its essence and does not depend on a different material substratum.

[11] Here, I think, the καί is epexegetic, since the objects of definition in every science are essences. So each science can be distinguished in terms of its characteristic essences and their relationship to matter.

[12] This independence must be conceptual because, although concavity is logically independent of sensible matter, it is definitionally dependent on some intelligible matter like extension.

Usually the matter is associated with a potency for change, which Aristotle sees as a characteristic aspect of all natural things, and this is the reason why he says that the science of physics deals for the most part with the kind of substance which, with respect to definition (κατὰ τὸν λόγον), is inseparable from matter (οὐ χωριστήν); cf. *Met.* 1025b27–28.[13]

Within the context of this whole discussion of the mode of being of physical forms, it is plausible to take χωριστήν as having a logical sense primarily.[14] Such a logical meaning seems to be what Aristotle has in mind above (1025b28–30) when he talks about the importance for every inquiry of clarifying the mode of being of the essence or of the definition. For instance, the whole character of physical inquiry is dictated by the fact that its objects have an internal principle of change which is related to the kind of matter that underlies physical forms.[15] By implication, therefore, the character of mathematical inquiry will also be dictated by the logical relationship which its objects of inquiry bear to matter. However, when Aristotle says (1025b33–34) that concavity is without sensible matter, it is unclear whether concavity is logically related to another kind of matter or whether it is completely unrelated to matter.[16] Given Aristotle's remarks elsewhere (in *Metaphysics* VII.10–12) about the relationship between intelligible matter and geometrical forms, the first alterna-

[13] Thus I take *Metaphysics* VI.1 to be differentiating between theoretical sciences in terms of the mode of being of the essences studied, as determined by their relationship to some kind of matter. In VII.6, with regard to the question of whether each thing and its essence are the same or different, Aristotle emphasizes that knowledge of each thing involves knowing its essence; cf. 1031b5–6, 18–21. Within the same context, he concludes (1032a4–5) that each primary and self-subsistent thing (such as Platonic Forms were held to be) is one and the same as its essence. This means that such essences can be known by first philosophy without reference to an independent material substratum, by contrast with physical and mathematical essences which involve some reference to matter either by addition or subtraction; cf. Cleary (1994b).

[14] By contrast, recent accounts of χωριστός emphasize its spatial sense (κατὰ μέγεθον) as a criterion of substance, whether that is interpreted as 'independent existence' (Fine 1984) or as 'outside the ontological boundaries of' (Morrison 1985a–b). But these senses are not so relevant to a discussion of theoretical objects, whereas a logical sense (κατὰ λόγον) fits better with talk about definition.

[15] From an historical perspective, the choice of snubness as a paradigmatic physical form may be Aristotle's little joke at the expense of the Platonists, perhaps reminding them that Socrates never separated the universal. I owe this suggestion to H.-G. Gadamer (1980) 212.

[16] In some places, Aristotle simply says that mathematical objects are without matter (ἄνευ ὕλης), but I think he must mean that such 'abstract' objects are without sensible matter.

tive seems the most plausible.[17] Later, he says (1026a15) that mathematical inquiry is about things which are perhaps not separable but have a mode of being as in matter (ὡς ἐν ὕλῃ). If intelligible matter is involved here, this might throw a different light on the tripartition of theoretical sciences.

In his preliminary discussion, therefore, Aristotle seems to distinguish between the three theoretical sciences in terms of the relationship which the essences studied by them bear to matter.[18] For instance, physical forms like snubness have a necessary relationship to sensible matter, as reflected in the typical definitions given in physics. By contrast, some mathematical sciences study their objects (μαθήματα) qua unchanged and separable (ᾗ ἀκίνητα καὶ ᾗ χωριστά), even though Aristotle notes (1026a7–10) that it is presently unclear whether these objects are really unchanged and separable. This remark may point towards the discussion in *Metaphysics* XIII.1–3 about the ontological status of mathematical objects, which concludes that their mode of being is analogous to that of matter.

But it is just at this point in the text that the tripartition of theoretical sciences is introduced in the following hypothetical manner:

> If there is something eternal and unchangeable and separable, then it is obvious that the knowledge of it will be theoretical. Yet it is not physics (because that science is about some changeable things)[19] nor is it mathematics but some science prior to both. For physics deals with inseparable though not unchangeable things, while some of the mathematical sciences deal with unchangeable things which perhaps are not

[17] Among the puzzles listed in *Metaphysics* XI.1, there is one which asks which science should explore the puzzles concerning the matter of mathematical objects (περὶ τῆς τῶν μαθηματικῶν ὕλης, 1059b14). Aristotle explains that this cannot be the task of physics since its whole subject-matter is concerned with things which have an internal principle of motion and rest. Thus he concludes that the puzzles about mathematical matter must be considered by first philosophy; cf. also XI.4.

[18] My hypothesis is not indebted to the Thomistic theory of separation (metaphysics), combined with two levels of abstraction (physics & mathematics), though I would not rule this out as a possible interpretation of Aristotle's text; cf. Thomas Aquinas, 'On the Division of the Theoretical Sciences' = Q. 5. from *Expositio super librum Boethii De trinitate*. However, I would discount Auguste Mansion's (1945) interpretation in terms of three degrees of abstraction, since it fails to account for the fact that Aristotle's use of abstraction terminology is confined to mathematical objects; cf. Cleary 1985.

[19] Both Jaeger and Ross have bracketed this explanatory clause in their editions, presumably as a marginal note that does not fit well with the structure of the whole sentence. While the clause could be an interpolation, one suspects that these editors' decision to bracket it is influenced by their acceptance of Schwegler's reading of the subsequent sentence, since περὶ κινητῶν would support the MSS readings.

> separable but are as in matter. First philosophy, on the other hand, is about separable and unchangeable things. For it is necessary for all causes to be eternal, but especially these since they are the causes with respect to the appearances of divine beings.[20]

It is clear from the *protasis* of the introductory conditional that the character of a science depends upon the mode of being of its objects of inquiry, since Aristotle concludes that some theoretical science must deal with eternal and unchanging things if these exist. Yet this science cannot be physics, which treats of changing or moving things, nor can it be mathematics but rather some science that is prior (προτέρας) to both.

To understand this claim about priority, however, we must see it in terms of the search for causes of unchanging things that are completely separable from matter. This is the significance of the final reference to the causes of divine things in the visible realm; i.e. the heavenly bodies whose motion is caused by unmoved movers. From that perspective physics is the least appropriate because it inquires about objects which are inseparable[21] and also changing.[22] By contrast, mathematics seems a better candidate because some branches of it deal with objects which are unchanging and hence eternal.[23] But Aristotle seems doubtful about whether mathematical forms satisfy the separation criterion because they appear to have a mode of

[20] *Met.* 1026a10–18: author's translation based on Apostle (1966).

[21] I am reading ἀχώριστα with the best MSS tradition (codd. & Alc), despite the fact that Schwegler's (1847–48) iv, 14–16 emendation (χωριστά) is widely accepted by modern editors like Christ, Ross & Jaeger. Against this emendation, see Apelt (1891) 231, Cousin (1940) 495–6, Trepanier (1946) 206–9, Gohlke (1951) 451n51, Owens (1951) 296n44, Wundt (1953) 49, Decarie (1954) 466–8, Halper (1989) 258–59, Zekl (1990) 85n94, & Cleary (1994b).

[22] Merlan (1954) 71–2 argues against taking μὲν ἀλλ᾽ οὐκ as 'et non' because that would ignore the sense of contrast in the text and so he rejects the MSS reading of ἀχώριστα. Similarly, Ross (1924) i, 355 thinks that the balance of the sentence demands Schwegler's emendation, otherwise it would contain a false antithesis. But Apelt (1891) 231 defends the traditional reading by arguing for a different contrast in the passage between the objects of first philosophy (separate and unchanging) and those of physics (inseparable and changing) together with those of mathematics (unchanging but not separable). Thus, as Decarie (1954) 466–8 points out, the general contrast would be between objects that are completely separable from matter and those which are not. It is possible to read ἀχωριστά because, according to Denniston (1970) 21, ἀλλά is not always adversative, even with μέν.

[23] Perhaps it is significant that Aristotle is careful to distinguish between the mode of being of objects for the different mathematical sciences, especially since he asserts elsewhere (*Met.* 1073b3–8) that astronomy deals with sensible but eternal substances, whereas arithmetic and geometry do not deal with any substances. Thus, contrary to my logical interpretation, his carefully qualified remarks seem to involve an on-

being that connects them with some kind of matter. Thus he concludes that there is a first science which deals with separable and unchanging objects, and this is subsequently called theology.

If this tripartite division of theoretical sciences is to remain consistent throughout, however, χωριστός must refer to logical separation whenever it is used in this passage, otherwise we shall have to accept Merlan's claim that it is inconsistent because it contains two principles of division, one logical and the other ontological. If we take χωριστός in a simple ontological sense,[24] Schwegler's conjecture about the text at 1026a14 seems apt because the sensible things studied by physics are usually composite substances. But it is more likely that Aristotle continued to use the word in the sense established by the previous discussion of physical forms which are inseparable from matter.[25] This would be consistent with its usage in reference to mathematical forms when he says (1026a15) that they are perhaps not separable but have a mode of being as in matter.

If he is referring specifically to sensible matter this clarification of the mode of being of mathematical objects seems to mean that some of them are not separate from physical things. Thus χωριστός would be used primarily in a spatial sense and, for the sake of consistency, we should have to assume that it was used in that way about physical objects also. On this interpretation, therefore, first philosophy deals with objects which are spatially separated and unchanging; whereas physics also deals with objects that are spatially separate but changing. However, according to such an interpretation, some of the mathematical sciences (like arithmetic and geometry) inquire about unchanging objects which are not separated in this sense but which

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tological meaning for χωριστός, especially in the case of heavenly bodies and their prime movers. But I will argue that the logical relationship that a given form has to matter is what determines the ontological status of the (formal) object of the relevant science. For instance, the heavenly bodies may be the material objects of all three theoretical sciences; i.e. of physics in so far as they move in perfect circles because of *aither*, of mathematics in so far as they embody geometrical forms, of philosophy in so far as they are eternal substances.

[24] At *Metaphysics* VIII.1, 1042a28–31, Aristotle distinguishes χωριστὸν ἁπλῶς as the mode of being of a sensible composite from λόγῳ χωριστόν, which is the mode of being of the substantial form according to definition. Since there is no trace of such a distinction in VI.1, one might agree with Fine (1984) that in its unqualified use χωριστός always has an ontological sense with spatial connotations. But this will not do for talk about essences and substantial forms.

[25] Cf. Cousin (1940) 495–6 and Bonitz (1870) 131 for parallel passages; e.g. *DA*. 403b18. Edward Halper (1989) 4 ff. has offered a similar reading of this passage, though for different reasons.

are dependent upon physical things. In addition, he seems to be referring to theology in his subsequent (1026a20–1) remark that the most honorable science will deal with the most honorable genus of objects; i.e. the heavenly bodies popularly regarded as divine.

While an ontological interpretation can be made consistent in this way, it introduces two distinct principles of division by using the criteria of separation and of change.[26] It also has the unfortunate effect of dislocating the tripartition of theoretical sciences from its broader context in *Metaphysics* VI.1 where Aristotle discusses how the mode of being of an essence is reflected in its definition. But if sensible matter is a potency for certain kinds of change then perhaps there is another way of understanding why completely unchanging things are described as being separated (from matter). Furthermore, if intelligible matter does not have the capacity for physical change, one may also describe mathematical objects as unchanging though not separated. This suggests that mathematical objects have more in common with the objects of physics than with those of first philosophy, since they are composites of form and intelligible matter.

At *De Anima* I.1, Aristotle also distinguishes between the theoretical sciences in terms of the types of definition that are appropriate to them. The context is a discussion of difficulties connected with the definition of the soul, and whether it is studied exclusively by physics. For him the decisive question is whether all the parts of the soul are dependent on the body or whether any of them are peculiar to the soul itself and hence independent of body. If the first option turns out to be the case then the study of soul belongs completely to physics which deals with properties insofar as they belong to bodies of a particular kind with a corresponding kind of matter. Thus the definition of the soul will follow the typical pattern of definitions in physics; i.e. it will include both the form of soul and the appropriate matter in which it is embodied. Aristotle uses the example of anger to illustrate how the physicist should define attributes that are inseparable (ἀχωριστά) from the physical matter of animals to which they belong.

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[26] But perhaps Aristotle intended to use these as two distinct criteria in such a way that they have four possible combinations, which also correspond to actual subjects of study; i.e. theology = separable + unchangeable, some mathematical sciences = inseparable + unchangeable, physics = inseparable + changeable, astronomy = separable + changeable. I owe this suggestion about Aristotle's procedure here to an informal discussion with Markus Woerner.

In this way a contrast with the other theoretical sciences is made as follows:

> The attributes which, though inseparable, are not regarded as properties of body of a given sort, but are reached by abstraction, fall within the province of the mathematician: while attributes which are regarded as having separate existence fall to the first philosopher or metaphysician.[27]

Perhaps the most significant feature of this passage is Aristotle's use of the terminology of abstraction (ἀφαίρεσις) to distinguish the attributes studied by mathematics from those studied by physics. This is necessary because both sets of attributes are not separate (χωριστά) from physical bodies, and so the difference in their modes of being consists in the way they are studied, which is indicated by the *qua* locution. For instance, concavity is a mathematical attribute because it is not considered as belonging to a particular kind of body, unlike snubness which can only be thought of as belonging in a nose. This careful differentiation of aspects is not necessary in the case of attributes studied by first philosophy because they are studied insofar as they are separated (κεχωρισμένα) from all body and hence from matter. But it is necessary in the case of physics and mathematics because they study attributes that are inseparable from body of some kind, as the example of the tangent shows; cf. *DA* 403a10–16.

For further clarification one may consult the extensive discussion of the distinction between the theoretical objects of mathematics and physics in *Physics* II.2, which refers back to a previous discussion of different senses of 'nature' for its guiding perspective.[28] It is precisely because Aristotle holds that mathematics is not about some supersensible substances but is rather about sensible substances in some way that it is necessary for him to distinguish it carefully from physics.[29] From an historical perspective, his opposition to the Platonic conflation of physics and mathematics in the *Timaeus* should also be

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[27] *De Anima* I.1, 403b12–16: tr. Hicks (1907).

[28] Since 'nature' has already been given two senses as form and matter, the effort to distinguish physics from mathematics is perhaps guided by the assumption that mathematics is also about a kind of form that may bear some relationship to a corresponding type of matter. On the other hand, the Greek connecting words (ἐπεὶ δὲ . . . μετὰ τοῦτο) can be read as indicating simply that this is the next task to be performed after having distinguished the senses of 'nature.'

[29] At *Physics* II.1, Aristotle says (193b8 ff.) that natural forms are not separable (οὐ χωριστόν), except in respect of their definition (κατὰ τὸν λόγον). Here he seems to distinguish between logical and ontological separation for natural forms, presumably

seen as a motivating factor. Thus, in explaining the need for such a distinction, Aristotle says that physical bodies have surfaces and solids and lengths and points about all of which the mathematician inquires.[30] If mathematical objects are ontologically dependent on physical bodies and theoretical sciences are distinguished by their objects, how does physics differ from mathematics?

The problem is exacerbated in the case of astronomy because that science is mathematical, though it appears to be dealing with physical objects in the heavens.[31] Perhaps this is why Aristotle raises the additional problem of whether astronomy is a distinct science or merely a part of physics.[32] Thus he says (193b26–28) that it would be absurd (ἄτοπον) to think that the physicist should know 'the whatness' (τὸ τί ἐστιν) of the Sun or the Moon, while not knowing which of their attributes belong to them *per se* (καθ' αὑτά). Another complication is the fact that those who inquire about nature are concerned with the shape of the Moon and the Sun. Hence natural philosophers inquire about the geometrical attributes of natural bodies, while the practice of Greek astronomers shows that these mathematicians concern themselves with physical bodies in the heavens. This makes it more difficult to draw a clear distinction.[33]

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because the Platonists failed to do so; see also 193b35 ff. The ontological inseparability of natural forms (from sensible matter) is linked with the fact that they are principles of motion and of rest, just as we would expect from the subject-matter of physics.

[30] καὶ γὰρ ἐπίπεδα καὶ στερεὰ ἔχει τὰ φυσικὰ σώματα καὶ μήκη καὶ στιγμάς, περὶ ὧν σκοπεῖ ὁ μαθηματικός, *Phys.* 193b23–25.

[31] Astronomy poses a problem for Aristotle's division of the sciences based on the ontological status of their objects because it is about substances that are moved, whereas the 'pure' sciences of arithmetic and geometry are about non-substances that are unmoved. This problem shows up later in the *Didascalicus* of Albinus who lists astronomy first as part of physics and later as part of mathematics.

[32] Contrary to the usual view, Wright (1973–4) argues that the astronomy of Eudoxus went beyond geometry in trying to give physical explanations for the motion of the heavenly bodies.

[33] Geminus is reported by Simplicius (*in Phys.* 290–1) to have explained that the astronomer and physicist often set out to prove one and the same point (e.g. that the earth is spherical) but that they proceed by different roads. While the physicist starts from substance, force or final cause, the astronomer proves his propositions from the attributes of figures or magnitudes or from quantity of motion or time proper to it. In this context Geminus mentions the hypothesis that the earth moves as a proposal of Aristarchus which is designed to save the irregularity of the sun's apparent motion. He excuses such a wild hypothesis on the grounds that it is not the business of the astronomer to know what kind of bodies are naturally at rest or in motion, since that is the task of the physicist. Having adopted such first principles from physics, however, the astronomer will try to save the phenomena by proving that the heavenly bodies move either in parallel or oblique circles.

It is with reference to this specific problem about how astronomy is to be distinguished from physics that Aristotle proposes the following solution:

> Now the mathematician, too, is concerned with these,[34] but not insofar as each is a limit of a physical body; nor does he investigate attributes qua existing in such bodies. That is why he separates them, for in thought they are separable from motion; and it makes no difference, nor does any falsity occur in separating them (in thought).[35]

The passage concedes (μέν) that the mathematician is concerned with the same things as the physicist presumably because Aristotle assumes that mathematical objects are ontologically dependent on sensible things. But if physics and mathematics are both concerned with the same sensible things then the difference between them must be found in their respective modes of inquiry. I take it that this is what Aristotle is trying to express by means of the *qua* (ᾗ) locution when he says that the mathematician deals with the shape of heavenly bodies but not insofar as each is the limit of a physical body.[36] In other words, if that shape turns out to be spherical then the geometer must inquire about its attributes, though not insofar as that shape is embodied in sensible matter.

Thus, as he says (193b33), the mathematician does not investigate the attributes of a shape as belonging to such kinds of bodies; i.e. he separates (χωρίζει) his objects of inquiry from physical bodies and their characteristics. Indeed such a procedure is defended (193b33–35) as not making any difference and as not generating any falsehood because mathematical objects are separated in thought (or by thought) from change.[37] Here Aristotle seems to be pointing out that

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[34] The reference of τούτων is ambiguous but, in the light of subsequent remarks, I think it must refer to the shapes or figures of the heavenly bodies rather than to these bodies themselves.

[35] *Phys.* 193b31–35: tr. Apostle (1969).

[36] ἀλλ' οὐχ ᾗ φυσικοῦ σώματος πέρας ἕκαστον, *Phys.* 193b32.

[37] χωριστὰ γὰρ τῇ νοήσει κινήσεώς ἐστι, 193b35. It is not clear here how much depends on whether we take τῇ νοήσει as a locative or as an instrumental dative (either of manner or of means). Fine (1984) 42–3 concedes that in this passage χωριστά refers to 'definitional separation' but she argues that this cannot be the central meaning of separation for Aristotle because his use of the complement τῇ νοήσει shows that it is not its simple sense. While Morrison (1985a) does not pay any particular attention to this passage, in general he thinks that Aristotle's claim about substantial forms being separate in definition is 'a philosophical dodge that borders on being a cheat.' But I take it as being central to his distinction between first philosophy and the other theoretical sciences.

mathematical objects can be conceptually separated from the kind of matter that accompanies change, without introducing falsehood into the science of mathematics.[38]

The point is taken at the expense of those Platonists who posit separated Forms and who are unaware (λανθάνουσι) that they are doing the same as mathematicians because they are separating physical forms which are less separable entities than mathematical forms.[39] Given the leading role which the mathematical sciences played in the Academy, this can hardly mean that the Platonists were unaware of the fact that they were imitating mathematics when they separated Forms as objects of inquiry. So it must mean that they failed to realize that physical forms should not be separated from sensible matter in the same way as mathematical forms. This is the logical fact which Aristotle emphasizes when he explains that 'physicals' are entities less separable than 'mathematicals,' and illustrates the point by means of the distinction between the concave and the snub. For it is clear that such typical mathematical objects as number, line, and figure, along with their *per se* attributes (i.e. odd & even, straight & curved) are defined without reference to change (ἄνευ κινήσεως).

By contrast, just as in the case of the snub, such physical compounds as flesh, bone, and man have definitions which involve an implicit reference to change in their material aspect. For instance, man may be defined as a rational mortal animal, while animal may be described as a sensible ensouled substance that is capable of locomotion.[40] Each of these defining descriptions includes a reference to some kind of change, and Aristotle considers this to be typical of all definition in physics. So he charges the Platonists with failing to notice this logical fact, presumably because they were so enamored of the mathematical approach to nature. Ross (1936, 507) thinks that this departs from Aristotle's usual charge against the Platonists; i.e. that they assign separate existence to the Forms. But, in the light of his distinction between physical and mathematical forms, perhaps separation should be understood in terms of greater or lesser dependence

[38] Contrary to Ross (1936) 506–7 I do not think the passage contains a theory of abstraction, though a logical method of subtraction may be implicit in this talk about separation; cf. Cleary (1985). One important difference is that subtraction does not require any idealizing contribution from the mind, whereas epistemological theories of abstraction have traditionally involved something like this.

[39] τὰ γὰρ φυσικὰ χωρίζουσιν ἧττον ὄντα χωριστὰ τῶν μαθηματικῶν, *Phys.* 193b36–194a1.

[40] τὸ γὰρ ζῷον οὐσία ἐστὶν ἔμψυχος αἰσθητικὴ καὶ κατὰ τόπον κινητική, Simplicius, *in Phys.* 294.18–19.

on sensible matter. Even in those passages where the question about separated forms is clearly ontological in character, it is consistently formulated as an issue about whether or not there are separate substances apart from sensible substances. At the end of *Physics* II.2, for instance, Aristotle still insists that it is the task of first philosophy to inquire about the mode of being of a separated form and about what it is.[41]

Finally let us consider a passage from this chapter which poses in its most difficult form the problem of distinguishing between the subject-matters of physics and mathematics:

> This is also clear in those parts of mathematics which are more physical, such as optics and harmonics and astronomy, for these are related to geometry in a somewhat converse manner. On the one hand, geometry is concerned with physical lines but not qua physical; on the other, optics is concerned with mathematical lines not qua mathematical but qua physical.[42]

From the general context it would appear that Aristotle is trying to clarify the distinction between mathematics and physics, even though this passage only mentions what were traditionally regarded as mathematical sciences. When he says that optics, harmonics, and astronomy are the more physical of the mathematical sciences, this might be taken to mean that Aristotle is redrawing the traditional boundaries. Ross[43] takes it to imply that these sciences are really parts of physics rather than being physical branches of mathematics, since

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[41] πῶς δ' ἔχει τὸ χωριστὸν καὶ τί ἐστι, φιλοσοφίας ἔργον διορίσαι τῆς πρώτης, *Phys.* 194b14–15. It is important to note that the leading question here (194b9 ff.) is posed as follows: Up to what point (μέχρι πόσου) should the physicist pursue knowledge of the form (τὸ εἶδος) and the 'what-is' (τὸ τί ἐστιν)? Drawing a parallel with productive arts like medicine and sculpture, Aristotle suggests that physics should confine its attention to things which are separable in form (χωριστὰ εἴδει) but have a mode of being in matter (ἐν ὕλῃ). Charlton (1970) 97 takes Aristotle to be saying that the student of nature should consider the form of a thing only from the point of view of its natural function and how its material parts contribute to that end. By contrast, the first philosopher can consider the same form from the point of view of its mode of being; i.e. whether it is separate from matter or not. In fact, however, Aristotle suggests that the first philosopher studies forms or essences that are separate from matter or 'without matter,' as he himself puts it rather ambiguously.

[42] *Phys.* 194a7–12: tr. Apostle (1969).

[43] Cf. Ross (1936) 507. Merlan (1954) 67 resists the suggestion that Aristotle is talking about a method of abstraction in physics here, since he regards the whole doctrine of degrees of abstraction as unaristotelian and even as non-Thomistic. Rather, he thinks, the context proves that Aristotle is charging the Platonists with overlooking the fact that their language implies that ideas of sensible things exist separately,

they are described as being "more physical than the mathematical sciences" (τὰ φυσικώτερα τῶν μαθημάτων).

If we take this description seriously, its implications are important for his view about the relationship between mathematics and the physical world. This view emerges more clearly in his explanation (194a9–12) of the inverse manner in which geometry is related to one of the more physical sciences like optics. While geometry inquires about a physical line, though not qua physical, optics deals with a mathematical line, though not qua mathematical but rather qua physical. The forced appositional style of this passage shows that Aristotle is striving to make the same distinction within the mathematical sciences themselves as he had previously made between mathematics and physics. The motivation for such efforts seems to be his ontological claim that all of these sciences deal with the same sensible things, except under different aspects. While geometry considers a line, though not as the boundary of a physical body, optics also inquires about a mathematical line but only insofar as that belongs *per se* to a ray of light. Contrary to some misconceptions, this means that for Aristotle the mathematical sciences can be applied to the sensible world, though their application is very limited.[44] Thus, to understand why a mathematical science of nature was ruled out by the Aristotelian tradition, one must note the special status given to mathematical objects as reflected in definition. For further elaboration of this point, let us turn to *Metaphysics* VII.

II. *The logical problem*

In *Metaphysics* VII the convoluted discussions of essence and definition bear directly on the distinction between physical and mathematical definitions because these reflect differences between the modes of being of physical and mathematical essences. Since Aristotle holds that a physical definition typically contains some reference to matter, it is reasonable for him to ask whether matter is part of the essence of a physical thing. And that seems to be the sort of question raised in

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whereas in reality they cannot. But his reading takes no account of the claim that 'physicals' are less separable than 'mathematicals' which, I think, can only be understood in terms of their respective relationships to sensible matter.

[44] See Lennox (1986) on the Aristotelian tradition of the so-called 'mixed sciences.'

VII.10–12 under the rubric of a general question about whether or not a definition of the whole should contain a definition of its parts. His reason for raising such a general problem is the following:

> Since a definition is a formula, and every formula has parts, and since a formula is related to the thing in a similar way as a part of the formula to the corresponding part of the thing, we may now raise the question whether the formulae of the parts should be present in the formula of the whole or not. For they appear to be present in some cases, but not in others. The formula of a circle does not have the formula of the segments of the circle, but the formula of a syllable does have the formula of its letters; yet the circle is divisible into segments just as the syllable into its letters.[45]

The *De Interpretatione* (16b26) account of definition seems to be the basis for Aristotle's assertion here that every definition (ὁρισμός) is a formula or account (λόγος) with parts (μέρη). But it is less easy to establish the grounds for his subsequent claim that there is a similar relation between the whole definition and the thing (τὸ πρᾶγμα) as there is between part of the definition and the corresponding part of the thing.[46]

An aporia subsequently emerges about whether or not the account of the parts should be present in (ἐνυπάρχειν) the account of the whole. Such an impasse is produced by a conflict between the different dispositions of parts and wholes in different things and their definitions. On the one hand, the definition of a circle does not contain any account of its segments (τὰ τμήματα) whereas, on the other hand, the definition of a syllable does contain an account of its letters (τὰ στοιχεῖα). Yet it would seem that the circle is divided (διαιρεῖται) into segments in the same way as the syllable into letters. Hence there is a problem to be resolved.

By way of compounding the impasse, Aristotle introduces a second conflict of views which depends upon using different criteria of priority:

> Moreover, if the parts are prior to the whole, and the acute angle is a part of the right angle and finger is a part of an animal, the acute would be prior to the right angle and the finger would be prior to the man. But the latter are regarded as prior to the former, for in each

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[45] *Met.* 1034b20–28: tr. Apostle (1966).

[46] As Apostle (1966) 334–5 points out, it is quite unclear in what sense the word 'part' is being used here, and so he considers the claim to be 'somewhat dialectical.'

> case, the formula of the part has the formula of the whole, and the whole is prior in existence to the part since it can exist without it.[47]

The first argument of the passage begins with the plausible assumption that the parts (τὰ μέρη) of something are prior to the whole (τοῦ ὅλου) and applies this general principle to particular instances which may be taken to illustrate mathematical and physical things, respectively. Thus, since the acute angle is a part of the right angle, it is also prior to it and, similarly, the finger will be prior to the man. But, on the other hand, it appears (δοκεῖ) that the latter member of each pair is prior to the former, both in the sense of priority in definition and priority in being. In setting up this opposition, Aristotle appeals briefly to what are recognizable as two distinct criteria of priority. First, he explains (1034b31) that the wholes are prior in account (τῷ λόγῳ) because the parts are spoken about (λέγονται) from these wholes (ἐξ ἐκείνων).[48] But there is also a weaker sense of priority in account by which an accident like whiteness can be prior to a composite whole like white man; cf. *Met.* 1018b34–36.

If the argument of *Metaphysics* VII.10 is to work, however, a stronger sense of priority in definition is required, and so the claim would be that the definition of an acute angle, for instance, presupposes that of a right angle but not *vice versa.* Yet, as Ross (1924 ii, 196) notes, Aristotle also makes the more dubious claim that the right angle can exist without the acute angle, whereas the reverse is not the case. While such a claim seems plausible enough for the case of the man and his finger, it is more difficult to see how natural priority can be applied to mathematical cases.[49] Aristotle attributes priority in being (τῷ εἶναι) to both the right angle and the man because they can exist without the acute angle and the finger, respectively. This looks like the criterion of priority in nature and substance attributed to Plato; i.e. whatever can exist without others (ἄνευ ἄλλων), while the others cannot exist without these (ἄνευ ἐκείνων). Given the aporetic character of VII.10, Aristotle may be playing off against one another the views of the natural philosophers (priority of part to whole) and those of the Platonists (priority of whole to part).[50]

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[47] *Met.* 1034b28–32: tr. Apostle (1966).

[48] I accept Ross's (1924) ii, 196 suggestion that 'the parts' be taken as the subject of λέγονται.

[49] The London Group raises similar difficulties about *Metaphysics* VII.10; cf. Burnyeat ed. (1979) 80.

[50] On the other hand, Frede & Patzig (1988) ii, 169–70 suggest that Aristotle

In a parallel passage at *Metaphysics* XIII.8, he adopts a similar strategy in generating an aporia about the separate existence of number by playing off against each other the same contrasting senses of priority, while once again using the example of the relationship of right to acute angles. Aristotle says (1084b7–8) that in one way (ἔστι μὲν ὡς) the right angle is prior to the acute because it is determinate and because of its definition. Yet in another way (ἔστι δ' ὡς) the acute angle is prior because it is into such parts that the right angle is divided. Aristotle then explains (1084b9–11) that it is as matter (ὡς ὕλη) that the acute angle and the element and the unit are prior, whereas it is with respect to the form and to the substance according to definition that priority is given to the right angle and to the whole compounded from the matter and the form.[51]

This passage throws some light on the distinction between different senses of priority made in VII.10 with reference to the right / acute angle and the man / finger examples. The first member of each of these pairs may be taken as prior in definable form. In support of the priority of the right angle in this sense, an additional factor is that it is determinate (i.e. 90 degrees) by contrast with an acute angle which is indeterminate (i.e. ranging from 0 to 90 degrees). Thus, in view of Plato's use of the Indefinite Dyad as a material continuum, I think *pace* Annas (1976, 182) that it is a plausible dialectical move for Aristotle to associate the acute angle with matter and the right angle with determinate form.

Adopting this distinction between senses of priority, let us consider the resolution of the impasse which Aristotle puts forward in VII.10. Predictably, he begins (1034b32–33) with the claim that 'part' is said in many ways and then illustrates one of these ways in terms of the measure with respect to quantity (τὸ μετροῦν κατὰ τὸ ποσόν). Since this covers both the first and second senses of 'part' listed in *Metaphysics* V.25, Aristotle seems to be referring to that catalogue of different ways in which 'part' is spoken about.[52] But he sets aside as irrelevant the quantitative sense of 'part' and focusses instead upon those parts

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means to resolve the previous aporia by distinguishing between these two intuitions about priority.

[51] ὡς δὲ κατὰ τὸ εἶδος καὶ τὴν οὐσίαν τὴν κατὰ τὸν λόγον, *Met.* 1084b10–11. See also *Physics* II.3, where the whole is closely linked with form whereas the part is connected with matter.

[52] A quantitative part seems to have both a loose and a strict sense. In the loose sense, a part is that into which a quantity is divided in any way (ὁπωσοῦν) because that which is subtracted from a quantity qua quantity is always called a part; e.g.

out of which the substance comes to be (1034b33–34). This may correspond with the third sense of 'part' in V.25 as "that into which the kind can be divided without reference to quantity."[53] It is in this sense that species are said to be parts of the genus.

But it appears that other senses of 'part' are also relevant to the discussion in VII.10. In V.25 (1023b19–20) Aristotle calls those things 'parts' into which the whole (τὸ ὅλον) is divided or out of which it is compounded, whether this whole is the form (τὸ εἶδος) or that which has a form (τὸ ἔχον εἶδος). This formulation suggests that the form itself, just like the compound, may have parts into which it is divided or out of which it is compounded. Yet the examples given by Aristotle seem to illustrate only compounds of form and matter, like a bronze sphere or a bronze cube. In fact, he explains (1023b21–22) that the bronze is a part of the whole because it is the matter in which the form is embodied. But it is still possible that he means to illustrate how something can be part of the form when he refers briefly to the angle as a part (ἡ γωνία μέρος).[54] Finally (1023b23), Aristotle clarifies the way in which things are parts of the whole because they are in the formula indicating each thing. It is in this sense that the genus is called part of the species, while in another sense the species is said to be part of the genus, as we have seen already (1023b17–19).

Thus, by using *Metaphysics* V.25 as the background for VII.10, we can see what Aristotle means when he says (1035a1–4) that in one way (ἔστι μὲν ὡς) matter is called a part, if there is a composite of matter and form; although, in another way (ἔστι δ' ὡς), matter is not called a part since it is not one of the elements out of which the account of the form is constructed. For example, as Aristotle explains (1035a4–5), flesh is not a part (in this sense) of concavity because this is only matter on which the form supervenes (ἐφ' ἧς γίγνεται). By contrast, however, flesh is a part of the snub because, as he has explained earlier in VII.5 (1030b31–32), it is defined as a concave nose.

Another Aristotelian way of making this contrast is to say that,

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two is part of three in that sense. Yet, in a stricter sense, two is not part of three because it is not one of its measures (καταμετροῦντα); cf. 1023b12–16.

[53] ἔτι εἰς ἃ τὸ εἶδος διαιρεθείη ἂν ἄνευ τοῦ ποσοῦ, καὶ ταῦτα μόρια λέγεται τούτου, *Met.* 1023b17–18.

[54] Apostle (1966) 315 suggests that Aristotle may have in mind the shape or form of the bronze figure, part of which might be called an angle. Kirwan (1971) 174–5 also takes it as referring to the shape.

whereas it is essential for the form of snubness to be embodied in the material constituents of a nose, this is not the case for concavity which may be embodied in many different sorts of matter. In conformity with his own theory of definition, Aristotle insists (1035a7–9) that what ought to be stated is the form of each thing insofar as it has that form, while the material element (τὸ ὑλικόν) should never be spoken about by itself (καθ' αὑτό).[55] Such a logical theory enables him to resolve the initial impasse by explaining (1035a9 ff.) why the definition of a circle does not contain any reference to its segments, whereas the definition of a syllable does contain an account of its letters (στοιχεῖα).[56]

Aristotle's solution is to claim (1035a11–12) that the letters are parts of the definition of the form and therefore are not parts of the matter, whereas the segments are parts of the circle in the sense that they are the matter in which the form comes to be embodied. Yet he also wants to claim (1035a13–14) that the segments as matter are closer to the form (ἐγγυτέρω τοῦ εἴδους) than is the case with bronze when it takes on the form of roundness (ἡ στρογγυλότης). This attempt to give some kind of ordering to the different sorts of matter, based on their proximity to form, has a parallel in XIII.8 (1084b12–13) where Aristotle says that the composite is nearer to the form (ἐγγύτερον τοῦ εἴδους) and to the object of definition, even though it is posterior in generation (γενέσει ὕστερον). In both cases it is clear that the form functions as an ordering principle but in VII.10 it is more clearly applied to the difference between kinds of matter. For instance, Aristotle differentiates (1035a14–17) between those (intelligible) letters which are parts of the account of a syllable and those letters in wax or in air, which are parts only as sensible matter (ὡς ὕλη αἰσθητή).

I think it is clear from the immediate context that by sensible matter Aristotle means the kinds of matter which embody forms that are accessible to the senses; e.g. letters formed in wax so that they can be seen or touched, and letters spoken or propagated through the air (cf. *Sens.* 446b6). These tokens are not parts of the essential nature of the syllable which is captured in a definition, even though

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[55] Cf. *Met.* 1035a8, 1036a8, 1039b27–30; *Rhet.* 1356b31; *GC* 332a25.

[56] The choice of syllable as an example of something whose form contains its elements can hardly be accidental, in view of the alphabetical connotations of στοιχεῖα. Frede & Patzig (1988) ii, 169 also attach significance to Aristotle's choice of example here.

the sensible syllable would not exist without some material embodiment. Similarly, as Aristotle points out (1035a17–20), even if a man disintegrates into bones and muscle and flesh upon his death, this does not imply that these are parts in the sense of substance (ὡς ὄντων τῆς οὐσίας μερῶν) but only in the sense of matter (ὡς ἐξ ὕλης). It is significant that he applies the same distinction to the relationship between a line and the parts into which it is 'destroyed' (φθείρεται) if it is divided (διαιρουμένη), since it shows that Aristotle regards a mathematical form (e.g. line) as having to some kind of matter a relationship analogous to that which a physical form (e.g. man) has to an appropriate sort of matter. Thus, as he explains, in both cases the material parts are parts of the composite (τοῦ συνόλου μέρη) but not of the form (τοῦ εἴδους) or of what is expressed by the definition. Thus the material aspects of such things are not included in the definitions of their essential forms.

Aristotle concedes (1035a22 ff.), however, that the account of a composite thing may refer to its material aspect when the thing is compounded from matter and form. This seems to introduce a further twist to the initial aporia about whether an account of the parts should be present in an account of the whole. The answer, as we might expect, is that it depends on what kind of whole we are inquiring about. Some wholes like the snub nose and the bronze circle are compounded from matter and form, as the elements into which they are also dissolved (1035a24–27). This is the reason why matter is included in the account of the kind of whole which Aristotle usually calls 'the composite' (τὸ σύνολον).

But we should be wary of identifying this entity too exclusively with what is sensible or individual, since he uses the term for anything which contains sensible or intelligible matter whether that be universal or particular.[57] By contrast with the composite whole, Aristotle claims (1035a28 ff.) that there are wholes without matter whose definitions are of the form alone. Such wholes are not destroyed at all or at least not by dissolution into their elements, since they are not compounds. When spoken of simply (ἁπλῶς λεγόμενος) the circle may be a whole of this kind, since Aristotle thinks (1035a33–

[57] Ross (1924) ii, 197 claims that the term σύνολον is applied to (1) to the intelligible individual, (2) to the universal answering to a set of sensible individuals (1035b29), (3) to the sensible individual. But Frede & Patzig (1988) ii, 177 dispute the second claim on the grounds that Aristotle speaks about σύνολόν τι at 1035b29 and not about σύνολον simply, which is reserved for the sensible individual.

b3) there is also a particular circle (ὁ καθ' ἕκαστον) which is a composite with matter that makes it divisible into segments.

This distinction between the universal circle and particular composite circles raises some ontological problems, even though Aristotle does not appear to be troubled by them in the present passage at VII.10. He merely remarks (1035b1 ff.) that 'circle' is applied homonymously (ὁμωνύμως) to the circle *simpliciter* (ἁπλῶς) and also to any particular circle, since there is no proper name (ἴδιον ὄνομα) for individual circles. While there are verbal similarities between the first kind of circle and the Platonic Form of the Circle Itself, it is safer to assume that Aristotle is referring to the circle as a universal which corresponds to a general definition like a plane figure which extends equally in every direction from the middle.[58] But what are we to make of his reference to the particular circle (ὁ καθ' ἕκαστον) which does not have its own proper name and is only homonymously called circle?

Although such talk reminds us of the Platonic way of describing the relationship between the Form and its particular instantiations, it is not clear that it has the same implications for Aristotle's conception of the relationship between the universal and the particular circle. From the previous discussion in VII.10, it would appear that he thinks of particular circles as composites of form and matter analogous to concrete compounds in the sensible world.[59] In the case of a bronze circle, for instance, it is plausible enough to think of the form of circularity as being embodied in a certain kind of sensible matter, even though it may be imperfectly realized. But it is not at all obvious what Aristotle means by talking (1036a2 ff.) as if the form of circularity were embodied in intelligible matter. It is quite possible that he is insisting upon the spatial reference involved in the definition of a circle, since this would be an intelligible composite analogous to the snub.[60] Furthermore he seems to treat the particular circle

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[58] ἐπιπέδον τὸ ἐκ τοῦ μέσου ἴσον, *Rhet.* 1407b27. See *Cael.* 286b13–16 for an alternative definition of the circle as a plane figure bounded by one (circular) line.

[59] Charlton (1970) concedes that Aristotle applies the form/matter analysis to individual mathematical circles, though he argues that such an analysis does not work because mathematical forms are simply universals. But this is to second-guess Aristotle rather than to elucidate what he had in mind.

[60] In a textually problematic passage at *De Anima* I.1 (403a10–16), Aristotle considers the possibility of soul being completely separated from matter by citing the mathematical example of a straight line touching a bronze sphere at a point. McKay (1979) takes this passage to say that the οὐσία of soul will be no more separable

as an intelligible individual which exemplifies the circularity relation in a particular space. But such entities begin to look very much like the so-called 'Intermediates' (τὰ μεταξύ) of the Platonists, whose very existence Aristotle is at great pains to deny.[61] Although he denies that such entities are independent substances either apart from or in sensible things, yet he must give some account of the mode of being of intelligible entities like the particular circle which are necessary for the science of geometry.[62]

Since Aristotle does not address this problem directly in VII.10, we must explore it indirectly through the explicit aporia with which he is concerned. Even though he claims to have resolved the difficulty about what parts are contained in a definition, he takes a fresh approach to the problem by restating (1035b4–6) his solution in terms of the rule that either all or some of the parts into which the definition is divided are prior. When applied to the geometrical example, this rule implies (1035b6–8) that the right angle is prior to the acute because the definition of the acute angle contains a reference to the right angle, whereas the reverse is not the case. The circle is similarly prior to the semi-circle because the latter is defined with reference to the former and the logical situation is just the same for the example of the finger and the man. By way of a general summary, therefore, Aristotle says (1035b11–14) that whatever are parts in the sense of matter (ὡς ὕλη) are posterior (ὕστερα); whereas those things which are parts of the definition of the substance are prior (πρότερα).[63]

The ontological implications of what is primarily a logical sense of priority become apparent with the subsequent application of the rule to the relationship between the soul and the body of a living thing.

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from its appropriate matter than a geometrical line is separable from its matter; i.e. thus-and-such magnitude. So the geometer's definition must be 'dyad in length' not 'dyad' or 'self-moving number.'

[61] Cf. *Met.* I, 991b29, III, 997b14, XI, 1059b6, XIII, 1077a1, XIV, 1090b36.

[62] Jones (1983) 97 claims that Aristotle individuates geometrical figures by assuming the existence of something called 'extension' (μέγεθος), which Jones (implausibly) links with Newton's notion of absolute space.

[63] It is unclear why Aristotle repeats the phrase ἢ πάντα ἢ ἔνια in connection with the priority of the parts of a definition of the substance; cf. 1035b5–6, b14, b19. Both Ross and Apostle think the point is that the last differentia of a species is neither prior nor posterior to it, whereas the London Group (1979) 82 is unhappy with that solution, though it does not supply a satisfactory alternative. See VII.12, 1038a19 for a parallel passage. Frede & Patzig (1988) ii, 188 plausibly suggest that the exception Aristotle had in mind may be the case of primary parts like the heart or the brain, which are simultaneous (ἅμα) with the living composite rather than prior or posterior.

According to Aristotle (1035b14–16), the soul is the substance with respect to definition (ἡ κατὰ τὸν λόγον οὐσία) or the form (τὸ εἶδος) or the essence (τὸ τί ἦν εἶναι) for such and such a body (τῷ τοιῷδε σώματι). It follows (1035b18–22), therefore, that the parts of soul are either all or some of them prior to the composite animal (τοῦ συνόλου ζῴου), whereas the body and its parts are posterior to such a substance. Furthermore, as Aristotle points out (1035b21–22), it is not the substantial soul but rather the composite thing which is divided into material parts. In the sense of matter, therefore, these parts are prior to the composite animal but in another sense they are not (prior) because they do not have a mode of being that makes them separable from the whole (1035b23–24). The latter explanation clearly invokes some criterion of priority in substance or actuality, according to which the material parts of a living composite are posterior. By way of illustration, Aristotle uses the finger example again to show that such a material part of a living animal is only equivocally called 'finger' when it is separated from the whole organism.

In some cases, however, the material part of a living thing is neither prior nor posterior but rather simultaneous (ἅμα) with the whole because of a reciprocal interdependence between the two. Examples of such principal (κύρια) parts may be either the heart or the head, if one or other of these is the first part in which the substantial form of the thing is present (1035b25–27). Here Aristotle seems to presuppose an ordering of different kinds of material parts, which ranges from the primary and most essential parts like the heart or the head all the way to accidental and secondary parts, such as hair and fingers and toes. Of course, these parts are ordered not with reference to their matter but in terms of their relationship to form. Thus, for instance, the heart is primary precisely because it is held to be the seat of the soul, which is the substantial form of the living body. By contrast, material parts like fingers and toes are secondary because their corresponding forms are accidental to the living thing.

In the light of these distinctions between different kinds of parts, we should now be able to follow Aristotle's differentiation (*Met.* 1035b27 ff.) of the corresponding wholes. Let us begin with the concrete individual which was called primary substance in the *Categories* and which is composed of this particular form (τουδὶ τοῦ λόγου) and of some proximate matter (τῆς ἐσχάτης ὕλης). Just as there are sensible composites like this man and this bronze circle, Aristotle claims (1036a1–4) that there are intelligible composites like the mathematical

circles which are studied by geometry. But he also holds (1036a5–6) that there is no definition of such things, so that they are known only by being thought or sensed (μετὰ νοήσεως ἢ αἰσθήσεως γνωρίζονται). Thus their ontological status seems rather precarious because, as Aristotle says, when the activity of thinking or sensing ceases it is unclear whether they exist or not.[64] If 'existence' is taken in the modern sense, he appears to be saying that the existence of these individual composites (whether sensible or intelligible) depends upon our sensing or thinking. On the other hand, his point may be that individual things are always spoken about and known through a universal definition, and so the question of whether any particular entity satisfies that definition is a matter for thinking and perceiving.[65] Even primary substance in the *Categories* must be known through some secondary substance, and this is more important for Aristotle's inquiry into the question of definition in *Metaphysics* VII. While the substantiality of the individual man or horse is not denied, it is secondary from the point of view of analysis precisely because it is a composite of matter and form.

From a general point of view, however, the composite turns out to be of more interest for Aristotle's analysis of sensible substances because their definitions have a composite structure of formal and material aspects. I think that this concern with definition is what leads him to talk about the composite as a universal (καθόλου), since he emphasizes that it is not a substance (οὐσία) but something composed of this form and that matter taken universally.[66] Thus it would seem that this concrete universal is the proper correlate for a definition such as 'man is a rational animal,' since this does not apply to the individual man nor even to the human soul, taken strictly as such. Yet, in view of Aristotle's trenchant criticism of hypostatized Forms and Intermediates, one might wonder how he can justify the postulation of concrete universals which seem to duplicate sensible reality in a Platonic fashion.[67] The evidence that Aristotle did accept such 'materiate universals' (Ross) or 'universal concretes' (Chen) in

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[64] ἀπελθόντες δὲ ἐκ τῆς ἐντελεχείας οὐ δῆλον πότερον εἰσὶν ἢ οὐκ εἰσίν, *Met.* 1036a6–7.

[65] This interpretation fits better with the role of ἔκθεσις and ἐπαγωγή in mathematics; cf. *EN* VI.8, 1142a26–30 for remarks about the perception (αἴσθησις) of ultimate figures in mathematics. See also *Metaphysics* IX. 9, 1051a 21–33 where the discovery of diagrams and constructions in geometry is attributed to the activity of nous.

[66] ἀλλὰ σύνολόν τι ἐκ τουδὶ τοῦ λόγου καὶ τησδὶ τῆς ὕλης ὡς καθόλου, *Met.* 1035b29–30.

[67] Cf. Chen (1964) 48–57.

his ontology seems undeniable, since he clearly distinguishes (1035b27–31) them from the pure form and the concrete particular.[68]

But why should Aristotle find it necessary to postulate such intermediate entities, despite his stern opposition to the ontological profligacy of the Platonists? Perhaps a clue is to be found in his criticism (at *Phys.* 193b36 ff.) of those who define physical forms as if they were separable from sensible matter. According to him, their mistake lies in thinking that the form can be defined without reference to the matter in which it is necessarily embodied. As a corrective, therefore, Aristotle insists that the definition of physical things must reflect the mode of being of their essences whose paradigm example is the snub; i.e. a certain kind of form in a certain kind of matter. But a definition in physics is not about a particular snub nose as such because individual things are indefinable, nor is it about the pure form of concavity, and so there must be some universal concrete like the snub. However, it is not clear whether Aristotle posits a similar kind of composite entity for mathematics.

In VII.11 Aristotle considers a related aporia about which sort of parts are parts of the form and which are parts of the composite (τοῦ συνειλημμένου). He explains (1036a28–29) that, unless this distinction can be made, each thing cannot be defined because the definition is of the universal *and* of the form.[69] Thus, if one does not differentiate clearly between material and formal parts, the definition of each thing will remain unclear. In the case of forms like circularity which can be embodied in different kinds of matter it seems obvious that bronze or stone or wood are not parts of the essence of the circle (τῆς τοῦ κύκλου οὐσίας) because the form can be separated (χωριζόμενα) from them.[70]

Yet even in the case of forms which are not seen to be separated (χωριζόμενα) from a particular kind of matter, such a material part may still not be part of the form though it is difficult to subtract it

[68] Driscoll (1981) suggests that Aristotle introduces a distinction between the formal cause and the 'universal composite' and that the latter is identical with the secondary substances of the *Categories*. But Michael Frede (1987) thinks that Aristotle drops 'secondary substances' entirely from the ontology of *Metaphysics* VII in favor of substantial form which does not satisfy the same criteria.

[69] It is unclear here whether the καὶ is epexegetic or disjunctive.

[70] It would appear from the context that χωρίζειν here has a logical sense primarily, though it has implications for the mode of being of the essence being defined. *Pace* Fine (1984), this involves an unqualified use of χωρίζειν for the separation of form from matter.

in thought (ἀφελεῖν . . . τῇ διανοίᾳ, 1036b2–3). For example, the form of man appears to be always in flesh and bones and other such material parts, though we should ask whether they are parts of the form and of the definition or whether they are merely matter. The second part of the question rests on a comparison with the circle because, as Aristotle explains (1036b6–7), we might simply be unable to separate (ἀδυνατοῦμεν χωρίσαι) the form of man from a particular kind of matter since we have not seen it embodied in any other way. Thus, as he already (1036a35–b1) pointed out, nothing prevents the form of man from being like the form of circle, if we imagine a state of affairs in which all circles that we have ever seen were made of bronze. From Aristotle's formulation of this aporia it would appear that neither our sense experience nor our thinking determines the logical (and hence ontological) relation between form and matter, as represented in the definitions of physical and mathematical things.[71]

In addition, there is the difficulty raised by previous thinkers who noted the similarity between mathematical and physical composites. They insisted that, since the relationship between the form of a circle and continuity is similar to that between the form of man and flesh, it is not correct to define a circle by means of lines and continuity, and so they reduced (ἀνάγουσι) all things to numbers and claimed that the definition of line is the same as the definition of two. This looks like a typically Pythagorean view, even though Aristotle represents it as being defended by means of an explicit comparison between mathematical and physical forms. Perhaps this is one implication of his frequent report that, unlike the Platonists, the Pythagoreans did not separate mathematical objects from sensible things. According to a scholiast on Euclid, the Pythagoreans treated the point as being analogous to the unit and the number two as being analogous to the line.[72] In fact, the tradition of representing the three dimensions in terms of the numbers 2, 3, and 4 can be traced back at least as far as Philolaus.[73] But here in VII.11 the crucial point is that some Pythagoreans do not regard continuous quantity as being part of the essence of the line, which they define strictly in terms of the number two.

[71] This follows from the ontological priority of the object of thought with respect to thinking, which is a basic principle of Aristotle's metaphysics and psychology; cf. *Categories* 12 & *De Anima* III.4–8.

[72] Cf. *Scholia on Euclid* 78.19 (Heiberg) & Alex. Aphr., *in Metaph.* 512.20–513.3.

[73] Cf. *Theologumena Arithmeticae* 62.17–22 Ast.

Aristotle subsequently (1036b14) indicates that some of these Pythagoreans formulated their view in terms of the identity of the Line Itself (αὐτογραμμή) with the Dyad. By contrast, there were others who, while they accepted that Two is the form of the line (τὸ εἶδος τῆς γραμμῆς), did not think that the form is the same as that of which it is the form, at least not in the case of the line; though perhaps it is so in the case of the number two. In other words, for them 'line' is ambiguous since it may be taken strictly in the sense of form and defined in terms of Twoness or it may be taken as a composite of matter and form, which itself may be described as something like twoness in length (δυὰς ἐν μήκει); cf. *Met.* VIII.3, 1043a33–34. From Aristotle's report it seems unlikely that the second group held continuous quantity to be an essential part of the definition of a line, although it would appear to be somehow essential for the differentiation of this form of twoness from that found in the number two. Perhaps this is what Aristotle means when he says that for these thinkers it follows that there is one form for many things whose form still appears to be different.[74] For example, two cows appear to be quite different from two abstract units and even more so from a line, yet their formal structure is the same.

This discovery of an identical mathematical structure in different material things was perhaps the central insight of the Pythagoreans whose ideas influenced Plato and Speusippus. In the differentiation of mathematical form through all kinds of material, for example, Plato found a conceptual model for the way in which many sensible things can participate in a single Form.[75] Thus, as Aristotle says (1036b19–20), Plato makes one thing (i.e. Form Itself) the form of all things and relegates sensible things to the status of non-Forms. If one pushes this line of reasoning to its ultimate conclusion (like some Platonists), there will be only one form (i.e. One Itself) from which all other things are somehow derived through a principle of material differentiation (e.g. the Indefinite Dyad). Indeed Aristotle's own thinking about the relationship between form and matter may have been provoked by the failure of such mathematical monism to make adequate formal differentiations which would avoid the difficulties that are generated by the Platonic-Pythagorean view.

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[74] ἕν τε πολλῶν εἶδος εἶναι ὧν τὸ εἶδος φαίνεται ἕτερον, *Met.* 1036b17–18.

[75] This is consistent with Aristotle's critical remark (*Met.* 987b10–13) that Plato merely changed the name for the relation between One and Many from 'imitation' (μιμήσις) to 'participation' (μέθεξις).

Having outlined the aporia about definition along with the opinions which gave rise to it, Aristotle searches for a solution by refuting some of these opinions. For instance, the Platonic-Pythagorean attempt to reduce everything to numbers and to eliminate matter (ἀφαιρεῖν τὴν ὕλην) is dismissed (1036b22–23) as 'useless labor' (περίεργον) because some things, at least, are essentially 'a this in a that' (τόδ' ἐν τῷδ') or 'these things being thus' (ὡδὶ ταδὶ ἔχοντα). What Aristotle seems to mean here is that the definitions of physical things must refer to a particular form in a particular kind of matter or to a specific subject having a specific attribute. Contrary to what Socrates the Younger used to maintain, he insists that the relation between the form of man and specific matter like flesh and bones is not the same as that between the form of a circle and bronze, for instance. What is misleading about such a comparison, according to Aristotle, is that it leads one to assume that the man can exist without his parts just as the circle can be without bronze.[76]

He proceeds (1036b28–29) to show the falsity of this comparison by arguing for the distinction between the case of the circle and that of the man. While the circle is something intelligible as the distinction implies, the animal is something sensible (αἰσθητόν) and cannot be defined without reference to motion. Therefore, Aristotle concludes (1036b30), the animal cannot exist without its parts existing in a certain manner (πώς). By way of clarifying the mode of being of these parts, he explains (1036b30–32) that the hand is not part of the man in every way (πάντως) but only when it can perform its function and is thus alive (ἔμψυχος). If it is not alive, however, and is therefore incapable of fulfilling its purpose then it is not a part of the man, strictly speaking, just as the severed finger is a 'finger' in name only.

While the argument does show why a living thing cannot be defined without some reference to motion, it is not immediately clear how this logical fact undermines the comparison of a man and his parts with a circle and its bronze parts. It seems that the distinction between sensible and intelligible things which is implicit in the argu-

[76] While one might be tempted to identify Socrates the Younger as one of the Pythagoreanizing Platonists mentioned previously in VII.11, the London group objects that such an identification is not quite warranted by the text, since Aristotle does not say that the comparison was incorrect on its own terms but merely that it leads one away from the truth. But I think that this hair-splitting is excessively subtle even for Aristotle, since he clearly does think that the comparison made by Socrates the Younger is false; cf. Burnyeat ed. (1979) 91–2.

ment does not undermine Socrates the Younger's claim, since the bronze circle is a sensible thing. Perhaps the point is that a circle, unlike a man, need not be perceptible and hence there is less basis in experience for treating bronze as part of the definition of a circle than for treating flesh and bones as parts of the definition of man. Yet Aristotle has previously admitted that sense experience does not establish the necessary connection between man and his material parts which would be characteristic of a definitional link. Therefore the weight of his argument must rest on a necessary connection between man as a living animal and his characteristic motion. It must be through such a logical connection that some material parts necessary for that motion are brought into the definition of man. In the case of a circle, however, there is no similar logical connection with any characteristic motion because mathematical objects are not in motion *per se* but only accidentally. This difference turns out to be crucial for Aristotle's distinction between physical and mathematical forms and their relationship to matter.

Such a distinction guides the subsequent discussion of parts and wholes with reference to mathematical objects:

> As for the mathematical objects, why are the formulae of the parts not parts of the formulae of the wholes? For example, why are the formulae of the semicircles not parts of the formula of the circle? For these objects are not sensible. But does this make a difference? For even some non-sensible things can have matter; for there is some matter in every thing which is not just an essence and a form by itself but is a this. Accordingly, as we said before, the semicircles will not be parts of a circle universally taken, but they will be parts of the individual circles; for some matter is sensible and some is intelligible.[77]

Many commentators and editors have treated this section as misplaced because it seems to go more naturally with the discussion (*Met.* 1034b24–1035a17) of the difference between the relation of the circle to its semicircles and that of the syllable to its letters.[78] But I agree with Ross (1924ii, 203) that this passage represents a continuation of Aristotle's argument against the view which conjoins a sensible thing like man and an intelligible thing like a circle with respect

[77] *Met.* VII.11, 1036b32–1037a5: tr. Apostle (1966).

[78] Ps.-Alexander thinks it belongs in VII.10 and Bonitz (1848–49) concurs. The London Group hedges its bets by hypothesizing that the section was a parenthetical part of an original VII.11 which was revised somewhat after the question was reworked in VII.10; cf. Burnyeat ed. (1979) 93.

to definition. While the logical relationship between a man and flesh is not comparable with that between a circle and bronze, perhaps it would parallel the relationship between a circle and its segments.

Thus Aristotle addresses the question as to why the formulae of the semicircles are not parts of the formula of the circle, since one would expect them to be if the case were comparable to that of man and flesh. One possible explanation for the lack of parallelism between the cases is that the circle and its semicircles are not sensible things (1036b34). Yet Aristotle appears to undermine this explanation when he asks whether being sensible makes any difference, since his point seems to be that it is insufficient simply to say that semicircles are not part of the definition of a circle on the grounds that they are not sensible things. He insists (1037a1–2) that some non-sensible things can have matter whenever they are individual things rather than essences or forms by themselves. Therefore, as he explains (1037a2–4), the semicircles as intelligible matter will be part of the particular circles (τῶν καθ' ἕκαστον) rather than of the circle as universal (κύκλου . . . τοῦ καθόλου). This is the basic reason why the formulae of the semicircles are not parts of the formula of the circle because the definition of the latter is of the universal rather than of the particular circle. What is noteworthy about this entire passage is that it seems to admit a class of intelligible particulars between sensible circles and the universal, which are somehow individuated by their relationship to intelligible matter. Perhaps these are pale ghosts of the old Platonic Intermediates, which are still required in some form by the mathematical practice of constructing particular figures to prove that they have definite relationships to each other.[79]

If Aristotle is still thinking of mathematical objects as intermediate entities, we may doubt whether VII.10–11 contains his final word on the problem about the ontological status of mathematical objects. Indeed, near the end of VII.11, he indicates that his discussion of sensible substance is preliminary to an inquiry into supersensible substance:

> Whether there exists, besides the matter of such substances, another kind of matter, and so whether we should look for other substances as the substances of these, such as numbers or something of this sort,

[79] Following Annas' argument (1975) with respect to Intermediates in Plato, Jones (1983) has argued that Aristotle too must accept such particular mathematical objects as being necessary for mathematical practice.

> should be considered later. For it is for the sake of this that we are also trying to describe sensible substances, although in some sense the investigation of sensible substances belongs to physics or second philosophy; for the physicist must know not only about matter, but also about substance according to formula, and even more so about the latter.[80]

Whether or not this section represents a later addition (as Jaeger thinks) to the original treatment of sensible substance in *Metaphysics* VII–VIII, it contains an implicit admission by Aristotle that the conception of mathematical objects as having a form / matter structure analogous to physical objects needs further examination. The leading question for such an inquiry is said to be whether, besides the matter of sensible substances, there is some other kind of matter.[81] An additional and related question is whether there is some other kind of substance, like numbers or such things. Contrary to what Apostle's translation suggests, I see no reason to take the latter as a question of whether numbers and suchlike things are the substances of sensible things. I think that the question is simply whether numbers and other mathematical objects exist as independent substances; i.e. the same question which is taken up in *Metaphysics* XIII.1–3.

Therefore, like Ross (1924 ii, 204) and other commentators, I see Books XIII and XIV as the subsequent inquiries to which Aristotle postpones the second question. This is consistent with the remarks at the beginning of XIII which clearly presuppose a completed inquiry into sensible substance. I find additional support for this construal in Aristotle's remark here that his inquiry into sensible substances is being completed for the sake of (χάριν) the subsequent inquiry into supersensible substances. The description of the first inquiry as physics or second philosophy (δευτέρας φιλοσοφίας) is quite consistent with his claim in *Metaphysics* VI.1 that physics would be first philosophy if there were no other substances besides physical substances. With regard to mathematics as a third theoretical science, it is now time for me to consider some psychological (and epistemological) problems associated with its ostensibly intermediate status between physics and first philosophy.

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[80] *Met.* 1037a10–17: tr. Apostle (1966).

[81] I follow Ross in supplying ὕλη after τις ἄλλη. Among the puzzles listed in *Metaphysics* XI.1, there is one which asks which science should explore the puzzles concerning the matter of mathematical objects (περὶ τῆς τῶν μαθηματικῶν ὕλης); cf. 1059b14 ff.

III. *The psychological problem*

Given his views on the mode of being of mathematical objects, Aristotle should explain how the human mind can grasp such objects and why they are so accessible even to inexperienced young people.[82] Such a task is incumbent on him because, along with rejecting Plato's ontology of separate Forms, he jettisons the sharp distinction between perception and thinking that goes with the theory of recollection. Instead, he claims that all knowledge is somehow derived from perception, though he gives only schematic accounts of how universals emerge from sense experience. So, before introducing post-Lockean assumptions about abstraction into Aristotle's psychology, we should try to reconstruct his account of how the mind grasps mathematical objects. First, we must consider Aristotle's general theory of sensation as the capacity of the soul to receive sensible forms, while paying special attention to the so-called 'common sense' whose characteristic objects include some of the quantitative attributes of sensible things. Then, with specific reference to mathematics, we must consider how the soul grasps intelligible objects with the help of perception.

III.1. *Perceptual capacities of the soul*

Aristotle's account of the soul[83] belongs to physics, as is clear from the extended review of opinions in *De Anima* I and from the different descriptions of soul given in II.1. In that review he includes the opinions of thinkers like Democritus and Anaxagoras who are usually classified as natural philosophers. Yet that is quite in order because physics is about motion, which is also one of the generally agreed marks of an animate body; cf. *DA* I.2, 403b25 ff.

Aristotle distinguishes between animate and inanimate natural bodies by appealing to a criterion of self-motion which treats self-nourishment along with growth and deterioration as signs of life; *DA* II.1, 412a11 ff. Thus he concludes that a natural body partaking of life will be a composite substance but that, contrary to many natural

[82] The word μαθήματα indicates that these objects were seen by the Greeks as preeminently learnable things; cf. Proclus, *in Eucl.* 44.25–47.7. See also Heidegger (1962) 50–83.

[83] Defined as 'the first actuality of a physical body with organs' (εἴη ἂν ἐντελέχεια ἡ πρώτη σώματος φυσικοῦ ὀργανικοῦ, *DA* 412b5–6).

philosophers, soul itself will not be a body either as a substratum or as matter. By a process of elimination, therefore, he determines the soul to be the substantial form of a natural body that has the capacity for life.[84] Since any such substance is an actuality (ἐντελέχεια), the soul may be defined as the first actuality of an animate body, and this has profound implications for Aristotle's psychology and epistemology.

Indeed he claims (413a11 ff.) that his definition of soul captures not only the fact (τὸ ὅτι) but also the reason why (τὸ διότι) contained in the cause (τὸ αἴτιον). Thus a body can be called animate if it has any one of the characteristic powers of soul; i.e. intellect, sensation, locomotion and rest, and even nutrition, which involves growth and diminution (413a22–25). According to Aristotle, there is a definite hierarchical ordering of these powers because the nutritive power is independent of the others in the case of plants, whereas in human beings the other powers of the soul cannot exist without the nutritive. While the nutritive capacity is a necessary condition of life in all animate beings, it is not sufficient for what are called 'animals' (ζῷα) because these are distinguished by their sensory capacities.

Just as among the powers of soul nutrition is prior because it is found separately in some living things, so also among the sentient powers touch is primary because it can exist apart from all the others. By way of evidence, Aristotle notes that some animals lack sight and hearing, whereas no animal is without the sense of touch. Therefore, since animals are distinguished from other living things by the power of sensation, the capacity for touch is the basic and definitive power of the animal soul. This fact influences the way in which Aristotle thinks about the characteristic access of the animal soul to its environment, just as does the fact that the nutritive power is basic for all living things. Speaking metaphorically, we might say that the sensitive soul 'feeds off' the sensible things in the world, just as the nutritive soul lives and grows by taking in food from its environment.[85]

In assimilating food from its surroundings, however, the living thing extracts from the food the nutritional elements which are akin to itself, and so for Aristotle 'like assimilates like' is the operational rule for the nutritional soul, just as 'like senses like' is the principle of

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[84] ἀναγκαῖον ἄρα τὴν ψυχὴν οὐσίαν εἶναι ὡς εἶδος σώματος φυσικοῦ δυνάμει ζωὴν ἔχοντος, *DA* 412a19–20.

[85] Kosman (1988, 186–88) develops the implications of this metaphor for Aristotle's conception of intellectual activity.

activity for the sensitive soul. Thus his account of sensation depends on the basic conceptual model of assimilation through contact, especially in the case of touch which is primary and hence definitive. The importance of this for his epistemology cannot be overestimated, since the intellect normally cannot 'grasp' anything that is not fed to it through the senses which themselves are nourished by the forms of sensible things. Even in the case of pure essences that are separate from matter, he claims that the intellect can only think them by 'touching' them directly.

After nutrition and reproduction (*DA* II.4), Aristotle investigates the psychic capacity called sensation (αἴσθησις). According to his classification of changes (*DA* II.5, 416b32 ff.), sensation is a kind of alteration (ἀλλοίωσις) and so the sensing soul depends on being moved (κινεῖσθαι) or on being affected (πάσχειν). For Aristotle the sentient part of the soul (τὸ αἰσθητικόν) by itself remains in a state of potency, just as fuel has the capacity to burn yet will not burn without fire as a moving cause. So the perceptual faculty is not self-activating in the sense that it could operate without an external sensible object as a moving cause. Therefore, Aristotle distinguishes two meanings of 'sensing' (τὸ αἰσθάνεσθαι): (i) to have the capacity for seeing or hearing, as when one is asleep and not exercising it; (ii) to be actually seeing or hearing. The point is that the faculty of sensation itself is purely receptive, so that its activation requires a sensible form as an external mover. With reference to the sense faculty, he says (417a14–16) that being affected and being moved and being in activity are the same. This means that every moved thing is moved by some maker (ὑπὸ τοῦ ποιητικοῦ) which is itself in activity (ἐνεργείᾳ).

Therefore Aristotle concludes (417a18–20) that in one way the sense faculty is affected by what is like, while in another sense it is affected by what is unlike because the mover is an active form, whereas the moved thing is a passive receiver. So, with one important exception, his general account of sensation is guided by the principle of 'like to like,' which he adopts from the tradition of Greek thought about the soul. The same principle also dominates his account of the workings of the intellect, except that in the case of sensation (417b19 ff.) the things which produce sensation are external (ἔξωθεν) to the soul, whereas this does not always apply to the intellect. The reason for this difference, as he explains it (417b21–23), is that actual sensation is of particular things (τῶν καθ' ἕκαστον), whereas knowledge is of universals (τῶν καθόλου) that are in the soul itself in a certain way.

Aristotle uses (417b23–25) this explanation to account for the fact that, whereas thinking may sometimes be voluntary, perceiving always depends on a sensible object.

Given his view of the sense faculty as a potency, Aristotle's whole account of sensation is oriented towards the sensible object because that is what the faculty is like when it is activated. In all he distinguishes (418a7 ff.) three different meanings of 'sensible object' (τὸ αἰσθητόν),[86] two of which apply to *per se* (καθ' αὑτά) sensibles and the other to *per accidens* (κατὰ συμβεβηκός) sensibles. Among the *per se* sensibles there is a further distinction between the sort that is proper (ἴδιον) and the sort that is common (κοινόν). Color exemplifies the first sort of sensible because it is the proper object of the visual faculty only, which cannot be mistaken about it (418a11–12). Aristotle consistently claims that there is no possibility of error when a sense faculty in good condition is perceiving its corresponding proper sensible object.[87] He holds, for instance, that the sense of sight is never mistaken about the colors which it perceives nor is the sense of hearing in error about the sounds it hears. Yet, as he acknowledges, each of these senses may be mistaken about *what* is colored or about *what* is sounding or about *where* these sensible things are. Such errors are possible because substance and place are not proper sensibles that may be infallibly perceived by any sense faculty.

Aristotle subsequently admits that sensation may also be mistaken about the common sensibles, even though there is one common sense faculty that perceives them along with proper sensibles. In fact, they are called common sensibles precisely because they can be grasped by more than one specific faculty. It is noteworthy that quantitative attributes are prominent in the following list of common sensibles: motion, rest, number (ἀριθμός), shape (σχῆμα), and magnitude (μέγεθος).[88] But Aristotle can hardly classify mathematical objects as common sensibles without dealing with the Platonic argument that sensible things cannot be objects of knowledge because they are

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[86] Modrak (1986) thinks there is a further ambiguity in Aristotle's use of τὸ αἰσθητόν to refer to the perceptible object itself and also to the complex of sensible characters which is the vehicle for the perception of an external object.

[87] Cf. *DA* 428a11, 428b18–19, 430b29, *Sens.* 442b8–10, *Met.* 1010b1–2, 1009b3–6.

[88] At *De Sensu* I.1, 437a8–10 Aristotle says that these common sensibles are perceived especially by sight, since all bodies are endowed with color which is the proper object of sight. Here physical body functions as the shared substratum for all the common sensibles.

imprecise and subject to change.[89] Yet his epistemological empiricism requires an account of mathematical knowledge that begins with the quantitative attributes of sensible things that are directly accessible to perception.

Since Aristotle's general account of how the sense faculty works provides the basic conceptual model for how the intellect grasps its proper objects, we must briefly examine *De Anima* II.12 where he says (424a17–19) that, in general, every sense is a receptor of sensible forms without the matter. His analogy with the wax receiving the impression of a signet-ring suggests that he is a 'tabula rasa' empiricist, though this does not quite fit with his description of the sense faculty as a capacity for discrimination.[90] In addition, each sense faculty is *potentially* identical with its appropriate sensible objects, and this means that the relevant sense organ must be constituted from the elements which are mixed in a certain proportion (λόγος) for each sensible form.[91] For instance, although the eye itself is not colored, it has in it all the elements that can be mixed to yield any color so that it can receive a definite proportion of these elements through the action of some external colored object.

This seems to be what Aristotle means by saying that the sense faculty receives the sensible form without its matter, as the form is nothing else but a determinate ratio (λόγος) of the elements. Once again, the crucial principle is the affinity of like to like involved in the definite ratio of elements that is actually the sensible form, since some other ratio of the same elements constitutes the sense faculty and makes it potentially identical with the external activating form. Thus each of the senses has a corresponding organ which is either partly or wholly constituted by the material elements that are arranged in a definite ratio when the organ is actually sensing. But,

[89] Plato (*Rep.* 523 ff.) claims that the soul is stimulated to engage in mathematical inquiry precisely because our perception of sensible quantities is infected with opposites. In *De Anima* III.3, Aristotle admits that the common sensibles like motion and magnitude are most subject to error in perception (428b23–25). Furthermore, he consistently treats mathematical objects as intelligible objects (νοητά), especially when he calls them 'the results of subtraction.'

[90] See *Posterior Analytics* II.19. Anachronistic interpretations may be resisted through a comparison with the wax-tablet and aviary metaphors used in Plato's *Theaetetus*. It may be significant for Aristotle's anti-Platonism that in his account of sense perception he chose to use one of the discredited metaphors from that dialogue.

[91] Barker (1981) argues that the slogan 'αἴσθησις is a λόγος' is used in two quite different senses: (i) the faculty of perception is a λόγος (*DA* 424a2 ff.); (ii) the actual perceiving is a λόγος only when the object of perception is a λόγος (*DA* 426a27 ff.).

when the organ is not actually sensing, these elements must be arranged in some other ratio which does not hinder in any way its capacity for receiving external sensible forms.

In general, Aristotle thinks of this constitutive ratio of elements as a mean between extremes like hot and cold, light and dark. If such a mean be equated with transparency in the eye and in the intervening medium, for instance, then vision can be explained as the capacity of the eye to take on the extremes of brightness or darkness which Aristotle associates with colors.[92] That also enables him to explain (*DA* 424a28 ff.) why excesses of heat or cold, light or darkness, can destroy the appropriate sense faculty. Yet this would not necessarily involve the destruction of the corresponding sense organ because, although these are numerically identical, the sense faculty is different in being (τῷ εἶναι) from the organ whose functioning it is.[93] Since it is an extended physical thing, the eye itself will remain even though its capacity for seeing may be destroyed through exposure to excessive light. By contrast, the intellectual capacity cannot be destroyed by excessively intelligible things partly because, according to Aristotle, the intellect does not have a corresponding physical organ. This is an important exception to the general parallel between sensation and intellection.

At the beginning of *De Anima* III (424b22 ff.), Aristotle gives a rather long and obscure argument to prove his empirical claim that there are five and only five special senses. This complements his subsequent claim (425a14) that there is no sense-organ which is proper to the common sensibles. The fact that there is no such sense-organ means that there is no *special* sense faculty for common sensibles but only what Aristotle calls (425a25–27) a common sense (αἴσθησις κοινή). It is characteristic for common sensibles like shape and magnitude to be sensed by more than one faculty (e.g. sight and touch) and hence Aristotle says (425a15) that they are sensed 'accidentally' (κατὰ συμβεβηκός) by each of the special faculties. Yet this does not mean that common sensibles are accidental sensibles like Cleon's son, because there really is a single faculty of perception which senses such things as shape, magnitude, and number through a synthesis of the special senses.

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[92] See *De Sensu* I.3 for a more physiological account of sight and its proper object, color.

[93] Cf. *DA* 424a25, 425b27, 426a16, 427a3, 431a14, a19, 432b1.

Indeed, Aristotle seems to regard motion as the most basic of the common sensibles because he claims that all of the others are perceived through the perception of motion. Thus he says that a magnitude is perceived by means of motion and so also is shape, as this is a kind of magnitude (425a16–17). Similarly, rest is perceived through the absence of motion, just as number is sensed by the negation of continuity.[94] In the absence of further explanation, however, we are left guessing as to whether Aristotle has in mind physical or mental motion. For instance, he might mean the motion of a sense organ (as distinct from its proper activity) like the eye which is felt to 'run around' the perimeter when it perceives a shape or to move in some other way when it senses a magnitude. Alternatively, some mental motion may be involved in the perception of number as the negation of continuity, even though counting is also seen as analogous to physical motion; cf. *Phys.* 263a25 ff.

This account of how the common sense perceives shape, magnitude, and number seems to suggest that for Aristotle mathematical objects are immediately sensible, but there are good reasons to doubt this suggestion. While one might argue that shape, magnitude, and number all belong to the subject-matter of some mathematical science, it is more likely that Aristotle is treating them here as quantitative attributes of sensible things. Furthermore, the fact that these common sensibles are all said to be grasped through the perception of motion is at odds with Aristotle's descriptions of mathematical objects as being separated from motion by (or in) thought. He consistently denies that any type of inference is involved in sense perception, even where we might expect it, for instance, in the case of common sensibles that are judged by different sense faculties. Finally, given the Platonic division between sense and intellect, one would expect a theoretical discipline like mathematics to have intelligible rather than sensible objects.

Aristotle appears to accept such a division for his account of the receptive capacities of the soul, even though he insists on a closer connection between sense and intellect. Whereas Plato's theory of recollection implies that the intellect grasps the Forms independently

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[94] Aristotle also adds that number is perceived by the special sensibles because each sense perceives one class of sensible objects; cf. *DA* 425a19–20. I presume that the explanation here depends on his anti-Platonic view that the One is not a separate substance but rather is a unit for counting that is relative to the kind of things being counted; e.g. man for the class of men, white for the class of color.

of the senses, Aristotle makes sensation the fundamental source of material for intellectual activity. Thus his epistemological account in *Posterior Analytics* II.19 should explain how the human soul can move from particular sensible objects to universal intelligible objects.

III.2. *Interpretive capacities of the soul*

Given his rejection of Plato's theory of recollection, the problem Aristotle must face is how the mind grasps universals if everything accessible through sensation is particular. But he seems curiously untroubled by this problem, since his two general summaries of an alternative epistemological approach can hardly be regarded as systematic. Let me briefly review these parallel accounts in *Posterior Analytics* II.19, and *Metaphysics* I.1, while focusing exclusively upon Aristotle's remarks about the transition from sense perception to knowledge.

It is important to notice that the inquiry in *Posterior Analytics* II.19 is guided by the dual questions (99b18–19) of how we become familiar with first principles and with what faculty we do so. By contrast, the discussion in *Metaphysics* I.1 serves as a general introduction to the science of wisdom, which is held to be accessible to human beings. This makes a difference to Aristotle's approach because within the second context the capacity of human beings to grasp universals is taken for granted, whereas in the *Posterior Analytics* it is at least put in question. Of course, he does not seriously doubt that knowledge of universals is possible, given the existence of sciences and arts which involve some kind of universal cognition.

Thus, at the end of a long treatise on the structure of demonstrative knowledge, the question about universals (99b26 ff.) is raised in the context of an inquiry about starting-points and how they are grasped by the soul. If one were to claim that we already have a distinct faculty for first principles then it would follow that we unwittingly possess powers of knowing more accurate than demonstration. Aristotle thinks that this is absurd and so he explores the other alternative; i.e. that we acquire knowledge of first principles with the help of some pre-existing capacity which is not superior in accuracy to demonstrative knowledge. So his way out of the Meno paradox is to posit (99b34–36) sense perception as an innate faculty of discrimination in all animals, which gives him a basis for an alternative account of 'firsts.'

Having posited sense perception as the natural basis for knowledge, however, Aristotle must account for the fact that only human beings appear to be capable of exercising the highest cognitive capacity, even though all species of animal share the capacity for sensation. His first step (99b36–37) is to distinguish between those species in which retention of the percept (τοῦ αἰσθήματος) comes about and those species in which it does not happen. In *Metaphysics* I.1 (980a27–29) this takes the form of a distinction between those animals in which memory (μνήμη) arises and those in which it does not. Given the linguistic and textual parallels, therefore, I think it is clear that in both passages Aristotle posits memory as the first stage in the process of acquiring knowledge from sense perception. Although he seems to overlook *phantasia,* I think one can correlate the interpretive functions of imagination and memory in Aristotle's psychology. In order to show this, one should compare the *Metaphysics* and *Analytics* texts, while glancing at parallel passages in *De Memoria* and *De Anima.*

The necessity of memory as a condition for acquiring knowledge is made clear in the *Posterior Analytics* where Aristotle claims (99b37–39) that for those animals which do not have memory there is no cognition (γνῶσις) beyond sense perception. In *Metaphysics* I.1 he says (980b1–2) that animals which have memory are more prudent (φρονιμώτερα) and more teachable (μαθητικώτερα) than animals which are not capable of remembering. But if memory is merely the recollection of individual sensations then it is not clear how it leads to universal knowledge, though a hint may be found in *Posterior Analytics* II.19. What the relevant passage suggests is that memory depends on some thing (perhaps an image) becoming established in the soul because only if this remains can there be recollection of it.[95]

Such a suggestion is confirmed by a passage near the beginning of *De Memoria* which states (449b26–28) that memory is not identical with either sense perception or belief but is rather some permanent disposition (ἕξις) or modification (πάθος) of them which is dependent

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[95] I agree with Barnes that reading ἐν οἷς at 99b39 makes no textual or grammatical sense and that it should be replaced by ἐνίοις δ' which would yield the expected contrast with ὅσοις μὲν at 99b37. But my reading of ἕν τι instead of ἔτι at 100a1 is more dubious, even though the text as it stands does not make complete sense. Barnes (1975) 252–3 thinks that neither word should stand in the passage which thus should be translated "but for some perceivers it is possible to grasp it (sc. the percept) in their minds." However, I think that his reading would make the whole contrast pointless, since the animals without memory also have the percept in their souls but not the permanent impression which memory supplies.

on the lapse of time. Except for the integral connection between memory and time, it is striking how similar this is to Aristotle's account of *phantasia*. In fact, within the same context, he refers to the account of *phantasia* given in *De Anima* III.3:

> An account has already been given of imagination in the discussion of the soul, and it is not possible to think without an image. For the same effect occurs in thinking as in drawing a diagram. For in the latter case, though we do not make any use of the fact that the size of the triangle is determinate, we none the less draw it with a determinate size. And similarly someone who is thinking, even if he is not thinking of something with a size, places something with a size before his eyes, but thinks of it not as having a size. If its nature is that of things which have a size, but not a determinate one he places before his eyes something with a determinate size, but thinks of it simply as having size.[96]

This passage throws some light on Aristotle's view of the role of *phantasia* in thinking about mathematical objects, but what interests me here is the connection with memory which emerges in a subsequent passage (450a13–14) where Aristotle declares that memory, even the memory of intelligibles, cannot exist without images. Yet it is difficult to reconstruct exactly how he arrived at that conclusion, since the line of argument is not at all clear. Sorabji (1972, 74) makes the plausible suggestion that the passage actually contains two separate arguments for the same conclusion; i.e. that memory is connected with the primary perceptive part of the soul. In both arguments, however, *phantasia* serves as a kind of middle term because Aristotle assumes that images are affections of the common sense and that memory cannot exist without images. But this, in turn, seems to rest on an assumption which G.R.T. Ross (1907, 252) attributes to Aristotle; i.e. that the imageable element (τὸ φανταστόν) of things is identical with the element of continuity (τὸ συνεχές).

Some traces of this assumption may be found in the subsequent passage (450a8–10) from *De Memoria*, where Aristotle postpones his explanation of why we cannot think (νοεῖν) anything without the continuous, though it is illustrated by the parallel with geometrical construction in the previous passage. There Aristotle declares that we cannot think without an image, and draws a parallel between what happens in the construction of a geometrical diagram and our

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[96] *Mem.* 449b33–450a8: tr. Sorabji (1972).

experience of thinking. From the universal point of view of geometrical proof, it is irrelevant that the constructed triangle has a determinate size, yet we must construct it with some definite size. Similarly, in the case of thinking, Aristotle claims (450a4–6) that a quantity is set before the eyes, even though we are not thinking of the thing as a quantity. Even for theoretical objects whose nature is quantitative yet indeterminate, he insists that their representations have a definite quantity though they are being considered theoretically only qua quantity.

Perhaps Aristotle has in mind here some theorem of Eudoxus, which deals with quantity in a completely general way while using as illustrations lines of a definite length.[97] What is noticed here was also remarked upon by British empiricists like Berkeley and Hume; i.e. that images are always particular even though the corresponding thoughts may be general. But the conclusions which Aristotle draws from that discovery are quite different, since he is not pursuing any so-called 'historical method' of accounting for the origin of ideas. Although he does want to base thinking on sense perception, he acknowledges a gap that *phantasia* helps to bridge. In this passage from *De Memoria* I think there is solid evidence that he gives an interpretive role to *phantasia* because Aristotle begins by asserting that there is no thought without *phantasia* and then treats the latter as being more akin to sense perception. Subsequently (450a11–12), in fact, he claims that the image is an attribute of the common sense, though this claim is part of his argument (450a10–14) that the primary faculty of sensation (τὸ πρώτον αἰσθητικόν) is what grasps continua like magnitude, motion, and time. Thus, since memory is characterized by an awareness of time, Aristotle concludes that it belongs *per se* to the common sense, even though it may belong *per accidens* to the noetic faculty.

The integral connection between memory and sense perception is used by Aristotle to explain (450a16 ff.) the fact that other animals besides man have the capacity for memory. If memory belonged essentially to the noetic parts of the soul, he argues, then many of the other kinds of animal would not have it. For Aristotle temporal awareness is an essential part of memory because, in remembering

[97] This is how the propositions are illustrated in Euclid's *Elements* V. A similar point about the unavoidable particularity of images is stressed by Berkeley in the Introduction to his *Treatise on Principles of Human Knowledge.*

something, we are also conscious that we have seen or heard or learned this thing at some prior time. He concludes (450a24–25) from all these considerations that memory belongs to the same faculty of the soul (i.e. the common sense) as that to which *phantasia* belongs. Therefore, he says (450a26–27), such things as are *per se* memorables are also *phantasta*, while those things which do not exist without *phantasia* are objects of memory *per accidens*.

In effect, Aristotle is claiming that all the objects of imagination are in themselves also objects of memory, whereas intelligible objects are held to be objects of memory incidentally, though they cannot be thought without *phantasia*. Thus, since memory and *phantasia* are both dependent upon sense perception, we might assume that the classes of their respective objects are at least coextensive for Aristotle, even though he underlines the awareness of time involved in memory. Such an assumption establishes a close parallel between memory and *phantasia* in terms of their respective functions within the cognitive transition from sensation to knowledge. For instance, whenever Aristotle talks about the role of memory in that transition, we can take it that he means to include *phantasia* also on account of his general claim in *De Memoria* that memory does not exist without images. From this perspective we can now see the significance of his statement in *Metaphysics* I.1 (980b25–27) that all other species of animals besides man live by appearances (ταῖς φαντασίαις) and memories (ταῖς μνήμαις) but participate very little in experience (ἐμπειρία). The unique cognitive abilities of mankind must be put down to experience because memory and imagination are shared by other species of animal, given the integral connection between these capacities and sense perception.

Thus it is the capacity for experience which distinguishes human cognition from that of the other species of animal. According to Aristotle experience arises from many memories of the same thing because, though the memories may be numerically many (πολλαὶ τῷ ἀριθμῷ), they constitute a single experience (ἐμπειρία μία); cf. *APst.* 100a4–6, *Met.* 980b28–981a1. Still it is not quite clear whether particulars or universals are grasped *in* experience.[98] When comparing men to other animals that participate very little in experience, Aristotle

[98] Drawing upon some parallels with *Physics* I.1, Robert Turnbull (1976) has suggested that Aristotle may have in mind 'sensory universals' which are closely tied to particulars.

says (*Met.* 980b25–28) that the race of men live by art and calculations. But, even though they look similar in many ways, art and experience differ because science and craft come to be *through* experience (981a1–5). Subsequently, he explains that art is generated in the soul when a single universal belief (μία καθόλου ὑπόληψις) arises out of many notions (ἐκ πολλῶν ἐννοημάτων) belonging to experience. For instance, he says (981a7–12), experience consists of beliefs about individuals like Socrates and about particular remedies which cured this disease in the past; whereas art involves having a universal belief about all persons of a single sort (κατ' εἶδος ἕν), like the phlegmatic or the bilious, who are cured of this kind of disease.

Now this illustration seems to suggest that art grasps the universal or general kind, whereas experience is confined to the particular. But that is not quite consistent with what Aristotle says in the *Posterior Analytics* (100a6–9) where he describes the transition from experience to science and art. There he claims that experience provides the starting-point for art with reference to generation and for science with reference to being, although it is not clear whether he means that the principle is grasped *in* experience or *through* experience, despite his consistent use of ἐκ. If we take this in the latter sense, however, we encounter difficulties in making sense of Aristotle's alternative description of the content of experience as "from the whole universal that has come to rest in the mind."[99] Should we interpret this as a description of what the soul grasps *in* experience or of what it grasps *through* experience? Perhaps the use of ἐκ weighs in favor of the latter, especially since the whole universal that comes to rest in the mind is characterized (100a7–8) as 'the one beside the many' (τοῦ ἑνὸς παρὰ τὰ πολλά) and as the unity (ἕν) which is identically the same (τὸ αὐτό) in all these things (ἐν ἅπασιν ἐκείνοις). If we could establish that ἐκείνοις refers to particulars as objects of experience, then we could confidently take Aristotle to mean that the first universal comes to be in the soul as a result of experience through some kind of induction.[100]

In the parallel passage at *Metaphysics* I.1, one can find support for this interpretation in his claim (981a12 ff.) that with respect to action

[99] ἐκ παντὸς ἠρεμήσαντος τοῦ καθόλου ἐν τῇ ψυχῇ, *APst.* 100a6–7.
[100] This process is called 'concept acquisition' by Barnes (1975, 252–4) but it might be better to talk about how the soul grasps (or receives) forms, which are universal when taken without matter.

there does not appear to be any difference between experience and art. As evidence for such a claim, Aristotle appeals to the well-known fact that in the field of action men of experience succeed better than inexperienced men with a theory. His explanation (981a15 ff.) of this fact is that experience involves acquaintance with particulars (τῶν καθ' ἕκαστον), whereas art is about universals (τῶν καθόλου). Therefore the experienced man is successful precisely because all actions and all productions are concerned with the particular. For example, the doctor does not cure 'man' (as a universal) but rather Callias or Socrates or some other individual whose essence is that of a man. Thus, if one has a λόγος without experience, one is liable to fail in applied disciplines like medicine because it is an individual like Socrates or Callias who must be cured.

His analysis of the epistemological condition of the exclusively theoretical person is revealing because it stresses that, while he knows the universal, he is unfamiliar with the particular which falls under it.[101] When we connect this analysis with previous statements, we have the materials for Aristotle's distinction between art and experience as cognitive orientations towards the universal and the particular, respectively. Another way of making the same distinction is to say (981a28–29) that men of experience know 'the that' (τὸ ὅτι) but do not know 'the why' (τὸ διότι), which is the goal of the theoretical man. In other words, according to Aristotle, both the artist and scientist know the causes or the reasons why something is the case, whereas the man of experience simply knows the fact that it is so.

Thus artists are thought to be wiser (σοφωτὲρους) because wisdom is attributed to men in virtue of their understanding (εἰδέναι) rather than their experience. For the same reason (981a30–981b4), the masters in any craft are held in higher esteem because they know the causes of the products and are therefore wiser than the manual workers who are like mere instruments or moving causes. Furthermore, if we treat the ability to teach as a general criterion of understanding, then art is more scientific than experience because the artist can teach; whereas the man of experience cannot since he only grasps the particular. This brings us to Aristotle's account of the transition from experience to scientific knowledge.

In *Posterior Analytics* II.19, unfortunately, Aristotle resorts to metaphor just at the point where we expect a clear explanation of that

[101] τὸ δ' ἐν τούτῳ καθ' ἕκαστον ἀγνοῇ, *Met.* 981a22.

transition. After giving his initial account of how the soul grasps universals, he claims (100a10–12) that this resolves the paradox of learning because it drives between the horns of a dilemma. The habits (ἕξεις) of soul through which we grasp first principles (whether these are universal concepts or propositions remains unclear) neither belong in us as determinate nor do they arise from other more cognitive habits but rather they come from sense perception. At this point (100a12–13), by way of illustration, Aristotle introduces the simile of a rout in battle presumably to show that the third option is plausible. When a rout occurs in battle, the process of re-forming ranks only begins after one man makes a stand (ἑνὸς στάντος) and then another and another until the army returns to its original starting-point (ἀρχή).[102] Obviously the simile is meant to illuminate a parallel cognitive process because Aristotle immediately says that the soul is such as to be capable of undergoing this.[103]

Given the relative obscurity of the parallel, he tries to restate more clearly the process of grasping universals from sense perception, while drawing upon the rout metaphor, by saying (100a15–16) that when one of the undifferentiated things makes a stand (στάντος . . . τῶν ἀδιαφόρων ἑνός) then there is a primitive universal in the soul. There has been some debate about whether τὰ ἀδιαφόρα refers to individual things or to infima species.[104] If we take this passage (100a6–7) as a restatement of his previous account, then the 'undifferentiated thing' must be equated with the first universal 'coming to a stand' (ἠρεμήσαντος) in the soul as a result of experience. But such an interpretation must also make sense of Aristotle's explanation (100a17–b1) that, although the particular is being perceived, perception is *of* the universal.

In order to address this interpretive problem, it may be useful to adopt Barnes' (1975, 255) suggestion that Aristotle is trying to meet

[102] Barnes (1975) 81 & 254 reads ἀλκήν for ἀρχήν without much MSS authority because he thinks that the latter makes no sense. Yet, even though it is hard to specify its exact meaning here, ἀρχή seems to me appropriate for the military simile and it has the right linguistic connotations for the comparison with cognitive activity, whereas ἀλκή does not fit at all.

[103] ἡ δὲ ψυχὴ ὑπάρχει τοιαύτη οὖσα οἵα δύνασθαι πάσχειν τοῦτο, *APst.* 100a13–14.

[104] Tredennick (1960) 259 thinks that it cannot mean infima species but must mean perceptible individuals, since Aristotle's account begins with them. Barnes (1975) 254–5 thinks the opposite holds because the 'stand' of a single individual cannot yield anything universal. Another possibility is Turnbull's (1976) sensory universal which is confused and hence undifferentiated.

a potential objection to the effect that his account involves a jump from particulars to universals. While it is possible for him to deny that there is a jump because perception gives us universals from the beginning, this would raise the question of why perception is not therefore identical with knowledge.[105] Perhaps this could be answered in terms of Aristotle's distinction (99b38–39) between some sort of initial acquaintance (γνῶσις) with universals gained through perception and the rational understanding (ἐπιστήμη) which is characteristic of scientific knowledge.[106] But, given his account of sense perception in *De Anima* and in *De Sensu*, we ought to share Barnes' puzzlement about Aristotle's illustration of the universal grasped through perception, when he insists (100b1–2) that it must be 'man' rather than 'a man, Callias.' Since the individual man is not a proper but rather an incidental object of perception, it is unlikely that the universal 'man' can be any kind of perceptual object; cf. *DA*. 418a21, 425a25, 430b29.[107]

Thus, if 'man' is neither a proper nor a common sensible, it is not directly implanted in the soul through the senses as Aristotle seems to suggest and we are owed an account of how such concepts are derived from perception.[108] While Barnes notes (quite correctly) that Aristotle nowhere gives us such an account, it may be possible to reconstruct one from his account of *phantasia* in *DA* III.3 as the basic interpretive capacity of the soul, which functions as an intermediary between sensation and intellection because it has a foot in both camps, so to speak. Furthermore, in keeping with its characteristic cognitive activity, Aristotle might say that *phantasia* enables the soul to perceive something *as* a man. Although he is never explicit on this point, still I think that *phantasia* is sufficiently connected with perception to yield a plausible interpretation of his actual claim here that perception is of the universal.[109] In a parallel passage from *Posterior Analytics* I.31

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[105] It is clear from *Metaphysics* IV (1010b2–26) that Aristotle takes sides with Plato against the Protagorean view that perception is identical with knowledge.

[106] In this regard, Burnyeat (1981) argues convincingly that these are two kinds of universal knowledge, one of which is informal and the other formal and axiomatic.

[107] Perhaps, as Turnbull (1976) suggests, Aristotle has in mind a confused sensory universal 'man,' which is related to the perceptible particular, Callias, who is a man.

[108] There is a similar problem for Aristotle with regard to how we grasp mathematical concepts like the straight, which are never directly accessible through experience, even though magnitude is a common sensible.

[109] ἡ δ' αἴσθησις τοῦ καθόλου ἐστίν, οἷον ἀνθρώπου, ἀλλ' οὐ Καλλίου ἀνθρώπου, *APst.* 100b1.

(87b28 ff.), he concedes that perception is of the 'such-and-such' (τοῦ τοιοῦδε) rather than of the 'this-something' (τοῦδέ τινος), even though he insists that what is being perceived must be a 'this somewhat' (τόδε τι) which exists at a particular time and place.[110] While that passage seems to rule out the possibility that a universal applicable to all things could be perceived by the senses, it should be read as an attempt to make a sharp distinction between sensation and demonstrative knowledge.

In *Posterior Analytics* II.19 he claims (100b2–3) that, just as primitive universals come to a stand in the mind, so also the ultimate universals or genera emerge from the infimae species; e.g. from such-and-such an animal comes animal in general. Thus, he concludes, it must be by induction (ἐπαγωγῇ) that we become familiar with 'the firsts' (τὰ πρῶτα), since perception also instils the universal in this way (100b3–4). Now it is clear that the explicit parallel being drawn here between the cognitive process of grasping universals from sense perception and the move from lower to higher universals is important, yet it is not easy to establish exactly what it involves.[111] Barnes (1975, 255) thinks that 'some process of abstraction' is required to move from the primitive to the higher universals, since he assumes that a similar process is involved in grasping universals through perception. But the usual terminology of abstraction is not used here or in *Metaphysics* I.1, and this significant linguistic fact has not been adequately explained by commentators.[112]

In my concluding section, therefore, I will take account of this and other linguistic facts while attempting to reconstruct, with special reference to mathematics, how Aristotle explained the cognitive process of thinking universals. One of the phenomena to be 'saved' is Aristotle's consistent description of this process as induction which, in its loosest sense, may refer to any cognitive movement from the less to the more general. Commentators are agreed that Aristotle is

[110] In *Metaphysics* XIII.10, Aristotle seems to have a different meaning of τόδε τι in mind when he says that potential knowledge is of the universal, which is indefinite, while actual knowledge is of something definite which is a 'this somewhat.'

[111] In the *Physics* (184a23 ff.) Aristotle seems to suggest that the mind moves from a generic and undifferentiated universal to a specific and differentiated one. For detailed analyses of this passage, see Konstan (1975) & Owens (1971).

[112] For example, in her major study of Aristotle's psychology, Deborah Modrak (1987) refers to 'abstraction' without further ado, as if it were a clear and unproblematic Aristotelian notion. In a later paper, however, Modrak (1989) gives a much more nuanced treatment of abstraction.

quite vague about what is involved in the inductive process of grasping universals, whether these be the infimae species or the highest genera. For instance, Ross (1949) thinks that the *Analytics* passage is not about concept acquisition but rather about how the mind grasps primitive propositions, though I think Barnes (1975, 255–6) is correct that Aristotle does not make any explicit distinction here between primitive concepts and propositions. Furthermore, the examples of universals which he gives suggest that he is thinking of terms rather than of propositions. Once the mind has grasped 'man' and 'animal,' it is no great mystery how it can grasp the definition of man as a rational animal.[113] But, given his psychological account of the activity of the intellect, the most difficult task facing an interpreter of Aristotle is to make sense of his general claim that the soul grasps universals through induction. It is to such a task that I turn finally in this chapter.

III.3. *The noetic activity of the soul*

In dealing with the part of the soul by which it knows (γινώσκει) and judges (φρονεῖ), it is clear that Aristotle is guided by his account of perception, especially at *De Anima* III.4 where he begins with the working hypothesis that thinking (τὸ νοεῖν) is like perceiving (τὸ αἰσθάνεσθαι) and draws several implications from this.[114] Despite some differences between them, Aristotle thinks (429a16–18) that the parallel between the faculties is sufficiently exact to support the following analogy: just as the faculty of perception (τὸ αἰσθητικόν) is related to sensible things (τὰ αἰσθητά), so also the intellect (τὸν νοῦν) is related to intelligible objects (τὰ νοητά). The analogy is intended to establish the capacity of the intellect for receiving intelligible forms, and for being potentially though not actually like any of them. In the parallel case, the sense faculty is potentially like all of the sensible forms which it is capable of receiving, though it is unlike them until

[113] Since propositions are given no ontological status by Aristotle, everything depends on the subject of the attributes which are predicated of it in the basic premisses of a science.

[114] Lowe (1983) thinks that, according to the rules of classical dialectic, this view must also be rejected because it is an opinion borrowed from predecessors. But, if I am right about the 'conservative' tendency in Aristotle's dialectical method, there is no reason why some of the similarities between thinking and perceiving should not be retained in his own definitive view of intellection.

it has been affected.[115] Similarly, prior to its exercise, the intellect is not like any of the intelligible forms with which it can become identical in actual thought. Since it must be potentially like all intelligible forms, the mind cannot actually be similar to any of them, although Aristotle describes it in Platonic terms as 'a place for the forms.'

The third inference (429a18–20) drawn from the analogy between sense and intellect is that the latter, since it thinks all things, must be unmixed (ἀμιγῆ) in the manner specified by Anaxagoras so that it may rule (ἵνα κρατῇ); i.e. in order that it may know (ἵνα γνωρίζῃ). The reference to Anaxagoras here is useful for ascertaining what is involved in describing the intellect as 'unmixed,' especially since a great deal of controversy surrounded this issue among the Greek and medieval commentators.[116] The report about Anaxagorean Nous in *DA* I.2 describes it as being impassive because it has nothing in common with any of the other things.[117] But this creates the problem of how it can think other things, and whether it is itself an object of thought. For Aristotle, however, nous is potentially thinkable, though it does not have its own unique form that might limit the range of objects which it can think.[118]

As a result of his extended analogy between sense and intellect, therefore, Aristotle concludes that this part of the soul (called *nous*) by which we think and believe *is* actually none of the things before it thinks.[119] Since he cannot be claiming that the intellect does not exist at all before it thinks, we must find some meaning of εἶναι that fits with the conclusions about intellect drawn from the analogy with sense perception. In that case the sense faculty is actually identical

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[115] Cf. *DA* 417a20, 418a3, 422a7, 422b15, 424a23 ff.

[116] Philoponus (*in APst.* 521.28 & 522.31) took it to mean 'unmixed with matter' and this line was adopted by Averroes and Aquinas, but Hicks (1907) 477 notes that such a reading is not supported by the text nor by the reference to Anaxagoras. More recently, however, Wedin (1988) 186 takes 'unmixed' to be a characteristic of productive mind and to mean that noetic activity is not the activity of any set of physical structures; cf. *GA* II.3, 736b22–9.

[117] καὶ κοινὸν οὐθὲν οὐθενὶ τῶν ἄλλων ἔχειν, *DA* 405b20–21.

[118] This conclusion is supported by *DA* 429b26–430a9 where Aristotle says that mind itself is an object of thought because, with regard to things without matter, that which thinks and that which is thought are the same. At *DA* 429b5–6, however, he seems to say that the mind thinks itself when it thinks *any* proper object of thought. *Metaphysics* XII.9 seems to provide a solution by distinguishing noetic activity from all other cognitive activities of the soul (e.g. dianoia, doxa & perception) as the unique activity in which the mind thinks itself because it is completely identical with its objects.

[119] οὐθέν ἐστιν ἐνεργείᾳ τῶν ὄντων πρὶν νοεῖν, *DA* 429a24.

with its proper object when perceiving it, and presumably the same is meant to hold for the intellect with respect to intelligible objects. While the 'is' of identity seems to fit best, yet εἶναι is temporally modified such that, prior to thinking some intelligible object, the intellect is potentially identical with all such objects, whereas it is actually identical with just one while thinking it.[120]

Having already argued that the intellect is impassive and unmixed, Aristotle now (429a24–25) concludes that this part of the soul has not been mixed with the body. The reason is that if it were mixed with the body then it would come to have a certain quality (ποιός τι), such as heat or cold, and so it would need some corresponding organ as does sense perception. But none of these things are true of the intellect, and so it is not mixed with the body.[121] Aristotle seems to be resisting the tendency to assimilate sense and intellect for which he faults previous natural philosophers like Empedocles. This may be why he praises those thinkers who call the soul 'a place of forms' (τόπον εἰδῶν), although he insists (429a27–28) that the description is more appropriate for the thinking part of the soul (ἡ νοητική) which contains the forms not actually but potentially.

The whole point of this discussion becomes clear in the subsequent passage (429a29 ff.) where Aristotle distinguishes the impassivity (ἀπάθεια) of the intellective faculty from that of the sense faculty by virtue of the fact that the latter is tied to sense organs. This fact makes all the difference because it means that the sense faculty has only relative impassivity, whereas the intellective faculty possesses it absolutely. He considers each of the sense faculties to be only relatively impassive because it may be impaired when its corresponding sense organ is exposed to extremes. By contrast, the intellect is absolutely impassive because, when it thinks something which is exceedingly intelligible, it is not impaired but rather enabled to think less intelligible things even better (429b3–4). Although Aristotle does not supply any examples of greater and less intelligible things, Hicks (1907, 483) assumes that he has in mind greater and lesser degrees

[120] This may be compared with the passage in *Metaphysics* VII.10 (1036a2–9) which seems to make the 'existence' of particular mathematical objects dependent on the activity of thinking.

[121] I agree with Kahn (1966b) that one should not take Aristotle to be introducing some kind of proto-Cartesian separation of the mind from the body. His radically different perspective is revealed by the question of whether there is any part of the soul that is independent of the body. Just as in the case of the Prime Mover, Aristotle assumes that one must prove the existence of such immaterial substances.

of abstraction. Aristotle connects (429b4–5) the absolute impassivity of intellect with the fact that it is separable (χωριστός) from body, whereas the limited impassivity of the sense faculty is due to the fact that it cannot exist without body.

After establishing the relative independence of intellect from body, Aristotle introduces (429b5 ff.) his distinction between the two different states of potency that pertain to the intellect. First, there is the potency$_1$ of the intellect for knowledge prior to it having learned or discovered anything; e.g. the mind of an untutored child. In such a state the intellect is most dependent on body because the correlative actuality$_1$ of learning something requires sensation, *phantasia,* and memory, which all require bodily organs to a greater or lesser degree. Thus the first actuality$_1$ of the intellect, corresponding to the primary potency$_1$, involves the acquisition of universal concepts in the manner described by Aristotle in *Posterior Analytics* II.19, and *Metaphysics* I.1.[122]

By contrast, the actuality$_2$ described in the present passage (*DA* 429b7 ff.) no longer requires the external stimulus of information received through the senses, since Aristotle says that the scientist is now able to be active through himself (ἐνεργεῖν δι' αὑτοῦ).[123] This represents the intellectual potency$_2$ of someone who has already grasped the universal concepts, and so is capable of rethinking them without recourse to experience.[124] Such a person is called a scientist (ἐπιστήμων) because he has acquired a habit (ἕξις) of thinking which can be reactivated at will and which is therefore different from first potency$_1$. Aristotle holds (*DA* 429b9) that it is only when the intellect is engaged in such activity$_2$ that it is also capable of thinking itself, and so

[122] Since Aristotle clearly developed his epistemology in response to the Meno paradox, it may be appropriate to compare this first actuality$_1$ of the intellect to the cognitive state of the slave boy when he has grasped for the first time that the square on the diagonal equals the sum of squares on the other two sides of a right-angled triangle. By contrast, the second actuality$_2$ may involve the slave boy recalling this proposition and perhaps showing why it is true.

[123] In connection with such thinking Aristotle introduces the projecting (βουλευτική) or intellective (νοητική) *phantasia* which is 'up to us' in the sense that we can project a particular situation in which some action would be appropriate or imagine some particular thing like a diagram to which the universal concept applies; cf. *Mem.* 449b31–450a7.

[124] Lowe (1983) distinguishes between *apprehensive* thinking about things having matter by means of sensation and *autonomous* thinking about things without matter by means of imagination. In the latter he includes the contemplative thinking of mathematics and natural philosophy, as well as thinking about concrete objects in their absence.

its unique feature is the capacity for self-activation; i.e. that it can begin at will to think a stock of concepts which it has acquired through induction.

Aristotle's discussion of the similarities and differences between sensation and intellection may provide some clues to an obscure passage in *De Anima* III.4 (429b10 ff.) where he distinguishes between composite things and their essences, and between the corresponding faculties by which we judge them. He illustrates this first distinction by means of the difference between magnitude (τὸ μέγεθος) and the essence of magnitude (τὸ μεγέθει εἶναι) and between water and the essence of water. Even though these are mathematical and physical concepts, respectively, Aristotle is not illustrating that distinction when he cites (429b13 ff.) the difference between flesh (ἡ σάρξ) and the essence of flesh (τὸ σαρκὶ εἶναι) as a representative example. With reference to that example, he claims that one judges (κρίνει) each of these either by a different thing (ἄλλῳ) or by the same thing differently disposed (ἄλλως ἔχοντι). The latter option is puzzling, in view of the previous indications that sense and intellect are different faculties, and in the absence of any prior hint of the possibility that sensation and intellection might be just different dispositions of the same thing.[125]

Perhaps there is a clue in his subsequent explanation (*DA* 429b13–14) that flesh cannot be without matter and that, like the snub, it is rather a 'this in this' (τόδε ἐν τῷδε). It seems fairly clear from the language used that flesh is a particular form in a particular matter and, therefore, that it is a composite object. Yet it is not so easy to decide whether this compound is an object of perception (i.e. a token of flesh) or an intelligible object; i.e. flesh as a universal compound (or type) having this kind of form and that kind of matter.[126] I think that the whole passage makes more sense, however, if we treat flesh as a sensible object, which is being distinguished from the essence of flesh as an intelligible object. In support of this reading consider the fact that Aristotle explicitly says (429b15–16) that it is by means of the faculty of sense perception that one judges the hot and the cold and those other sensible qualities of which flesh is a certain proportion (λόγος τις). Although flesh may be neither a proper nor a common sensible but rather an incidental sensible, this

[125] Aristotle's point may be that these faculties are just different dispositions of the soul.
[126] Chen (1964) finds such concrete universals in *Metaphysics* VII.10–11.

proportion may still be a sensible form, which is judged by the same thing as that by which one judges the essence of flesh, which is quite clearly an intelligible form.[127]

Therefore, Aristotle holds that the proportion (λόγος) of sensible qualities constituting flesh is a sensible form that is grasped (or judged) by the soul through perception. So the first possibility is that flesh and the essence of flesh are judged by the same thing (soul) differently disposed, just as a bent line is related to itself when straightened out.[128] The alternative possibility is that the essence of flesh is judged by something separate (χωριστῷ) from the faculty which grasps the composite. Hicks (1907, 489) argues that χωριστός here must be understood as 'absolutely separable' in the sense that the intellect is wholly independent of bodily activity and its related faculties. If this is correct then the first possibility, as expressed metaphorically, implies that the soul is dependent upon or integrally related to the sensory activity of the body.

In this connection, we should not lose sight of Aristotle's distinction between sensation and intellection as different activities of the soul. Perhaps their ultimate unity is to be found in consciousness, which Kahn (1966a, 43–8) thinks is assigned to the sentient faculty because this characterizes man as an animal. But yet we must consider the possibility that intellect is something separated (χωριστός), whether we take this in an absolute or in a relative sense. This is necessary for clarifying how the intellect grasps mathematical objects, since Aristotle's account is guided both by the ontological status of these objects and by the nature of the faculty judging them.

So much is apparent from a passage in *DA* III.4 which follows directly upon the one just discussed:

> Again, in the case of those things which exist in abstraction, the straight corresponds to the snub, for it involves extension; but 'what it is for it to be what it was,' if what it is to be straight and the straight are different, is something else; let it be duality. We judge it, then, by something different or by the same thing differently disposed. In general, then, as things are distinct from matter, so it is too with what concerns the intellect.[129]

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[127] Cf. Barker (1981) for a discussion of proportions and sense perception.

[128] *DA* 429b16–17. See Berti (1978) on the intellection of 'indivisibles' by contrast with that of compounds, and von Fritz (1971) on the 3 different types of induction corresponding to physical, mathematical and metaphysical inquiries, respectively.

[129] *DA* III.4, 429b18–22: tr. Hamlyn (1968).

This passage continues the previous discussion about the distinction between concrete compounds, such as flesh or the snub, and their essences. In fact, with reference to 'the beings in abstraction' (ἐπὶ τῶν ἐν ἀφαιρέσει ὄντων), Aristotle draws (429b18–19) an explicit parallel between the snub (τὸ σιμόν) and the straight (τὸ εὐθύ) because the straight presupposes extension. From the logical distinction between the straight and what it is to be straight, Aristotle draws ontological and epistemological conclusions.

In fact, the argument may be summarized as follows: if the straight and the essence of straight are different (just like the snub and the essence of snub) then they will be judged either by different faculties or by the same faculty differently disposed. When these options from the previous passage are now formulated in terms of faculties, we can see the issue more clearly. The first option is more likely to be chosen by Aristotle if the contrast is between sensible and intelligible objects, unless he means that it is the soul itself which is differently disposed in judging the straight and its essence. On the other hand, if these are two distinct kinds of intelligible object then they will be grasped by the intellectual faculty in different dispositions.

Our chief interest in the passage, however, is to clarify what it says about how the soul grasps mathematical objects. Although he refers to such objects in talking about 'the beings in abstraction,' it is difficult to know exactly what is implied by such terminology. Whereas most commentators assume that Aristotle is referring to abstract entities arising from some epistemological process of abstraction, I hold that the reference is to logical entities that result from an established Academic method of 'subtraction,' which Aristotle uses to determine the primary logical subject of any given attribute.[130]

If one adopts such a logical perspective, one can explain why such terminology is used only with reference to mathematical objects. In the present passage, for instance, it is clear that the straight is held to belong among 'the beings in abstraction,' while it is also said to be like the snub because it is together with extension, presumably in the same way that the snub involves a nose. Yet abstraction from matter is not the key to understanding this parallel because neither the snub nor the straight can be logically separated from their corresponding kinds of matter. Instead these are parallel concepts because just as snubness is an attribute that belongs primarily to a

[130] Cf. Cleary (1985).

nose, so also straightness belongs primarily to a line. Therefore the snub and the straight are similar universal composites, even though each of them is linked with a different kind of matter through their primary subjects. So just as the snub involves flesh, the straight involves a continuum in one dimension, which itself is a kind of intelligible matter. And it is precisely this logical connection between straightness and some kind of matter which enables Aristotle to distinguish between the straight and its essence in the same way that he previously distinguished between flesh and its essence. Thus, by appealing to the Pythagorean approach to mathematics, Aristotle can define straightness in terms of duality so as to support his distinction. According to a typical Pythagorean schema, 1 determines a point, while 2 defines a line, 3 defines a plane, and 4 defines a solid; cf. *Met.* 1036b12, 1043a29.[131]

Hence Aristotle seems to adopt this Pythagorean definition of a line for the purpose of illustrating his own distinction between the straight and its essence. But, in fact, there is some evidence that this way of thinking about the mathematical line in terms of the number 2 exercised a great deal of influence on the mathematical tradition which is preserved in Euclid.[132] The purpose of the example in this passage, however, is to distinguish between the form and the compound in the case of mathematical entities, just as in the case of physical entities. So Aristotle can say (*DA* 429b20–21) for each parallel case that the composite and its essence are judged either by different faculties or by the same one differently disposed.[133] As a general conclusion, therefore, he says that just as things are separable from matter, so also is it the case with the intellect.[134] In this rather enigmatic conclusion Aristotle seems to claim that the intellect conforms to things insofar as they are distinct from matter.

There is some ambiguity in the phrase τὰ περὶ τὸν νοῦν since it

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[131] Proclus (*in Eucl.* 97.18 ff.) reports that this schema is Pythagorean in origin, and it seems to have been used by Speusippus (Fr. 28.34–35 Taran). See *Phys.* II.3, 194b26–29 for a parallel example that gives the essence of an octave as number (i.e. ratio 2:1) and see II.6, 198a17–18 which uses the straight line to show that in mathematics the formal cause is contained in the definition.

[132] Cf. *Parm.* 137E, *Met.* 1036b12, 1043a29, *DA* 404b20–22, & Heath (1921, 158–60).

[133] Aristotle's rationale for offering these two alternatives may be the recognition on his part that a composite like 'the straight line' may be either particular or universal and hence may function as a cognitive object for either perception or intellection; cf. *Metaphysics* VI.10–11.

[134] ὅλως ἄρα ὡς χωριστὰ τὰ πράγματα τῆς ὕλης, οὕτω καὶ τὰ περὶ τὸν νοῦν, *DA* 429b21–22.

could refer either to the intellectual faculty or to its objects. If Hicks (1907, 492–3) is correct about it being a periphrasis for νοῦς then the whole statement may be interpreted as follows: according to the degree to which a cognitive form is involved in matter or is free from it, so it is also with the faculty which judges that form. Thus, for instance, in grasping the snub as a compound the intellect is oriented towards sense perception and will think the concept in conjunction with a concrete image. Just as with the snub so also with the straight, the intellect will think it with the help of some image like a diagram. On the other hand, snubness and straightness are 'without matter' and so these essences will be grasped directly by the intellect. But, since the snub and the straight both involve matter, even though one is sensible and the other is intelligible, the intellect grasps them indirectly through perception and imagination.

Although I hold that Aristotle does not have any epistemological theory of abstraction, I think one can reconstruct from the *De Anima* some psychological account of abstraction in terms of the cognitional activity of the soul. I take it that the terminology of abstraction has a basic logical meaning that determines the manner in which abstract objects are grasped by the intellect. From that perspective, let us see how we might make sense of the above passage. As I have already indicated, the straight may be taken as an example of an abstract entity because it involves the logical isolation of an attribute, straightness, together with its primary subject, a line. Despite the subtraction which yields this entity, however, the straight is similar in logical structure to the snub because that also involves the isolation of an attribute, snubness, together with its primary subject, a nose. By virtue of their similarity as logical compounds, Aristotle can distinguish in each case between the composite and its essence which are said to be judged either by different faculties or by the same faculty in different dispositions. Yet there is also a difference between these compounds in that, through their respective logical subjects, they involve distinctive kinds of matter. For instance, the snub is logically tied to sensible matter because its primary subject presupposes flesh, which Aristotle holds (*DA* 429b15–16) to be a ratio of qualities like hot and cold that are judged by the faculty of sense perception.

This may well be the reason why he consistently cites the snub as a paradigmatic example of the objects studied by the science of physics, which is concerned with things subject to change by virtue of their

natures. Incidentally, such theoretical objects are sometimes referred to as τὰ ἐκ προσθέσεως, which I take to mean that they are products of a logical method of addition that is the inverse of subtraction.[135] By contrast, abstract objects like the straight, which are typically studied by mathematics, logically involve not sensible but rather intelligible matter such as continuous magnitude or the units of mathematical number. From this logical perspective we can now offer a very different interpretation of Aristotle's general conclusion to the effect that, just so far as things are separable from matter, so also are the powers of intellect that judge them. Since the distinction between the straight and its essence is like that between the snub and its essence, he concludes that in each case a different cognitive capacity judges the composite and its essence. In both cases, some form is being separated from its appropriate matter and grasped directly by the intellect.

The point is clarified further by *DA* III.6 where Aristotle distinguishes between the way in which the intellect grasps indivisibles directly or not at all, and the way in which it grasps compounds as either true or false. Among the so-called 'indivisibles,' he includes indivisibles in quantity and in species, as well as things with contraries such as limits and privations, and things with no contraries like substance. According to him, the intellection of indivisibles happens in an indivisible time and it involves essences that are 'without matter' (ἄνευ ὕλης). Comparison with *Metaphysics* IX.10 shows that these are incomposite things, which are contrasted with composite things put together by the mind. For instance, the triangle is an indivisible kind that is either grasped or not, whereas the incommensurability of the diagonal is an intelligible composite that is either true or false. Therefore, knowledge of the essence of each indivisible is always true, whereas knowledge of what belongs to something else may be either true or false.

So, when Aristotle talks about the intellection of indivisibles, he means knowledge of the essence of any universal, whether that be substance, quantity, quality, or of any other category. As Berti (1978) rightly points out, all such essences are immaterial because they are logically separable from the material composite; so that Aristotle need not be referring only to immaterial substances. This is especially the case for the mathematical examples of a line and a point, since these are not immaterial substances but rather conceptually separated as-

[135] Cf. *De Caelo* 299a15–17. For a similar interpretation see Philippe (1948).

pects of material realities. Thus Aristotle uses the metaphor of the bent line and the straightened line to describe the two related uses of the intellect in grasping an intelligible composite and its essence; e.g. the straight and the essence of straight.

According to my interpretation, therefore, the passage quoted above from *DA* III.4 provides no support for a general epistemological theory of abstraction in Aristotle, although it does enable us to understand the familiar terminology in an alternative logical way. In order to show that this interpretation stands up, I will briefly survey some of the other passages in the *De Anima* where such terminology occurs in discussions of how the mind grasps mathematical objects. At *DA* III.7, for instance, there is a parallel passage which goes as follows:

> Those things which are spoken of as in abstraction one thinks of just as, if one thought actually of the snub, not qua snub, but separately qua hollow, one would think of it apart from the flesh in which the hollow exists—one thinks of mathematical entities which are not separate, as separate, when one thinks of them. In general, the intellect in activity is its objects. Whether or not it is possible for the intellect to think of any objects which are separate from spatial magnitude when it is itself not so separate must be considered later.[136]

Despite the disjointed appearance of the whole chapter, I think that Aristotle's summary conclusion (beginning with "In general, . . .") shows that the context in which this passage should be understood is the continuing discussion of how the mind thinks its objects. In fact, the general principle which consistently guides that discussion is repeated here; i.e. that when the intellect is in a state of activity it is identical with its object. Furthermore, as he says earlier (431b1–2), it is the forms (rather than the matter) which the intellects thinks in images.

In view of this reference to imagination, we can understand the point of the final sentence (431b18–19) which mentions the aporia about whether or not the mind can think anything among the things that are completely separated (τῶν κεχωρισμένων), if it is not itself separated from bodily magnitude. While such an aporia might be relevant to thinking about an unmoved mover, it is hardly applicable to thinking about mathematical objects because Aristotle denies that they are separated in reality from bodily magnitudes. This is implied in the above passage when he says that the intellect thinks the

[136] *De Anima* III.7, 431b12–19: tr. Hamlyn (1968).

objects of mathematics as if they were separated (ὡς κεχωρισμένα), even though they are not separated in reality. We may equate these mathematical objects with 'the things spoken about in subtraction' (τὰ ἐν ἀφαιρέσει λεγόμενα), whose manner of being thought is the main topic of the passage. Aristotle tries to explain this mode of thinking by means of a rather elaborate comparison with how the intellect thinks about physical objects like the snub. Despite possible corruptions in the text, the main point of the comparison is clear; i.e. thinking about mathematical objects is like thinking about snubness without reference to the sensible matter in which it is embodied.[137]

The most significant thing about the whole passage, however, is Aristotle's extensive use of the 'qua' (ᾗ) locution to capture this rather difficult comparison between the ways in which the intellect thinks mathematical and physical objects. He says (431b13–14) that one thinks of the so-called things in subtraction just as if one were to think of the snub not qua snub but rather separately (κεχωρισμένως) qua concave (ᾗ κοῖλον).[138] If one were to think of the snub in this way then one would think of it without the flesh in which concavity is embodied in the case of a sensible compound. The counterfactual conditional structure used here suggests that, normally, one would not think in this way about the snub but that, if one did, one would be thinking of a mathematical object. Thus Aristotle concludes (431b15–17) that when one thinks mathematical objects, one similarly thinks as separate what are not separated in reality. For our understanding of the whole passage, therefore, it is important to decide on how the meaning of 'separate' is related to the 'qua' locution.

In line with my interpretation of *Metaphysics* XIII, I take its primary meaning here to be a logical one that has both ontological and psychological implications. However one reads the Greek in the first part of the passage, I think that the adverb κεχωρισμένως must be taken in conjunction with ἄνευ τῆς σαρκός to establish its basic logical meaning. Snubness is logically inseparable from flesh because it is an attribute that belongs primarily to a nose, whereas concavity is

[137] There is a necessary relation between snubness and flesh (as matter) which does not hold for concavity and flesh; cf. *Met.* VII.10, 1035a4, 11, 1036b23. See also Balme (1984).

[138] In a comprehensive logical analysis of the 'qua' locution, Bäck (1979) argues that the qualification introduced by 'qua' belongs to the predicate rather than to the subject. This logical fact also facilitates the conceptual separation of mathematical attributes from the sensible subjects to which they actually belong, according to Aristotle's ontology.

separable from sensible matter because its primary subject is a line. One of the functions of the 'qua' locution, therefore, is to indicate the primary subject involved by indicating the particular aspect of the snub nose which is being considered; e.g. qua nose it is snub, whereas qua line it is concave. It is on account of this logical situation (rather than the ontological relation of concavity to the flesh in which it is embodied) that the intellect can think of the concavity of the snub nose without reference to flesh, whereas this is impossible for snubness. As evidence for the primacy of the logical over the ontological situation with respect to thinking, one should consider the fact that Aristotle maintains the possibility of thinking mathematical objects as separable (logically), even though they are not separated (ontologically) from sensible magnitudes. Hence it is the logical method of subtraction that makes possible the conceptual activity typical of mathematics, so that its objects are rightly called 'the results of subtraction.'

The final passage I want to consider finds its context in *DA* III.8 within a general summary of Aristotle's inquiry about the cognitive powers of the soul. At the beginning of the chapter, for instance, he sums up this inquiry by saying that the soul is in a way all beings.[139] The identity of the soul with its cognitive objects is explained by Aristotle in terms of a broad division of beings into sensible (αἰσθητά) and intelligible (νοητά) entities, which are grasped through sense and intellect. Even though this is familiar from previous discussions, Aristotle feels it necessary to explain further how knowledge (ἐπιστήμη) is in a way identical with knowable things (ἐπιστητά) and, similarly, how sensation (αἴσθησις) is somehow identical with sensible things. Thus he makes (431b25–26) an internal division within knowledge and sensation which corresponds to things in the sense that potencies correspond with potencies, while actualities answer to actualities. What Aristotle seems to mean by this is that the distinction between potentiality and actuality applies correlatively to cognitive capacities and their objects. On the cognitive side, we have already met the distinction between a faculty and its exercise, whether this be the power of knowing in the soul as distinct from the activity of knowing or the power of sensing as different from actual sensing.

But the corresponding distinction for the realm of objects seems to involve a division between potential and actual sensibles and,

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[139] ἡ ψυχὴ τὰ ὄντα πώς ἐστι πάντα, *DA* 431b21.

similarly, between potential and actual intelligibles. Within the present context I do not see how it can be taken as a division between actual and potential *existents*, as some scholars think.[140] In fact, what it seems to mean is that with respect to sensible things we should distinguish between the compound thing which is potentially sensible and its form which is actually sensed. Similarly, with reference to the sense faculty itself, we must distinguish between the power of perception and the activity of perception. Now the sense faculty is potentially identical with the corresponding sensible form but not with the sensible compound because, as Aristotle puts it (*DA* 431b29), it is not the stone which is in the soul (ἐν τῇ ψυχῇ) but rather the form (τὸ εἶδος). However, it is only when the sense faculty is in activity that it is actually identical with any one of these sensible forms which are received into the soul, so to speak. In modern terms we might say that the faculty itself is a permanent capacity for perception, just as the sensible compound can always be perceived through its form. Perhaps that is the basis for the correlative divisions that Aristotle wants to make within the cognitive and objective spheres. If so, it would throw a different light on his account of how the soul grasps cognitive objects, either through the senses or the intellect.

Aristotle's general view of the cognitive activity of the soul is summed up in a striking analogy which he subsequently (*DA* 432a1–3) draws between it and the human hand: just as the hand is an instrument of instruments (ὄργανον ὀργάνων), so the intellect is a form of forms (εἶδος εἰδῶν) and the sense faculty is a form of sensibles (εἶδος αἰσθητῶν). The comparison which grounds the analogy is particularly appropriate because of the obvious connection between human intelligence and the use of tools for which the hand is indispensable. Contrary to the opinion of Anaxagoras that the possession of hands is the cause of man being the most intelligent (φρονιμώτατον) of animals, Aristotle insists that a man has hands because he is the most intelligent animal; cf. *PA* 687a8 ff. Behind this difference of emphasis lies his conception of nature as acting prudently in the distribution of organs to animals according to their capacity to use them. Aristotle sees (*PA* 687a12–15) the characteristic activity of nature as the perfecting of inherent powers, like giving a flute to one who has the ability to play. Therefore he concludes (687a17–19) that man does not owe his superior intelligence to his hands but rather he has

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[140] Cf. Hicks (1907) 544 & Hamlyn (1968) 149.

been given hands because of his superior intelligence. Since the mark of that intelligence is his ability to put organs to good use, the hand is not simply a single organ but has multiple uses in that it is, as it were, 'an instrument for instruments' (ὄργανον πρὸ ὀργάνων); cf. *PA* 687a20–21.

This clear parallel in terminology shows that the proper context for understanding the analogy in *De Anima* III.8 is a natural biological one, where man is seen as being endowed with appropriate instruments for the realization of certain inherent capacities for learning. Thus we might view both the intellective and sensory capacities of man as being analogous to the hand in that they are natural instruments which can 'grasp' external objects so as to use them as tools.[141] Perhaps we could even distinguish between intelligence as a natural instrument in us and artificial instruments such as the arts and sciences produced by intellect. Whether or not this was Aristotle's intention, it is clear that functional talk about instruments is particularly appropriate for both the intellective and sense faculties, which were previously said to be impassive and unmixed. Indeed, it is a functional notion of the faculties of intellect and perception that leads him to call them 'forms' of intelligible and of sensible forms, respectively. In the case of composite objects, the intelligible forms are only potentially known until they are separated from their matter, whereas essences are directly and actually known.

Even though *DA* III.8 is not completely transparent, I believe that the discussion of the relationship between the soul's cognitive capacities and their characteristic objects provides the appropriate context for the subsequent passage which is of greatest relevance for the mathematical sciences:

> Since there is no actual thing which has separate existence, apart from, as it seems, magnitudes which are objects of perception, the objects of thought are included among the forms which are objects of perception, both those that are spoken of as in abstraction and those which are dispositions and affections of objects of perception. And for this reason unless one perceived things one would not learn or understand anything, and when one contemplates one must simultaneously contemplate an image; for images are like sense-perceptions, except that they are without matter. But imagination is different from assertion and denial; for truth and falsity involve a combination of thoughts. But what distinguishes the first thoughts from images? Surely neither these

[141] Cf. *Problems* XXX, 5, 955b23 ff.

> nor any other thoughts will be images, but they will not exist without images.[142]

While the syntax of the Greek is just as convoluted as Hamlyn's translation, the meaning is tolerably clear and it supports my general claim that Aristotle does not have a special epistemological theory of abstraction to explain how mathematical concepts are grasped by the intellect. Instead, we find 'the things spoken about in subtraction' (τὰ ἐν ἀφαιρέσει λεγόμενα) being described as intelligible objects (τὰ νοητά) along with the dispositions and attributes of sensible things, all of which are contained in sensible forms (ἐν τοῖς εἴδεσι τοῖς αἰσθητοῖς). And this follows from the claim that besides sensible magnitudes nothing exists as a separated thing (πρᾶγμα κεχωρισμένον).

Whether or not Aristotle holds this view, it is typical of him to draw epistemological conclusions from such ontological claims.[143] From the claim that there exist only sensible magnitudes, for instance, he concludes (432a7–8) that one who is not perceiving something could neither learn nor understand anything. This conclusion is confirmed by the *Posterior Analytics* (I.18, 81a40–b2) where Aristotle claims that the loss of any one of the sense faculties entails the loss of a corresponding portion of knowledge. In that passage the two modes of learning are specified as induction (ἐπαγωγή), which begins from particulars (ἐκ τῶν κατὰ μέρος), and as demonstration (ἀπόδειξις) which starts from universals (ἐκ τῶν καθόλου). But for Aristotle it is impossible to contemplate (θεωρῆσαι) the universals (which include objects of mathematics) except through induction that begins from sense-perception. Therefore he concludes that if any of the sense faculties has been lost there will be a corresponding loss of the appropriate universals; e.g. a blind man cannot have a concept of color.

For my purposes the importance of Aristotle's conclusion is its presupposition that there are only two ways of learning; namely, induction and demonstration. Thus mathematical objects must be grasped in one or the other way or perhaps in both, since mathematics is for him the leading paradigm of a demonstrative science. In the *Posterior Analytics* passage (81b3–5), however, he claims that

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[142] *De Anima* III.8, 432a3–14: tr. Hamlyn (1968).

[143] In view of what he says about the separation of the Prime Mover in *Metaphysics* XII, ὡς δοκεῖ may indicate that he is accepting the assumption here merely for the sake of dialectical argument, even though some characteristic Aristotelian conclusions are drawn from it.

'the things spoken about as a result of subtraction' are made familiar (γνώριμα) through induction because some of them belong to each genus of things qua such-and-such (ᾗ τοιονδὶ), even though they are not separate (χωριστά). Whichever way we construe the syntax of that passage, I do not see how one could take abstraction to be a third way of learning, even if one treats it as a sub-species of induction like some commentators.[144] I think it is clear from the Greek that 'the results of subtraction' are being treated as intelligible entities whose mode of being is such that they are accessible through induction from sense perception; i.e. they are not separate but rather belong to each genus of sensible things insofar as these are qualified in a certain way. Most likely, Aristotle would connect this kind of induction with the use of diagrams in mathematics and with the practice of beginning from an hypothesis, since these were characteristic procedures of ancient mathematicians.

Thus I suggest that we take mathematical objects to be intelligible entities that are the products of a logical process of subtraction which isolates them along with the primary subject to which they belong, without separating them ontologically. The 'qua' locution here indicates the general aspect of sensible things that is being isolated as the logical subject of mathematical properties. In a similar way we can make sense of the claim in *De Anima* passage that 'the things spoken about in subtraction' are intelligible entities which become accessible to the soul through sensible forms. The fact that they are not ontologically separated from sensible magnitudes has direct epistemological implications; e.g. that one cannot grasp them except through sensation.

A second implication which Aristotle draws (*DA* 432a8) is that when one contemplates such intelligible entities one must do so together with some image (ἅμα φάντασμά τι). Presumably he thinks this is the case because such concepts originate in sense perception and hence *phantasia* serves as an intermediary between sensation and intellection. In fact, he seems to be referring to some such theory when he subsequently (432a9–10) explains that images (φαντάσματα) are like our sensations (αἰσθήματα), except that they are without matter (ἄνευ ὕλης). This suggests that images are the forms of perceptual content which are somehow 'abstracted' from matter, as in the case of perception itself. Similarly in his account of *phantasia,* Aristotle distinguishes

[144] Hicks (1907), Ross (1949) & von Fritz (1971).

its typical function from that of other cognitive activities like sensation, belief, and intellection; cf. *DA* III.3, 428a27 ff.[145]

Conclusion

Let me conclude this survey of Aristotle's views on the intellective capacity of the human soul by drawing out some of the general implications for the mathematical sciences and their characteristic objects. Since the term 'mathematics' signifies that its objects are preeminently learnable and teachable, it is not surprising that he should appeal to the capacity of the intellect for grasping theoretical objects such as those exemplified in the mathematical sciences, thereby placing himself squarely within the Platonic tradition of the *Meno* and *Republic*. Even though he does not accept Plato's epistemological theory of recollection, Aristotle does recognize the peculiar aptitude of the human mind for grasping 'pure' theoretical entities such as those studied by the mathematical sciences. In the *Nicomachean Ethics*,[146] for instance, he acknowledges that young men can learn mathematics, even though they are not suitable students for ethics and politics, and it is worth noting his explanation for this puzzling discrepancy between the acquisition of theoretical and practical wisdom.

The reason why young men do not become practically wise, according to Aristotle, is that prudence (φρόνησις) concerns particular things (καθ' ἕκαστα) with which one becomes familiar as a result of experience (ἐξ ἐμπειρίας) and it is precisely this that young men lack, since experience can be acquired only with time. By contrast, a young man can become a mathematician because the objects with which that science are concerned can be made familiar through subtraction (δι' ἀφαιρέσεως), whereas the principles of ethics and even of physics are grasped through experience; cf. *EN* 1142a16–19. Within the

[145] With respect to this passage, Wedin (1988) 115–6 argues that Aristotle cannot be read as claiming that one contemplates an image along with an object of thought, since there is no room in his psychology for objects of imagination (φανταστά). In other words, *phantasia* is functionally incomplete and hence is not a distinct faculty of the mind. So the image is just the means by which the thought is represented to the subject or, as Aristotle puts it, the objects of thought are forms in images. Even if Wedin is right about *phantasia* not being a distinct faculty of the soul for Aristotle, it may still function along with memory as an intermediary in the transition from sensing to thinking.

[146] Cf. *EN* VI.8, 1142a11 ff.

context of this discussion, it would seem that 'abstraction' is being proposed by Aristotle as a special epistemological process for grasping mathematical objects, in contradistinction to induction from experience which yields the principles of a practical science like ethics and even of a theoretical one like physics.

But, since this conflicts with what Aristotle says elsewhere (e.g. *Posterior Analytics* I.18), I think we must give an alternative interpretation of the *Ethics* passage which contrasts those sciences that seem to require experience with those which appear to be relatively independent of experience. We should notice also that this contrast cuts across the distinction between theoretical and practical sciences, since Aristotle claims that physics also requires experience. As far as I can see, the key to an adequate interpretation of the passage is the implicit contrast between sciences like ethics and even physics, which must concern themselves with particulars (καθ' ἕκαστα), and sciences like mathematics which are primarily concerned with universals (καθόλου). So the first kind of inquiry depends upon experience for its principles, whereas the second is relatively independent of experience.

According to Aristotle (*EN* 1142a18–19), the crucial difference between ethics and mathematics is that the former science gets its first principles from experience, whereas the latter obtains them through subtraction. As I have suggested already, subtraction is a logical method of identifying the primary subject of any given attribute and so it yields the immediate premises which are needed as first principles in any demonstrative science like mathematics. But this logical method also has epistemological implications because 'the things spoken about from subtraction' (τὰ ἐξ ἀφαιρέσεως λεγομένα) are more translucent to the intellect by virtue of being intelligible rather than sensible entities. Thus, as the example of the slave boy in the *Meno* dialogue illustrates so clearly, young men can learn mathematics because even an untutored intelligence can become acquainted with mathematical objects through diagrams. Yet, although he cannot deny this phenomenon, Aristotle refuses to adopt the epistemological theory which the illustration was designed to support. Neither does he give an alternative theory of 'abstraction' which, as the terminology suggests, would be designed primarily to explain how the intellect grasps mathematical objects. If he were to give such a privileged status to these objects, he would be reverting to the kind of Platonism in mathematics which he resists so strenuously in *Metaphysics* XIII & XIV.

Thus I think that Aristotle's account of how the soul grasps mathematical objects is consistent with his general epistemology which accepts only two ways of learning, namely, induction and deduction. Given the mode of being of quantity as an attribute of sensible substances, he should say that mathematical objects are originally grasped through sense-perception. In fact, he almost says as much in *Posterior Analytics* I.18 when he claims that it is impossible to grasp universals except through induction and mentions 'the things spoken about from subtraction' as examples of such universals. Yet in *Nicomachean Ethics* VI.8, he contrasts 'the things through subtraction' with the principles of ethics and physics that are obtained through experience. How are we to reconcile these apparently conflicting statements? Perhaps a lead may be found in the *Analytics* passage where he introduces mathematical entities with ἐπεὶ καὶ as if they would *not* normally be considered the results of induction and so he must argue for this with reference to their mode of being as non-separated aspects of sensible things.[147] I think that the reason he feels compelled to give an argument is that normally the Greek mathematician assumed the existence of such intelligible entities, so that their ultimate basis in sense experience is obscured by the method of hypothesis; cf. *APst.* I.10, 76a31 ff. According to Aristotle's epistemology, however, concrete universals like the line or the plane are ultimately grounded in sensation, just like the snub which is the paradigmatic object of physics.

As evidence for the ultimate dependence of mathematical universals upon sense experience, he would probably point to the fact that mathematicians have recourse to images even though they are conducting a 'pure' theoretical inquiry. This fact about mathematical practice was recognized even by Plato (*Rep.* 509–511) who conceded that, unlike dialecticians, mathematicians rely on images and begin with hypotheses. For him, however, this made mathematics a transitional discipline between the sensible and intelligible realms, since he understood its practitioners to be using images as likenesses of the real things which were the true objects of the science. By contrast, Aristotle denied that there exist independent entities such as appear to be posited by mathematics, and so he is forced to give an alter-

[147] Another way of resolving the apparent conflict might be the distinction between three kinds of induction outlined by von Fritz (1971), corresponding to physical, mathematical and metaphysical inquiries, respectively. For instance, in mathematics a single diagram can yield insight into the universal triangle, but this still involves induction though not experience; cf. Kal (1988).

native account of the mode of being of its intelligible objects. By making mathematical quantities depend upon sensible substance, therefore, he can construe the use of images by the scientist as reflecting the mode of being of his theoretical objects, since it indicates that the ultimate origins of his concepts lie in sense perception. As the passage in the *Ethics* shows, however, Aristotle also recognizes that mathematical concepts can be acquired without experience and, consequently, that mathematical inquiry can be conducted *as if* its theoretical objects were separate substances. Yet for him the logical method of subtraction underpins the psychological abstraction of mathematical objects, which is performed so easily by the intellect with the aid of diagrams.

Thus, Aristotle's explanation of how the mind grasps such objects is consistent with his general epistemology of two ways of learning, induction and deduction. But the concrete universals studied by mathematics are logically different from those studied by physics and this makes a great difference in learning the respective sciences. While concepts like snubness (which exemplifies the objects of physics) are logically tied to a particular kind of sensible matter and hence to sense experience, mathematical concepts like concavity are not tied to experience in the same way and, as Aristotle puts it, involve only intelligible matter. Another way of formulating this point with reference to the practice of the sciences is to say that mathematicians are concerned primarily with universals, so that particular images or diagrams function only to represent some universal characteristic. Since the human mind has a special affinity for such universals, young men can learn mathematics more easily than ethics but it is difficult for them to become practically wise because the universals in that sphere hold only for the most part and are gained from long experience. While the generalizations that apply to practical affairs are obscured by circumstances and are dependent on experience, mathematical universals are transparently intelligible and are not temporally indexed. Thus for Aristotle, just as for Plato, they are paradigmatic objects of knowledge.

In this final chapter I have raised doubts about whether Aristotle had a consistent philosophy of mathematics in any modern sense. Within the ancient problematic, however, I have concluded that it is possible to talk about his philosophical treatment of a cluster of problems surrounding mathematics and its characteristic objects. For instance, with respect to the epistemological problem of how the human

soul grasps mathematical objects, I reject the traditional view that Aristotle has some special theory of abstraction apart from induction and demonstration as modes of knowing. As a positive resolution, I argue that both mathematical and physical principles are grasped through induction, while conclusions in both sciences are established through demonstration. This means that the crucial differences between the objects of these sciences must be found in their logical and ontological characteristics. In this respect, I claim that Aristotle's method of subtraction is decisive because it enables him to determine the logical relationship between any given attribute and its primary subject. This is what lies behind his use of the snub and the concave as paradigmatic objects of physics and mathematics, respectively. But the logical relationship between these paradigmatic forms and particular kinds of matter also has ontological implications. For instance, it means for Aristotle that mathematical forms cannot be ontologically separate from the sensible world because the intelligible matter correlated with them is dependent on sensible things. This result is consistent with his rejection of the implications for cosmology of mathematical Platonism, and with his own notion of a hierarchical order culminating in a completely separate unmoved mover. So it represents the triumph of physics over mathematics as the central science for Aristotle's cosmology and metaphysics.

GENERAL CONCLUSION

My overall purpose in this book has been to analyse the complex role of mathematics in Aristotle's cosmological and metaphysical thought. On the historical level, I have argued that his opposition to Pythagorean cosmology in Plato's Academy lies behind his concern with the question about whether or not the mathematical sciences reveal the essential structure of the sensible universe. Consequently, Aristotle held mathematical objects to be ontologically posterior, in contrast to the prior mode of being attributed to them by Platonists and Pythagoreans. Far from being a special problem in the philosophy of mathematics, I have claimed that this has wider implications for Aristotle's teleological view of the universe. Furthermore, on the methodological level, I have shown that Aristotle has a characteristic dialectical way of treating all such problems in first philosophy and that following this procedure carefully enables us to understand the precise details of his solutions.

Throughout the book I have confined myself very much to the ancient context in discussing Aristotle's views, and this focus may seem excessively narrow and even pedantic, considering the lively debates about related issues in modern philosophy of science. In my introduction I have briefly indicated how contemporary discussions in the philosophy of mathematics may be misleading for any attempt to understand Aristotle's views on the foundations of mathematics. By way of conclusion, therefore, let me draw some comparisons between his views and some views espoused in contemporary debates, so as to get some sense of the similarities and differences. In the interests of brevity, I will focus upon the distinction between physics and mathematics, with reference to the question of the infinite.

Aristotle's rather sharp distinction between physics and mathematics runs counter to the modern scientific tradition stemming from Galileo which treats mathematical physics as the central discipline for understanding the physical world. In the 20th century this tradition has been carried on by scientists like Werner Heisenberg,[1] who was himself inspired to study physics on reading Plato's *Timaeus*. Despite the dependence of modern physics on mathematics, however,

[1] Cf. Heisenberg (1953 & 1959).

questions have been raised about the assumed coherence between an empirically-defined world and mathematical processes. By way of answer, some people claim that mathematics has only a descriptive function as a kind of language of the physical sciences, but this fails to explain the prescriptive use of mathematics in suggesting and verifying physical hypotheses.

On the other hand, the prescriptive view of mathematics tends to reduce it to the logic of the sciences, and such a tautological view overlooks the frequent use of complex numbers and of surds. Given their concern with consistency and epistemological grounding, for instance, the intuitionists reject the use of infinity in mathematics, though it seems to be necessary for the calculus. Such difficulties lead people like Carnap to take the pragmatic view that we use mathematical systems because they are effective, and that we should not endow them with any real ontological status. If the pragmatic view were correct, however, it would mean that we can adopt as many different mathematical systems as suit our purposes; whereas the tendency in modern physics is towards greater unification and simplification. For instance, the so-called Grand Unified Theory (GUT) appears to assume that there should be only one true mathematical system.

While Aristotle's finitism in mathematics seems to ally him with intuitionists like Brouwer, I think there are insuperable obstacles to such an alliance. In a well-known passage from the *Physics* (III.7, 207b28 ff.), Aristotle insists that his rejection of an actual infinite in the direction of increase does not hinder mathematical inquiry because mathematicians only need to extend a finite line as far as they wish. Leaving aside the possible conflict with Euclid's 5th postulate, we might take Aristotle to be making the plausible claim that geometers operate with finite constructions, which they can extrapolate at will to the very large and to the very small because of the indefinite divisibility of a homogeneous continuum. For the same reason, he claims that number is potentially but not actually infinite, since it is always possible to think of a greater number, just as it is possible to make another division of a continuous magnitude (207b10–15). Similarly, time as the measure of motion is potentially but not actually infinite, because the measured motion is primarily that of the heavenly bodies. Although there is little doubt about the mind-independent existence of such motion, Aristotle raises the question (IV.14, 223a22 ff.) of whether or not time would exist if no soul existed to do the

numbering. It seems that here he comes closest to the point of view of modern intuitionists who make the existence of mathematical objects depend on some construction or finite procedure like counting.

But a closer look at the assumptions of intuitionism, as these are elucidated by Michael Dummett (1983), undermines the apparent parallel with Aristotle's views. The basic standpoint of modern intuitionists is that classical mathematics employs forms of reasoning which are not valid on any legitimate way of construing mathematical statements. Thus they tend to assume that the meaning of a mathematical statement is exhaustively determined by use rather than by reference to some independently existing set of objects. In contrast, a platonistic interpretation of a mathematical statement takes the central notion to be that of truth as correspondence with such objects. Given the theory of meaning which underlies platonism, our grasp of the meaning of a mathematical sentence consists in our knowledge of the conditions required for it to be true, although we cannot in general recognize when such conditions obtain. For instance, when the sentence is effectively undecidable, as in the case of the infinite, the conditions which must obtain for it to be true are not accessible to us. So the meaning of such a mathematical sentence can only be grasped by implicit knowledge that is not exhausted by the capacity to state what is known.

However, on the assumption that use exhaustively determines meaning, the ascription of implicit knowledge is meaningful only when someone can fully manifest that knowledge in suitable circumstances. Thus the platonistic theory of meaning cannot be one in which meaning is fully determined by use. But if to know the meaning of a mathematical statement is to grasp its use then proof rather than truth becomes central for a theory of meaning of mathematical statements. In other words, grasping the meaning of a statement consists in a capacity to recognize a proof of it when it is presented. In the case of statements about the infinite, however, such a proof will always consist in finite procedures which undermine the truth of any claims about an actual infinite. So modern intuitionists talk about a potential infinite, though it is not clear that this means the same as what it means for Aristotle, whose realist assumptions seem to put him in the platonist camp.

We can see this clearly if we consider the intuitionistic thesis that mathematical statements do not relate to an objective mathematical reality existing independently of us. In other words, mathematical

objects are creations of the human mind whose mode of being depends on being thought. Such a thesis is quite compatible with a realist view concerning the physical universe and statements about its physical properties. But, with some notable exceptions, Aristotle treats mathematical and physical statements in a similar realist manner by assuming that they correspond with objective features of the physical world. A conception of meaning as determined by truth-conditions is available for any statements which relate to an independently existing reality, since we can assume that each statement has a determinate truth-value. But when the statements do not relate to such an external reality, the supposition that each of them has a determinate truth-value is empty, and so their meaning cannot be given by their truth-conditions.

According to Dummett (1983, 110), the difficulty in modern philosophy of mathematics concerns how we are to decide whether mathematical objects are the creations of human thought before deciding what is the correct model for the meanings of mathematical statements. One is faced with the metaphysical question of whether mathematical objects are human creations (constructivism) or whether they are independently existing abstract objects (platonism). The platonist picture likens mathematical inquiry to astronomy, which investigates mathematical structures just like galaxies that exist in another realm which we do not inhabit but which we can observe. Indeed, this picture is very apt for both Plato and Aristotle who regarded astronomy as one of the most important mathematical sciences precisely because it investigates the most exalted substances in the universe. By contrast, the constructivist picture likens mathematical activity to that of an artificer fashioning objects through his imagination. While some scholars are inclined to interpret Aristotle's theory of abstraction in terms of such a picture, I have argued that this theory is based on realist assumptions about the conformity of the intellect with objective features of the sensible world. Therefore, in terms of the modern distinction, his view of mathematical objects is more platonistic than constructivist, although such a simple dichotomy does scant justice to his complex position.

But, if Frege is right, we cannot refer to any object except by saying something about it. Hence any thesis about the ontological status of certain kinds of objects must also involve a thesis about the truth of statements referring to such objects. In other words, we cannot separate the question of the ontological status of a class of objects

from the question of the correct notion of truth for statements about those objects. As we have seen, the correspondence theory of truth tends to support platonism, while intuitionism relies on the notion of proof and its associated truth-conditions. But substantial disputes between these two models of meaning only emerge over statements that are effectively undecidable; for instance, those involving quantification over infinite totalities like the natural numbers. Whereas platonists are willing to accept such statements as true or false, intuitionists deny that the law of the excluded middle applies to incompletable procedures. It is possible for someone to accept a platonistic view of the existence of mathematical objects, yet to reject a similar view of the objectivity of mathematical statements. Thus he may conclude that quantification over a denumerable totality cannot be construed in terms of our grasp of the conditions under which a quantified statement is true but rather in terms of our ability to recognize a proof or disproof of such a statement. Such a person will reject a classical logic for number-theoretic statements in general, and admit only intuitionistically valid arguments for them. Here claims for knowledge of number are given priority over claims to truth.

This tension between truth conditions and knowledge in contemporary philosophy of mathematics signals a more fundamental problem about the existence of mathematical objects like transfinite sets. According to platonists like Frege who tried to reduce arithmetic to logic, the existence of an actual totality of integers is a necessary assumption for the logical principle of the excluded middle to be applicable to number theory. Similarly, the impredicative definition of a real number depends on assuming that the totality of integers exists, since a real number is a Dedekind cut involving an infinite series of rational numbers. But such assumptions have been undermined by the notorious paradoxes of set theory and by the vicious circularity associated with impredicative definition.

In order to avoid the snares of extreme platonism, Carnap[2] applies the rule that in mathematics only what has been proved in finitely many steps may be taken to exist. This brings him some way towards the intuitionist position that every logical-mathematical operation or proof or definition must be finite. But intuitionists like Heyting[3] go much further by claiming that the existence of mathematical objects

[2] Cf. Carnap (1983a).
[3] Cf. Heyting (1983).

and their properties is guaranteed only insofar as they are determined by human thought. Against such anthropological mathematics, however, the formalists[4] object that the human mind does not have exact images of straight lines or of very large numbers, so that these entities cannot depend on thought for their existence. Of course, they concede that the mathematician can methodically develop a series of relations for these entities, yet they insist that these relations are independent of the mental significance we attach to them. Such a meaningless series of relations gains mathematical existence only when it has been represented in spoken or written language, together with the mathematical laws upon which its development depends. David Hilbert insists that the infinite as a completed totality is an illusion that must be replaced by finite procedures that produce the same results in mathematical deductions. He points out that the infinite divisibility of the continuum, for instance, is an operation which exists only in thought, since modern physics shows that nature does not provide a suitably homogeneous continuum. Similarly, even though Euclidean geometry assumes that space is infinite, it remains an open question whether physical space is Euclidean or elliptical.

A brief review of conflicting views on the question of mathematical existence shows how difficult it is even to find common ground for an answer. Thus empiricists in the tradition of Locke and Hume tend to reject abstract entities, while giving a nominalist account of language that purportedly refers to them. For instance, in the case of a mathematician who talks about infinite numbers and sets, they would insist that he is talking about meaningless symbols which are manipulated according to formal rules. The old problem about abstract entities has been given new currency by fresh developments in semantic theories of meaning and truth. While some semanticists hold that scientific expressions designate not only concrete material things but also abstract entities, others object that such claims go beyond empirical science into the realm of Platonic metaphysics. From a philosophical perspective, however, it is difficult to mediate between metaphysical views that differ so radically as the empirical assumption that only sensible physical things exist, and the platonist view that there are abstract entities.

One modern way of eliminating the problem is through Carnap's[5]

[4] Cf. von Neumann (1983) & Hilbert (1983).
[5] Cf. Carnap (1983b).

distinction between internal and external questions about existence. The former are questions about the existence of new kinds of entities within a given framework, which may be routinely answered either by logical or empirical methods, depending on the character of the framework. For example, the question of whether there is a prime number greater than 100 can be answered through logical analysis based on the rules of the number system. In contrast, the question of whether or not numbers exist is problematic for Carnap because it is unclear whether it is an internal or an external question. It can hardly be an internal question, however, since that would amount to nothing more than whether or not the framework of numbers is empty of content. So he infers that it must be taken as an external question about the existence of the system of entities as a whole; i.e. the question of whether or not numbers have a metaphysical character called 'reality' or 'subsistence.' Whichever way one formulates the question, however, Carnap finds it to be meaningless because it is not formulated in the common scientific language and so lacks cognitive content. Therefore, he brands it a pseudo-question that is masquerading as a theoretical one, since it boils down to the practical problem of whether to accept the new linguistic forms that constitute the framework of numbers.

I have drawn attention to Carnap's argument as a typical example of the radical challenge to traditional metaphysics that was thrown down by the logical positivists. Despite their fastidiousness about metaphysical excess, however, they must give some explanation of the reference to abstract entities to be found in the language of mathematics and physics, which they place on the same methodological level. Thus, even hard-bitten reductionists like Frege and Russell felt compelled to accept the extralinguistic existence of abstract entities such as numbers and sets. But Carnap accuses these philosophers of making the same old metaphysical mistake of treating the acceptance of a system of entities as a theoretical question, when it is actually an external and practical question. Yet he does not provide any satisfactory method of deciding such practical questions, especially where the truth of statements about abstract entities is concerned. Presumably Carnap would reject any type of correspondence theory of truth in favor of some instrumentalist or conventionalist theory, yet he is quite vague on the whole issue.[6]

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[6] Carnap (1983b) 250 says that the acceptance of a new linguistic form cannot be

However, despite his positivist background, Karl Popper[7] concedes that foundational questions about the sciences involve a metaphysical dimension when he posits a so-called 'world 3,' consisting of objective contents of knowledge. Although he denies that this is the same as Plato's theory of Forms, he does seem committed to some kind of platonism when he likens his world 3 to Frege's objective content of thought. In some ways, Popper's approach to the physical sciences is similar to that of Frege to the mathematical sciences. For instance, just as Frege criticises abstractionist accounts which appealed to psychological states, so also Popper resists the subjective psychologism in epistemology which was typical of post-Cartesian 'belief-philosophers.' He does this by making a distinction between knowledge or thought in the subjective sense (world 2) and in an objective sense (world 3). While he regards the first sense of knowledge as being largely irrelevant for a theory of knowledge, Popper claims that every epistemology must concern itself with the second sense of knowledge 'without a knowing subject.' In this respect, therefore, Popper is justified in calling his position 'realism' in the classical Greek tradition of Plato and Aristotle.

By contrast, Brouwer's intuitionism represents an ingenious solution to a problem inherited from Kant, who minimized the role of discursive arguments in mathematics because he thought that the basic axioms and every step of a proof were grounded in pure intuition. Brouwer's solution is to distinguish between mathematical activity as mental construction based on the pure intuition of time, and its linguistic expression or communication. Thus he resolves the epistemological problem about the source of mathematical certainty by appealing to intuition, while explaining the nature of mathematical proof as a construction of constructions, each of which is ultimately based on intuition. Similarly, ontological problems about the nature of mathematical objects and their mode of existence are resolved by Brouwer's two-pronged doctrine of constructivism and mentalism. In other words, he holds mathematical objects to be nothing more than constructions of the human mind, whose objectivity depends entirely on repeating their construction. By contrast with formalists like Hilbert

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judged as being either true or false, since it is not an assertion, but only as being more less expedient or fruitful. Against Quine's notion of ontological commitment, Carnap insists that the acceptance of a linguistic framework must not be taken to imply any metaphysical doctrine concerning the reality of the entities in question.

[7] K.R. Popper (1972, Ch. 3–4).

for whom exact mathematical objects exist only on paper, the intuitionists locate such objects in the human intellect.

Finally, with respect to methodological problems, there is an obvious parallel with the ancient Academic debate as to whether mathematical proofs are theorems or problems. If a mathematician is mainly interested in theorems (i.e. in the truth or falsity of mathematical propositions), he is presupposing the existence of mathematical facts that are waiting to be discovered. By contrast, the mathematician who concentrates on proofs tends to regard them as the only mathematical objects. Thus, for instance, Brouwer regards the constructions which constitute proofs as not only creating and establishing mathematical objects but as being themselves mathematical objects. For this reason he claims that mathematical existence depends on actual construction which brings to light, as it were, the mathematical 'object' in question. In this context, the object terminology may even be misleading, since Brouwer is consciously anti-platonist in denying mathematical objects any mode of existence independent of our constructive activity. Hence it is clear that, despite some superficial resemblances, Aristotle's account of mathematical objects in terms of abstraction is not the same as modern intuitionism, since he acknowledges that such objects exist independently of the mind.

What should be clear from this brief and superficial survey of some issues in contemporary philosophy of mathematics is that there is more discontinuity than continuity with the ancient problematic that I have been discussing throughout my book. On the face of it, the ontological and epistemological problems connected with mathematical objects look very similar but, at a deeper level, the post-Cartesian assumptions about the primacy of the knowing subject which govern modern philosophy dictate that epistemology takes precedence over ontology. With regard to the infinite, for instance, the modern approach is to ask whether a finite human mind can prove that such a thing exists; whereas Aristotle insists that its existence does not depend on human thinking. Instead for him the question is whether the universe is such as to contain the infinite under any one of its guises, and his answer is a qualified one, as we might expect. He denies that there is an actually infinite body, since he argues on many grounds that the universe must be finite in extension. However, he does accept that there is a potential infinite by division because he thinks that the finite body of the universe is an indefinitely divisible continuum. Furthermore, the series of numbers is also held to be

potentially infinite, since one can always add another one to the series, just as one can always make another division in any continuum. The reasons by which Aristotle justifies his complex position on the infinite show that the cosmological dimension of this thinking is one of the essential differences between it and modern approaches to similar problems in the philosophy of mathematics.

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